Chaos Synchronization in Coupled Delayed Two-stage Colpitts Circuits for UWB Communications

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Abstract—The paper presents chaos synchronization in coupled delayed two-stage Colpitts circuits for the first time. Numerical simulation demonstrates and verifies the effectiveness of the proposed model. Furthermore, the spectrum analysis shows the potential utilization for UWB communications.

1. Introduction

The Colpitts oscillator has been attractive for many years due to the application in telecommunications [1,2] as a source of oscillation. Chaotic Colpitts oscillators have received increasing attention because they can generate chaotic signal at the frequency range of kilohertz [3,4], UHF [5,6], VHF [5], and very recently of UWB communications with the range from 3.1GHz – 10.6GHz [7]. However, Colpitts circuit is not easy to generate chaotic signals at the frequency range of 3.1GHz to 10.6 Ghz. With inclusion of time delay, the system is much easier in generation of chaotic behavior in compare with conventional ones [8]. Moreover, due to the complexity in dynamics, delay systems have a potential application in secure communications.

So far, there are several types of synchronization in coupled systems. Synchronization represents correlations between the master and slave systems i.e. schemes of complete [9,10], generalized [11], anticipating [10,12], lag [10,13], and phase synchronizations [13,14]. In oscillation circuits, synchronization is to adjust frequencies of weakly interacting periodic oscillators at the side of slave [2,3].

In this paper, for the first time we investigate a delay two-stage Colpitts oscillator by means of incorporating delay to conventional one. Also, synchronization in coupled delay two-stage Colpitts circuits is studied and hardware design for the system is presented. Simulation result shows the evidence of chaotic behavior and synchronization in frequency range of UWB (3.1GHz – 10.6GHz) communications.

2. Circuit Design

Fig. 1 depicts the conventional two-stage Colpitts oscillator [6]. The circuit includes one conductor L, one resistor R, three capacitors C1, C2, C3 and two transistors, Q1, Q2. In comparison with the classical single-transistor Colpitts oscillator [15,16], the two-stage modification includes an extra transistor Q2, and a capacitor C3 to obtain oscillation at higher fundamental frequency, up to $f = \frac{1}{2\pi}\sqrt{\frac{C_2C_3}{L(C_2+C_3)}}$ (1).

The fundamental frequency $f$ of a two-stage Colpitts oscillator can be estimated as

$$f = \frac{1}{2\pi}\sqrt{\frac{C_2C_3}{L(C_2+C_3)}} - \frac{R^2}{\pi^2}$$

The circuit schematic for a delay two-stage system is as shown in Fig. 2. It consists of three bipolar junction transistors (BFG 425W, Phillip Co. Ltd.), cut-off frequency of 25GHz) which is biased in the active region by means of Vee, Re and Vcc. The Q1 and Q2 compose for the intrinsic two-stage Colpitts oscillator while Q3 is an emitter follower inserted to buffer the influence of the measuring devices. Other components require an inductor L with series resistance R, and three capacitors C1, C2 and C3. The bias emitter current I0 can be manually changed by varying the voltage source Vee. A delay line consists of 20 pairs of LCs, and the time delay can be estimated by $\tau_d = n\sqrt{L/C}$ seconds. With the presence of delay-line, chaotic behavior is more easily generated. The delayed signal is feedbacked to the collector of Q1.
In Fig. 3, Master and Slave are coupled each other through the resistor \( R_c \) for impedance matching. Master’s and Slave’s circuits are identical as depicted in Fig. 2. The output signal from Master is fed to the emitter of Q2 at the Slave side. Full synchronization is achieved with \( x_m(t) \approx x_s(t) \).

The block diagram for synchronization using two Colpitts oscillators is as in Fig. 3. One Colpitts oscillator plays a role of Master and the other does as Slave. We will denote subscript “m” and “s” for the Master and Slave, respectively.

### 3. Synchronization model

As depicted in Fig. 2, the time delay signal \( V_{C1} \) is feedbacked to the collector of Q1, so state equations for Master can be expressed as

\[
\begin{align*}
C_{1,m} \frac{dV_{C1,m}}{dt} & = I_{L,m} - I_{EQ1,m}(r,V_{C2,m},V_{C3,m}) + \frac{1}{R_{d,m}} V_{C1,m} \\
L_m \frac{dI_{L,m}}{dt} & = V_{C1,m} - V_{C2,m} - V_{C3,m} - RI_{L,m} \\
C_{3,m} \frac{dV_{C3,m}}{dt} & = I_{L,m} - I_{EQ2,m}(r,V_{C2,m}) \\
C_{2,m} \frac{dV_{C2,m}}{dt} & = I_{L,m} - I_{b,m}
\end{align*}
\]

It is clear to observe from Fig. 3 that the signal from Master is connected to the emitter of Q2 of Slave’s circuit, so we can describe state equations of Slave as

\[
\begin{align*}
C_{1,s} \frac{dV_{C1,s}}{dt} & = I_{L,s} - I_{EQ1,s}(r,V_{C2,s},V_{C3,s}) + \frac{1}{R_{d,s}} V_{C1,s} \\
L_s \frac{dI_{L,s}}{dt} & = V_{C1,s} - V_{C2,s} - V_{C3,s} - RI_{L,s} \\
C_{3,s} \frac{dV_{C3,s}}{dt} & = I_{L,s} - I_{EQ2,s}(r,V_{C2,s}) \\
C_{2,s} \frac{dV_{C2,s}}{dt} & = I_{L,s} - I_{b,s} + \frac{1}{R_C} (V_{C2,s} - V_{C3,s})
\end{align*}
\]

Assumed that the forward current gain of transistors (denoted as \( \alpha \)) is equal to 1 as given in Eqs, (2) and (3). In the forward active mode, the intrinsic resistance (denoted as \( r \)) is the differential one of the base-emitter junction and the break-point voltage (denoted as \( U_T \)) is around 0.7 voltage. Above expressions can be converted to dimensionless by using following assumptions

\[
\begin{align*}
\tau = \frac{t}{\sqrt{L_1C_1}} , \tau_d = \frac{t_d}{\sqrt{L_2C_3}} , \tau_n = n\sqrt{L_4C_d} \\
a = \frac{\rho R}{r} , b = \frac{R}{\rho} , \epsilon = \frac{C_{2,s}}{C_1}, \alpha = \frac{1}{R_C}, \omega = \frac{1}{R_d} \\
F_1(a, z, v) = \begin{cases} 1 - a(z + v), & a(z + v) < 1 \\ 0, & a(z + v) \geq 1 \end{cases} \\
F_2(a, z) = \begin{cases} 1 - az, & az < 1 \\ 0, & az \geq 1 \end{cases}
\end{align*}
\]
From Eqs. (4), (5) and (6), Eq. (2) for Master can be represented in the form of dimensionless as

\[
\begin{align*}
    x_m &= y_m - F_{1,m}(a, z, v) + dx_{m,r} \\
    y_m &= -x_m - z_m - v_m - by_m \\
    \varepsilon_{1,m} v_m &= y_m - F_{2,m}(a, z) \\
    \varepsilon_{2,m} z_m &= y_m - 1
\end{align*}
\]

and Slave from Eqn. (3) becomes

\[
\begin{align*}
    x_s &= y_s - F_{1,s}(a, z, v) + dx_{s,r} \\
    y_s &= -x_s - z_s - v_s - by_s \\
    \varepsilon_{1,s} v_s &= y_s - F_{2,s}(a, z) \\
    \varepsilon_{2,s} z_s &= y_s - 1 + c(z_m - z_s)
\end{align*}
\]

4. Simulation result

The numerical simulation is carried out for Eqs. (7) and (8) with \(a = 20, \ b = 0.8, \ c = 0.02, \ d = 0.02, \ \varepsilon_{2,3} = 1\). The result for \(-y\) versus \(x+y+z\) shown in Fig. 4 presents the strange attractor or chaotic behavior is exhibited. Simulation result for the circuit in Figs. 2 and 3 using Advanced Design System 2008 Update 2 (ADS) is carried out with the value of parameters chosen as \(R_0 = 100\, \Omega, \ R_1 = 510\, \Omega, \ R_2 = 3k\, \Omega, \ R_3 = 5.1k\, \Omega, \ R_4 = 3k\, \Omega, \ R_5 = 200\, \Omega, \ R_e = 1.5k\, \Omega, \ C_0 = 47nF, \ C_4 = 1pF, \ C_5 = 270pF, \) (other parameters of the tank elements, namely \(R, L, \) and \(C\) depend on the chosen fundamental frequency \(f\)). In order to achieve the frequency band of UWB communication (3.1 GHz - 10.6 GHz), the microwave transistor BFG425W (Phillip Co. Ltd) is chosen. The value for the delay line consisting of 20 pairs of LC circuit is
adopted as $L = 1\text{nH}$, $C = 1\text{pF}$, and $R_d = 50\text{Ω}$. The specific value of the supply voltages $V_{cc}$ and $V_{ee}$ is adjusted to achieve the desired chaotic performance of the oscillator. Based on the previous result of numerical integration for the dimensionless system as given above, we choose $R = 1\text{Ω}$, $L = 1\text{nH}$, $C_1 = C_2 = C_3 = 5\text{pF}$, $V_{cc} = 12\text{V}$, $V_{ee} = 30\text{V}$, and $R_c = 50\text{Ω}$. Hence, the fundamental frequency is $f \approx 3.1\text{GHz}$.

It is observed from Figs. 5 (a) and (b) that random-like, non-periodic signals generated by Master and Slave. This is chaotic signal [17]. The power spectrum in Fig. 5 (c) and (d) is obtained by analyzing the signal $V_m$ and $V_s$. In the range of the frequency beyond $f \approx 3.1\text{GHz}$, the power average is less than -41 dB. This is very suitable for the application of UWB communications. The synchronization in Master and Slave can be seen in Fig. 6 is the amplitude relation between output signals of Master and Slave; $V_s$ versus $V_m$. In addition, synchronization error fluctuated around zero as displayed in Fig. 7. It means that the asymptotical synchronization is obtained.

![Figure 6. Simulation results of synchronization between Master and Slave](image1)

![Figure 7. Synchronization errors between Master and Slave](image2)

5. Conclusion

In this paper, we have demonstrated the chaos synchronization in the coupled time delay two-stage Colpitts circuits. The analysis on power spectrum has shown that time delay Colpitts circuit generates chaotic signals in the frequency range from 3.1 Ghz to 10.6 Ghz and the power is less than -41 dB. In summary, the synchronization in time delay Colpitts circuits can be utilized in UWB communications.

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References