

# A Simple Interpretation of the Repeating WPT Characteristics

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**Abstract** Nowadays, the study of extending transmitting distance of wireless power transmission is drawing attention. Using a repeating coil is regarded as one of the most effective solutions improving the magnetic resonance wireless power transmission distance. Many researchers are studying the design of repeating coils in order to improve transmission distance. However, the mathematical estimation of wireless power transmission efficiency using repeating coils is difficult due to the various arrangements of the repeating coils. Thus, we will give a simplified analysis to help in estimating the efficiency. Our analysis is based on the simple equivalent circuit of wireless power transmission and shows the importance of the dense arrangement in interval distances between the repeating coils for improving transmission efficiency.

**Keyword** WPT, Repeater, Resonance, Magnetic Coupling

## 1. BACKGROUND

We are studying HEMS (Home energy management System). We are developing novel approach for HEMS which consists of sensor network nodes. All sensor network nodes needs continues energy for work [1]. We are considering that the energy should be supplied by wireless power transmission. Then we are studying wireless power transmission, especially magnetic coupling resonance power transmission because our system needs relatively high power and farther transmission [2]. The introduction of repeating coils for magnetic coupled wireless power transmission is useful method. So many researcher are proposing various method for the optimization of farther transmitting characteristics. Almost all of the analysis are the description on complex situation [3], [4]. For example that is related about multi pass transmission by switching and so on. In this paper we will consider the way of the simple outlook of the relayed wireless power transmission by using simple model.

## 2. EXPERIMENT ON REPEAT TRANSMISSION

At first, our question has begun through the power transmission measurement when we introduce the repeating coils. Our measurement view is shown in Fig.1. We have calculated the transmitting efficiency by measuring  $S_{21}$  with Vector Network Analyzer in the case of the containing both one repeating coil and no repeating coils. We have believed the transmitting efficiency with one repeating coil is higher than the efficiency without repeating coil in all over distance. But the result is contrary with our expectation. There is the exchanging point where the higher efficiency exchanges for each situation (Fig.2).

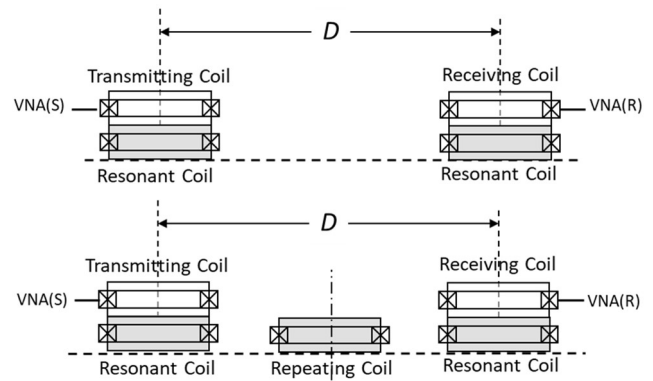


Fig.1. Transmitting Efficiency Measurement View (Upper: Non Repeating, Lower: One Repeating)

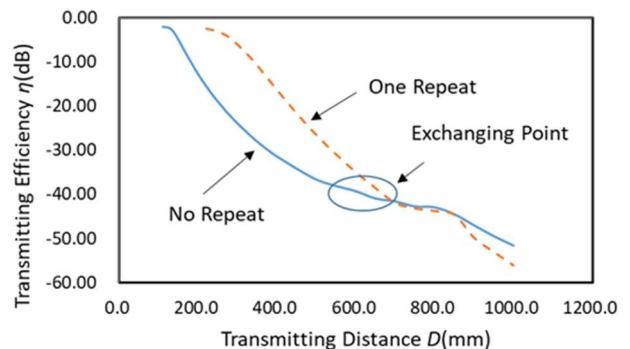


Fig.2. Transmitting Efficiency with and without Repeater

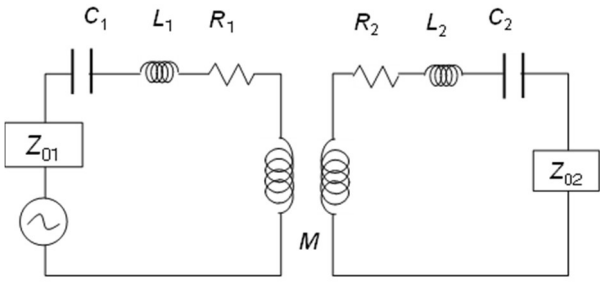
We consider this result shows that there are no advantage domain in farther transmission even by introducing repeating coils. We can understand the advantage by introducing repeat coils is limited in the

restricted distance.

So we wonder how to know the limit distance, and we went back to the fundamental equivalent circuit for the wireless power transmission.

**3. CONCEPT FOR INTERPRETATION**

The fundamental equivalent circuit is shown in Fig.3



$Z_{01}$  Source Impedance  $Z_{02}$  Load Impedance  
 $L_1, L_2$  = Resonant Coil Inductance  
 $C_1, C_2$  = Added Capacitance  
 $R_1, R_2$  = Resonant Coil Resistance  
 $M$  Mutual Inductance

Fig.3. Fundamental Equivalent Circuit for WPT

Under resonant frequency condition, solving this equivalent circuit, we can derive the following simple equation (1) of the efficiency  $\eta$ .

$$\eta = \frac{(kQ)^2}{(1 + \sqrt{1 + (kQ)^2})^2} \tag{1}$$

Here  $k$  is the coupling coefficient and  $Q$  is the external  $Q$  of the resonant coil.

In considering this phenomena, we have to take care about the following two points, one is that the efficiency decreases with the power of distance, and the other is that the efficiency for the repeating decreases with the product of the number of repeating. Now we introduce the following model to calculate the transmitting characteristics (Fig.4).

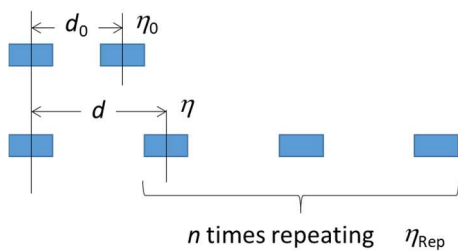


Fig.4. Calculation Model for Considering Repeating Transmission Characteristics

The efficiency is denoted as  $\eta_0$  at the unit distance  $d_0$ ,

the efficiency is denoted as  $\eta$  at the distance  $d$ , and the efficiency is denoted as  $\eta_{Rep}$  at the end point of  $n$  times repeating. The focusing transmitting distance range is most important in the consideration of this problem. In our HEMS model, the focusing transmitting distance is from 0.5[m] to 1[m]. In such condition, using one turn square coil with a side length is 100[mm], coupling coefficient  $k$  is assumed to be inversely proportional to the cube of the distance  $d$  in eq.(2) then  $kQ$  product changes from 8 to 1. So the transmitting efficiency  $\eta$  is proportional to  $kQ$  product shown in Fig.5, after all, the transmitting efficiency  $\eta$  is inversely proportional to the cube of  $d$  in eq.(3). The repeating total efficiency is summarized in eq.(4) with  $n$  times repeating.

$$k \propto d^{-3} \text{ (for example)} \tag{2}$$

$$\eta \propto d^{-3} \text{ (} 1 < kQ < 10 \text{)} \tag{3}$$

$$\eta_{Rep} = \left\{ \eta_0 \left( \frac{d}{d_0} \right)^{-3} \right\}^{n+1} \tag{4}$$

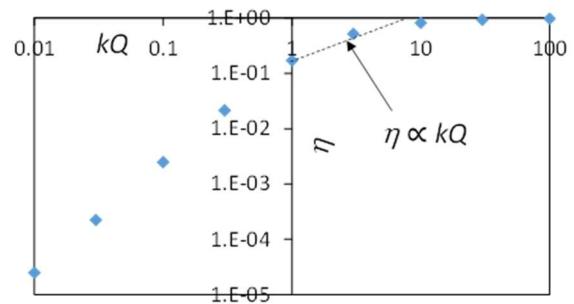


Fig.5. Relationship between  $\eta$  and  $kQ$  product

**4. VALIDITY FOR DENSE ARRANGEMENT**

The calculation is performed with eq.(3) and eq.(4) about various arrangement array, rough, mid, and dense in the case of  $D_N=1$  shown in Fig.6.

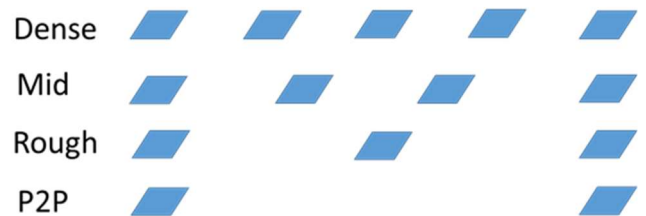


Fig.6. Calculation Model for Various Arrangement Of the Repeating Coils ( $D_N=1$ )

Here we use the following value  $\eta_0$  is 0.9 at  $d_0=1$  (normalized unit distance). The results according to the extended distance by increasing the number of normalized distance  $D_N$  are shown in Fig.7.

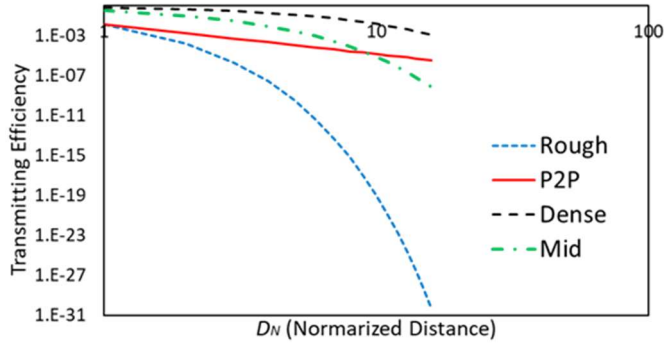


Fig.7. Transmitting efficiency calculation for various arrangement of repeating coils.

As the result, we can confirm the following points in Fig.7.

- (1) Higher efficiency exchanging point exists in our calculation same as the experimented data shown in Fig.2.
- (2) In higher density arrangement of repeating coils higher transmitting efficiency can be maintained in the extent of farther transmitting distance.

We have experimented for verifying the relevance of our interpretation. The measurements was performed for the dense packing transmitting characteristics as three pattern, no repeater, one repeater, two repeater in Fig.8.

We can examine the fact that the higher density arrangement can maintain higher transmitting efficiency even at farther transmission distance by this measurement as showing in Fig.9.

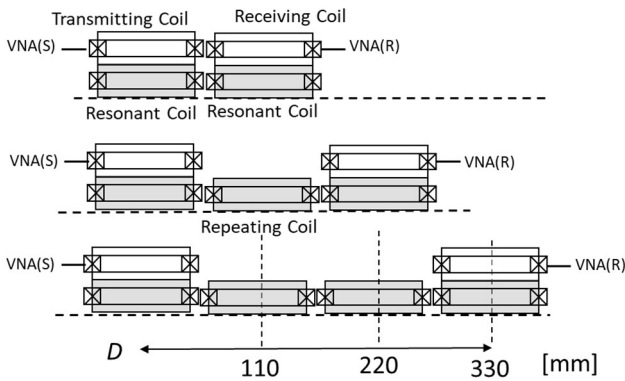


Fig.8. Dense Repeater Arrangement Model.

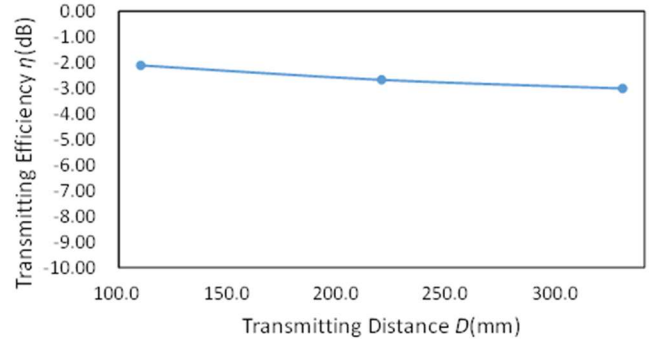


Fig.9. Measured Results for Three Distance Model.

### 5. NUMERICAL EXAMINATION

We have tried to examine the numerical validity of the distance at exchanging point in Fig.2. In Fig.2 the distance at exchanging point can be read around 600[mm] and the transmitting efficiency can be read as -20dB around 300[mm] as a half distance of the exchanging point. The situation is described in Fig.10.

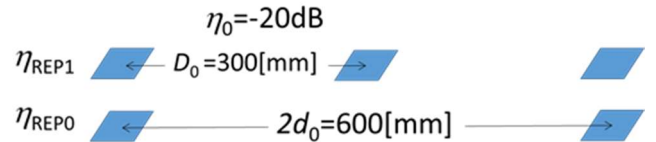


Fig.10. The Experimental Condition around Exchanging Point in the Fig.2.

Fig.11 is logarithmic conversion in the horizontal axis of Fig.2. The transmitting efficiency with no repeating coil in Fig.11 is inversely proportional to the sixth power of the transmitting distance.

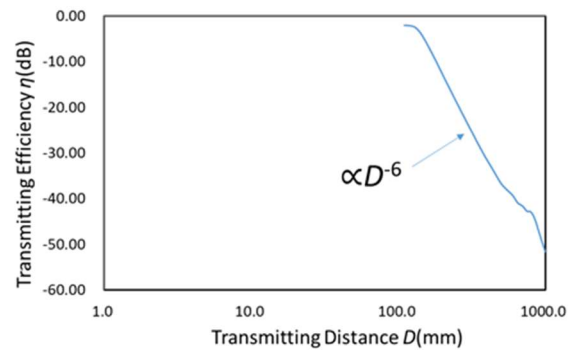


Fig.11. No Repeating Transmitting Efficiency Expressed in Logarithmic Scale on Transmitting Distance.

Considering eq. (4), the transmitting efficiency of no repeating coils and one repeating coil is shown in eq.(5), eq.(6) using  $\eta_0$ . Here  $\gamma$  is the power decay with

respect to the transmitting distance.

$$\eta_{REPO} = \eta_0 2^\gamma \tag{5}$$

$$\eta_{REP1} = \eta_0^2 \tag{6}$$

The relationship between  $\eta_0$  and  $\gamma$  at the exchanging point is derived in Eq.(7) from eq.(5) and eq.(6).

$$\eta_0 = 2^\gamma \tag{7}$$

Applying  $\eta_0=0.01$  ( $\eta_0=-20$ dB) in eq.(7),  $\gamma$  is derived as  $-6.67$ . This value ( $\gamma = -6.67$ ) is roughly matched with the experimental value ( $\gamma = -6$ ) in Fig.11. This result says we only measure the coefficient  $\gamma$  as power decay with respect to the transmitting distance, we can give the  $\eta_0$  and finally get the value of the transmitting efficiency and the exchanging point roughly in the case of many times repeating. Moreover we can know the exchanging point by the relationship in eq.(8) and eq.(9) using the following diagram in Fig.12. Here  $\eta_{pp}$  is the point to point transmitting efficiency and  $\eta_n$  is the  $n$  times repeater transmitting efficiency.

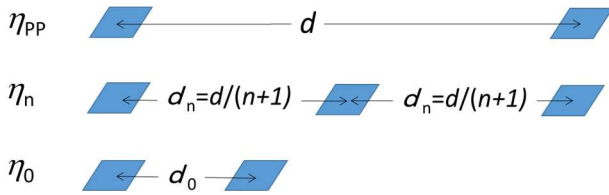


Fig.12. Diagram for the Calculation of the Transmitting Efficiency for  $\eta_{pp}$  and  $\eta_n$ .

$$\eta_{PP} = \eta_0 \left(\frac{d}{d_0}\right)^\gamma \tag{8}$$

$$\eta_n = \left\{ \eta_0 \left(\frac{d}{(n+1)d_0}\right)^\gamma \right\}^{n+1} \tag{9}$$

Using the relationship that  $\eta_{pp}$  is equal to  $\eta_n$ , eq.(10) is obtained.

$$d = d_0 \eta_0^{-\frac{1}{\gamma}} (n+1)^{n+1} \tag{10}$$

By substituting the parameter  $d_0=300$ [mm],  $\eta_0=0.01$ ,  $n=1$ ,  $\gamma = -6.0$  into eq.(10),  $d$  is derived as  $557$ [mm]. That value is a just smaller comparing to the experimental value around  $600$ [mm]. The reason is considered to neglect the multi path power transmission. Although we can get the exact value with our approach, we can get the approximate value using easy calculation based on physical theory. This method is very useful to know the rough estimation

for establishing the transmitting line with the repeaters. And it is thought that this theory can be applied to a two dimensional array by drawing a point and a point.

In addition to that, eq.(10) shows that the exchanging point is farther with smaller  $\gamma$  and smaller  $d_0$ , indicating the strong advantage of dense array arrangement under the condition of low loss interval distance ( $kQ$  product is large).

## 6. CONCLUSION

In our analysis, we derived the equations of the estimation in the wireless power transmitting efficiency when using the repeating coils array arrangement. Using this theory, it is possible to roughly estimate the total transmission efficiency without making a difficult numerical calculation. This can be done, in the case of multiple repetitions, by only measuring the power decay of transmitting efficiency with respect to the transmitting distance between only one pair of coils. Furthermore the calculated and experimented results show the fact that the dense coils array arrangement can maintain much higher transmitting efficiency comparing to the rough coils array arrangement.

## References

- [1] Takashi Yoshikawa, Ikuo Awai, "A Novel Design for HEMS consisting of Sensor Network Nodes with Energy Harvesting and Wireless Power Transmission", AIEM(Advances in Industrial Engineering and Management), Vol.2, No.1, PP.11-15, 2012.
- [2] Takashi Yoshikawa, Shota Saraya, "HEMS Assisted by a Sensor Network Having an Efficient Wireless Power Supply", IEEE Trans. on Magnetics, Vol.49, Issue 3, PP.974-977, 2012.
- [3] Y. Naruse, et al, "Impedance Matching Method for Any-Hop Straight Wireless Power Transmission Using Magnetic Resonance", in Proc. IEEE RWS, Jan. 2013, pp.20-23.
- [4] T. Imura, "Equivalent Circuit for Repeater Antenna for Wireless Power Transfer via Magnetic Resonant Coupling Considering Signed Coupling ", in Proc. ICIEA, 2011 pp.1501-1506.