A Study on Adaptive Notch Filter Using Fourier Sine Series Expansion

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1. A brief introduction

Adaptive IIR notch filter has been widely used in various applications such as detection of sinusoids in noise or canceling a periodic interference from signal measurements [1]-[3]. As one of the kind of the adaptive notch filter, there is the cascades notch filter implemented using second order all-pass filter [4]-[6]. The method to update the weights of the all-pass filter composed by IIR filter is known well. However, these adaptive notch filters have not been put to practical use since the inevitable stability problem of IIR filter and the problem concerning the determination method of the degree of IIR filter exist. In this paper, we propose a new adaptive notch filter that realizes the phase shift of the all-pass circuit by the Fourier sine series expansion and the adaptive algorithm. The proposed adaptive notch filter is always stable since the all-pass filter is composed by an exponential filter. Moreover, the adaptive algorithm is simple gradient based algorithm and the convergence of weights is guaranteed. The degree of the all-pass filter only depends on the required steepness of the elimination characteristic and does not depend on the number of narrowband signals that should be eliminated. Moreover, it can be said that the proposed method is a suitable method for practical use since the elimination characteristic hardly influenced by the broadband signal added to the narrow band signal. Finally, we confirm the convergence characteristics of the proposed method in comparison with the conventional method through the computer simulation.

2. Preview of conventional method

The transfer function of a notch filter implemented using cascaded second order all-pass filter, each of which has been realized by second order all-pass filter is given by

\[ H(z) = \prod_{i=1}^{M} \left\{ \frac{1}{2} \left\{ 1 + H_{a_i}(z) \right\} \right\} \quad (1) \]

where \( H_{a}(z) \) is a second order notch filter and \( M \) is number of cascaded second order all-pass filters. \( H_{a}(z) \) can be realized using the direct form II all-pass filter and expressed as

\[ H_{a}(z) = \frac{\rho^2 + a_1 z^{-1} + z^{-2}}{1 + a_1 z^{-1} + \rho^2 z^{-2}} \quad (2) \]

Realization of the \( i \)-th cascade stage based on direct form II allpass filter is shown in Fig.1 (b). From this figure, the transfer function the \( i \)-th cascade stage can be given by

\[ H_i(z) = \frac{1 (1 + \rho^2) + 2a_1 z^{-1} + (1 + \rho^2)z^{-2}}{2 \left[ 1 + a_1 z^{-1} + \rho^2 z^{-2} \right]} \quad (3) \]

If we define the radian frequency \( \omega_c \) along the unit circle on the z-plane such that \( z = e^{j\omega_c} \), then the radian frequency \( \omega_{a_i} \), which corresponds to the zero of \( H_{a}(\omega) \) will be given by \( a_i = -(1 + \rho^2) \cos \omega_{a_i} \). This radian frequency \( \omega_{a_i} \) is referred to as the notch frequency of the \( i \)-th stage of the cascaded notch filter. The value of \( \rho \) determines the bandwidth of notch filter [4]. Conventionally, \( \rho \) is set to the close to one. The simplified adaptive algorithm used to update this kind of notch filter is given by

\[ a_i(n + 1) = a_i(n) - \mu u_i(n - 1) e_i(n) \quad (4) \]

for \( i = 1,2,\ldots, M \), where \( \mu \) is the step size of adaptation, \( e_i(n) \) is the output of the \( i \)-th stage second order notch filter and \( u_i(n-1) \) is given in Fig. 1(b). This algorithm can be considered as minimizing the mean square of \( e_i(n) \) based on the forward coefficient \( a_i(n) \). In order to minimize the bias, the value of \( \mu \) must be kept close but less than one. However, as explained earlier, which this kind of setting, the convergence speed reduces significantly due to the small ranges of step size that ensures the convergence.
The ith cascade stage of second order notch filter and the detailed circuitry of the ith cascade stage, which has been implemented using a direct form second order all-pass filter.

### 3. Proposed method

To solve this problem, we propose a new adaptive notch filter that realizes the phase shift of the all-pass circuit by the Fourier sine series expansion and the adaptive algorithm. The fundamental structure of an adaptive notch filter is shown in Fig. 2. The transfer function of the notch filter is given by

\[ H(z) = \frac{1 + H_A(z)}{2} \]  

where \( H_A(z) \) is a transfer function of the all-pass filter which is composed by an exponential filter. \( H_A(z) \) is given by

\[ H_A(z) = \exp \left[ - \sum_{m=1}^{M} h_m (z^{-m} - z^{m}) \right] \]  

Using \( z = \exp(j\theta) \), we can rewrite Eq. (6) as follows:

\[ H_A(j\theta) = \exp \left[ - j \sum_{m=1}^{M} h_m \sin m\theta \right] \]  

From equation (7), the phase shift of the notch filter is described as

\[ \xi(j\theta) = - \sum_{m=1}^{M} h_m \sin m\theta \]  

The input signal \( x(n) \) of the notch filter at discrete time \( n \) is assumed to be given by

\[ x(n) = w(n) + s(n) \]

\[ s(n) = \sum_{i=1}^{N} a_i \cos(n\theta_i + \Phi_i) \]  

where \( s(n) \) is narrow band signal and \( w(n) \) is broadband one. And the estimation error of the notch filter is obtained by

\[ e(n) = \frac{x(n) + u(n)}{2} \]  

We define the cost function \( D \) as follows:

\[ D = e^2(n) \]  

From Fig. 2, we can obtain

\[ \frac{\partial u(n)}{\partial h_i} = - \frac{1}{2} q_i(n) \]  

\[ q_i(n) = u(n-k) - u(n+k) \]  

Using above equations, we can get the following partial derivative:

\[ \frac{\partial D}{\partial h_k} = \frac{1}{2} e(n)q_k(n) \]  

From Eq. (14), the adaptive algorithm used to update notch filter is given by

\[ h_k(l + 1) = h_k(l) - \mu e(n)q_k(n) \]  

where \( \mu \) is positive step size for updating \( h_k(l) \). As the step size \( \mu \) of the proposed method is used and given by

\[ \mu = \frac{\alpha}{\sum_{m=1}^{M} q_m^2(n-M)} \]  

where \( 0 < \alpha < 16 \).

If it satisfies the upper expression, the adaptive algorithm will converge.

The practical structure of an adaptive notch filter, which is a causal realization, is shown in Fig. 3. Moreover, the practical adaptive algorithm can be rewrite as follows:

\[ h_k(l + 1) = h_k(l) - \mu e(n-M)q_k(n-M) \]

\[ q_k(n-M) = u(n-k-M) - u(n+k-M) \]  

\[ \mu = \frac{\alpha}{\sum_{m=1}^{M} q_m^2(n-M)} \]  

where \( 0 < \alpha < 16 \).
4. Simulation results

In this section, we verify the performance characteristics by the computer simulation. The input signal \( x(n) \) is the sum of narrow band signal \( s(n) \) and broadband signal \( w(n) \).

In the first simulation, the input signal and the proposed method for simulation had the following parameters:
\[
\theta_1 = \pi/3, \alpha_1 = 1.0, \Phi_1 = \pi/4, \quad M=10, \quad N=1
\]  

We used a performance function defined using function EA(Estimation Accuracy). EA is given by
\[
EA(l)=10\log_{10} \frac{s^2(n-NL)}{[s(n-NL)+s_k(n)]^2} \quad \text{(dB)}
\]  

where, \( s_k(n) \) is the component of the narrow band signal \( s(n) \) within the output signal \( u(n) \) of all-pass filter and \( h_z(n) \) is the impulse response of the all-pass circuit. In addition, broadband signal was generated using a white signal and a coloring filter \( H_c(z) \) given by
\[
H_c(z) = \frac{1}{1+0.8z^{-1}}.
\]  

The signal to noise ratio (SNR) was defined by
\[
\text{S/N} = 10\log_{10} \frac{s^2(n)}{w^2(n)} \quad \text{(dB)}.
\]

The convergence characteristic for different value of \( \alpha \) was given by Fig.4. The result showed that the proposed algorithm could not converge when \( \alpha \) set to 20. And using the other values of \( \alpha \), we observed that the proposed algorithm converged.

In the next simulation, we verify the performance characteristic of the proposed method to compare with the conventional method by the computer simulation. Figure 5 showed a result obtained.

From this result, proposed method exhibited the superior estimation accuracy than the conventional method. And furthermore, the degree of the all-pass filter only depended on the required steepness of the elimination characteristic and did not depend on the number of narrowband signals that should be eliminated.
5. Conclusions

In this paper, new adaptive notch filter that realized the phase shift of the all-pass circuit by the Fourier sine series expansion and the adaptive algorithm was proposed. The proposed adaptive notch filter was always stable since the all-pass filter was composed by an exponential filter. The degree of the all-pass filter only depended on the required steepness of the elimination characteristic and did not depend on the number of narrowband signals that should be eliminated. Moreover, it could be said that the proposed method was a suitable method for practical use since the elimination characteristic hardly influenced by the broadband signal added to the narrow band signal. From the simulation result, the proposed method exhibited the superior estimation accuracy in comparison to the conventional method.

References


