Numerical Analysis of the Two-dimensional Pillar-type Photonic Cristal Waveguide Devices Using Spectral-Domain Approach

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Abstract—This paper presents a formulation of two-dimensional pillar-type photonic crystal waveguide devices. The device structures are considered as cascade connections of straight waveguides. Decomposing the structure into layers of the cylinder arrays, the input/output properties of the devices are obtained using an analysis method of multilayer structure. The periodicity cell that makes up the photonic crystal waveguide device has imperfect periodicity in the direction perpendicular to wave propagation. The field transformed by PPFT has a periodic property in terms of the transform parameter. To solve the integral equation, we introduced a discretization in the transform parameter. The input/output properties of the devices are obtained by recursive calculation of scattering matrix with each layer.

I. Introduction

Photonic crystal is a periodic structure consisting of high contrast dielectrics, in which the electromagnetic wave cannot transmit in a specific wavelength range. It is therefore known that, if localized defects are introduced in the photonic crystal, the electromagnetic fields are strongly confined around the defects. For example, point defects in the photonic crystal work as resonance cavities and line defects work as straight waveguides. Decomposing the structure into layers of these waveguides. Also, appropriate arrangements of the defects function as photonic crystal waveguide devices (PCW), such as branching filter, resonator filter. This paper presents a formulation of two-dimensional pillar-type photonic crystal waveguide devices using the spectral-domain approach. For the straight waveguides, the structure maintains the periodicity in the propagation direction, and the Floquet theorem asserts that the electromagnetic fields in the structure can be expressed by superposition of the Floquet-modes [1]. The Floquet-modes of the photonic crystal waveguides (PCW) are obtained by the eigenvalue analysis of the transfer matrix for one periodicity cell in the propagation direction. The periodicity cell that makes up the PCW has imperfect periodicity in the direction perpendicular to wave propagation. Therefore the fields in the structure have continuous spectra. The present analysis uses the pseudo-periodic Fourier transform (PPFT) [2] to consider the discretization scheme in the wavenumber space. The PPFT and its inverse are formally given by

\[ \tilde{f}(x;\xi) = \sum_{m=-\infty}^{\infty} f(x-md)e^{imd\xi} \] (1)

\[ f(x) = \frac{1}{k_d} \int_{-k_d/2}^{k_d/2} \tilde{f}(x;\xi)d\xi \] (2)

where \( d \) is a positive value usually chosen to be equal with the structural period, \( \xi \) is a transform parameter, and \( k_d = 2\pi/d \) is the inverse lattice constant. General structures of the PCWD are considered as cascade connections of the straight waveguides.

II. Setting of the Problem

This paper considers the guided Floquet-modes propagating in a PCWD schematically shown in Fig.1. The structure consists of identical circular cylinders that are infinitely long and described by the radius \( a \), the permittivity \( \varepsilon_s \), and the permeability \( \mu_s \). The cylinders are situated in a surrounding medium with the permittivity \( \varepsilon_r \) and the permeability \( \mu_r \). The cylinder axes parallel to the \( z \)-axis are located at \((x,y) = (nd,(q+1/2)h)\) for any integer \( n,q \), though some cylinders are removed. To indicate the removed cylinders, we introduce a notation \( \mathcal{G}^{(l)} \), which is a finite subset of the integer set \( \mathbb{Z} \). If an integer \( l \) is an element of \( \mathcal{G}^{(l)} \), the cylinder whose center is at \((x,y) = (ld,(q+1/2)h)\) is removed.

The structure under consideration is divided into three sections by two planes \( y = 0 \) and \( y = Qh \). The regions \( y < 0 \) and \( y > Qh \) are the input/output sections consisting of straight waveguides, while the other region \( 0 \leq y \leq Qh \) is a transition section. The transition section is composed of \( Q \) layers stacked in the \( y \)-direction, and arbitrary cylinders can be removed if the fields are well confined in the \( x \)-direction. The fields are supposed to be uniform in the \( z \)-direction. Then, the problem becomes two-dimensional, and the fields are decomposed into the transverse magnetic (TM) and the transverse electric (TE) polarizations, in which the magnetic and the electric fields are...
perpendicular to the z-axis, respectively. We consider time-harmonic electromagnetic fields assuming a time-dependence in $e^{-i\omega t}$.

III. OUTLINE OF FORMULATION

A. Transfer Matrix for Each Layer

The incident field for $q$th-layer periodicity cell consists of the waves propagating in the negative y-direction from the plane $y=qh$ and the waves propagating in the positive y-direction from the plane $y=(q-1)h$. Therefore, the incident field transformed by the PPFT $\tilde{\psi}^{(l)}(x,\xi,y)$ can be expressed in the plane-wave expansion [2] as

$$\tilde{\psi}^{(l)}(x,\xi,y) = f^{(l)}(x,y+(q-1)h;\xi)\tilde{\psi}^{(-)}(\xi,qh) + f^{(l)}(x,y+qh;\xi)\tilde{\psi}^{(+)}(\xi,(q-1)h)$$

(3)

where the column matrices $f^{(l)}(x,y;\xi)$ are generated by the plane-waves whose $n$th-components are given by

$$\left(f^{(l)}(x,y;\xi)\right)_n = e^{\alpha_n(x)1\times2(x,y)}$$

(4)

with

$$\alpha_n(\xi) = \xi + nk_d, \quad \beta_n(\xi) = \sqrt{k_n^2 - \alpha_n(\xi)^2}.$$ (5)

$\tilde{\psi}^{(\pm)}(\xi,y)$ denote the column matrices of the amplitude corresponding to the plane-waves propagating in the positive and the negative y-direction, respectively and given by

$$\tilde{\psi}^{(\pm)}(y) = \begin{bmatrix} \tilde{\psi}^{(\pm)}(\xi_1,y) \\ \vdots \\ \tilde{\psi}^{(\pm)}(\xi_L,y) \end{bmatrix}$$

(6)

where $[\xi_l]_{l=1}^L$ are sample points for integration scheme. $k_d$ denotes the wavenumber in the surrounding medium. We can obtain a relation between the amplitudes of plane-waves $\tilde{\psi}^{(\pm)}((q-1)h)$ and $\tilde{\psi}^{(\pm)}(qh)$ as

$$\begin{bmatrix} \tilde{\psi}^{(+)}(qh) \\ \tilde{\psi}^{(-)}(qh) \end{bmatrix} = F(q) \begin{bmatrix} \tilde{\psi}^{(+)}((q-1)h) \\ \tilde{\psi}^{(-)}((q-1)h) \end{bmatrix}.$$ (7)

The matrix $F(q)$ is the transfer matrix for the $q$th-layer. A highly precise method is suggested to calculate the transfer matrix $F(q)$ in Ref.[3]. This method use the spectral-domain approach because fields in the structure have continuous spectra. The present analysis uses the PPFT to consider the discretization scheme in the wavenumber space, and the scattering from cylinders are calculated by the recursive transition-matrix algorithm (RTMA) [4]. The transfer matrix $F(q)$ is given as

$$F(q) = \begin{bmatrix} S^{(q)}_{11} & S^{(q)}_{12} & S^{(q)}_{13} & S^{(q)}_{14} \\ S^{(q)}_{21} & S^{(q)}_{22} & S^{(q)}_{23} & S^{(q)}_{24} \\ S^{(q)}_{31} & S^{(q)}_{32} & S^{(q)}_{33} & S^{(q)}_{34} \\ S^{(q)}_{41} & S^{(q)}_{42} & S^{(q)}_{43} & S^{(q)}_{44} \end{bmatrix}$$

(8)

with

$$S^{(q)}_{11} = \bar{B}^{(q)}(\xi)\bar{C}^{(q)}(\xi)\bar{A}^{(-)}$$

(9)

$$S^{(q)}_{12} = \bar{B}^{(q)}(\xi)\bar{C}^{(q)}(\xi)\bar{A}^{(+)} + \bar{V}$$

(10)

$$S^{(q)}_{21} = -\bar{B}^{(-)}(\xi)\bar{C}^{(q)}(\xi)\bar{A}^{(-)}$$

(11)

$$S^{(q)}_{22} = -\bar{B}^{(-)}(\xi)\bar{C}^{(q)}(\xi)\bar{A}^{(+)}$$

(12)

$$S^{(q)}_{31} = \left(A^{(q)}(\xi_1)F(h/2;\xi_1)^{\dagger} - 0 \right.$$

$$A^{(q)}(\xi_1)F(h/2;\xi_1)^{\dagger} 0$$

(13)

$$S^{(q)}_{32} = \left(F(h/2;\xi_1)B^{(q)}(\xi_1)^{\dagger} - 0 \right.$$\n
$$F(h/2;\xi_1)B^{(q)}(\xi_1)^{\dagger} 0$$

(14)

$$S^{(q)}_{33} = \left(\begin{array}{cc} M^{(q)}_{1,1} & \cdots \\ \vdots & \ddots & \ddots \\ M^{(q)}_{L,1} & \cdots & \cdots & \cdots & \cdots \end{array} \right), \quad \bar{C}^{(q)} = \left(\begin{array}{cc} C^{(q)}_{1,1} & \cdots \\ \vdots & \ddots & \ddots & \cdots \\ C^{(q)}_{L,1} & \cdots & \cdots & \cdots & \cdots \end{array} \right)$$

(15)

$$S^{(q)}_{41} = \left(0 \right.$$\n
$$F(h;\xi_1) 0$$

(16)

$$S^{(q)}_{42} = \left(0 \right.$$\n
$$F(h;\xi_1) 0$$

(17)

$$C^{(q)}_{ij} = \delta_{i,j} + \frac{\omega L^{(q)}(\xi_1)}{k_d}$$

(18)

$$M^{(q)}_{ij} = \delta_{i,j} M^{(q)}(\xi_1) + \frac{\omega L^{(q)}(\xi_1)}{k_d}$$

(19)

$$L^{(q)}(\xi_1) = \sum_{\xi_1} G^{(l)}(-Ld,0;\xi_1)e^{i\theta(\xi_1)}$$

(20)

where $|\xi_l|_{l=1}^L$ that related to $|\xi_l|_{l=1}^L$ denote the weight factors determined by the appropriate numerical integration scheme, $\delta_{i,j}$ denotes the Kronecker delta, $\rho(x,y) = \sqrt{x^2 + y^2}$, and $\phi(x,y) = \text{arg}(x + iy)$. The matrices with tilde denote large matrices consisting of submatrices associated with individual cylinders. The superscript Z indicates the type of the cylindrical functions. $Z = J$ denotes the Bessel function and $Z = H^{(1)}$.
denotes the Hankel function of the first kind. The \((n,m)\)-th components \(T\) are given by
\[
(T)_{n,m} = \delta_{n,m} \\
\times \frac{\xi_n J_n(k\alpha) J'_m(k\alpha) - \xi_m J'_n(k\alpha) J_m(k\alpha)}{\xi_n H_n^{(1)}(k\alpha) J_m(k\alpha) - \xi_m H_m^{(1)}(k\alpha) J'_n(k\alpha)}
\]
(24)
for TM-polarization, and
\[
(T)_{n,m} = \delta_{n,m} \\
\times \frac{\xi_n J_n(k\alpha) J'_m(k\alpha) - \xi_m J'_n(k\alpha) J_m(k\alpha)}{\xi_n H_n^{(1)}(k\alpha) J_m(k\alpha) - \xi_m H_m^{(1)}(k\alpha) J'_n(k\alpha)}
\]
(25)
for TE-polarization. The \((m,n)\)-entries of \(F(y,\xi), A^{(\pm)}(\xi)\) and \(B^{(\pm)}(\xi)\) are given by
\[
A^{(\pm)}(\xi)_{m,n} = \frac{\pm \beta_{m}(\xi)}{\pm \beta_{n}(\xi)}
\]
(26)
\[
B^{(\pm)}(\xi)_{m,n} = \frac{\pm \beta_{m}(\xi)}{\pm \beta_{n}(\xi)}
\]
(27)

**B. Scattering Matrix for Composite Structure**

Let \(b_0^{(q)}(y)\) and \(r_n^{(q)}\) be \(n\)-th-eigenvalues and the associated eigenvectors of \(F^{(q)}\), respectively. We define a column matrix \(b^{(q)}(y)\) by
\[
b^{(q)}(y) = R^{(q)\dagger} \begin{pmatrix} \psi^{(q)}(y) \\ \psi^{(q)}(y) \end{pmatrix}
\]
(29)
\[
R^{(q)} = \begin{pmatrix} r_1^{(q)} & \cdots & r_{N+1}^{(q)} \end{pmatrix}.
\]
(30)

Then, from Eqs. (7) and (29), we obtain a relation:
\[
b^{(q)}(q h) = \beta^{(q)} b^{(q)}(q h - 1).
\]
(31)

This relation implies that \(b^{(q)}(q h)\) gives the amplitudes of the Floquet-modes propagating in the photonic crystal waveguide at \(y = q h\). We classify the propagation directions of the Floquet mode in the following rules.

- if \(\beta^{(q)} < 1\), the corresponding mode is the evanescent one propagating in the \(+y\)-direction.
- if \(\beta^{(q)} > 1\), the corresponding mode is the evanescent one propagating in the \(-y\)-direction.
- if \(\beta^{(q)} = 1\), the corresponding mode is the guided one. When the modal power carried in the \(y\)-direction is positive (negative), the corresponding mode propagates in the \(+y\) \((-y)\)-direction.

The eigenvalues and the eigenvectors are sorted in such a way that the eigenvalues \(\beta^{(q)}(y)\) and \(\beta^{(q)}(y)\) correspond to the Floquet-modes propagating in the \(+y\) and \(-y\)-directions, respectively. Then, \(b^{(q)}(y)\) and \(b^{(q)}(y)\) are rewritten by \(b^{(q)}(y)\) and \(b^{(q)}(y)\), respectively. We also use column matrices \(b^{(q)}(y)\) generated by the amplitudes \(b^{(q)}(y)\). From Eq. (31), the propagation constants of the Floquet-modes are given as
\[
\eta^{(q)}_{m} = -\ln(b^{(q)}_{m})
\]
(32)

for \(m = 1, \ldots, 2N + 1\), where \(L_n\) is the principal natural logarithm function. Using the propagation constants \(\eta^{(q)}_{m}\), we derive the following relations:
\[
b^{(q)}(n h) = b^{(q)}(n h)e^{\eta^{(q)}_{m}(n - m h)}
\]
(33)
where \(n\) and \(n'\) are arbitrarily integer, respectively. S-matrix of the device is derived by the multilayer technique. We define S-matrix of the region \(0 < y < q h\) as
\[
\begin{pmatrix} b^{(0,-)}(0) \\ b^{(0,+)}(q h) \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} b^{(q,+)}(0) \\ b^{(q,-)}(q h) \end{pmatrix}
\]
(34)
where \(S_{11}, S_{12}, S_{21},\) and \(S_{22}\) are \((2N+1)\times(2N+1)\) square submatrices. The boundary conditions at \(y = q h\) \((q = 0, \ldots, Q)\) are matched by equating the Fourier coefficients of tangential field components in both sides, and yield
\[
\begin{pmatrix} b^{(q,+)}(q h) \\ b^{(q,-)}(q h) \end{pmatrix} = \begin{pmatrix} G_{11}^{(q)} & G_{12}^{(q)} \\ G_{21}^{(q)} & G_{22}^{(q)} \end{pmatrix} \begin{pmatrix} b^{(q,+)}(q h) \\ b^{(q,-)}(q h) \end{pmatrix}
\]
(35)
with
\[
G_{11}^{(q)} G_{12}^{(q)} G_{21}^{(q)} G_{22}^{(q)} = R^{(q)\dagger} R^{(q)}.
\]
(36)

Also, Eq. (33) gives the following relations:
\[
b^{(q,+)}(q h) = D^{(q)} b^{(q,+)}((q - 1)h)
\]
(37)
\[
b^{(q,-)}((q - 1)h) = D^{(q)} b^{(q,-)}(q h)
\]
(38)
\[
(D^{(q)})_{n,m} = \delta_{n,m} e^{\eta^{(q)}_{m}}.
\]
(39)

From Eq. (35) for \(q = 0\), the initial S-matrices are derived as follows:
\[
S_{0,11} = G_{11}^{(q)}
\]
(40)
\[
S_{0,11} = -S_{12} G_{21}^{(q)}
\]
(41)
\[
S_{0,21} = G_{11}^{(q)} + G_{12}^{(q)} S_{0,11}
\]
(42)
\[
S_{0,22} = G_{12}^{(q)} S_{0,12}.
\]
(43)

When S-matrices \(S_{q-1,11}, S_{q-1,12}, S_{q-1,21},\) and \(S_{q-1,22}\) are given, S-matrices of the region \(0 < y < q h\) are derived from Eqs. (35), (37), (38) as follows:
\[
S_{q,12} = S_{q-1,12} D^{(q)} W_{q-1,q}^{-1}
\]
(44)
\[
S_{q,11} = S_{q-1,11} - S_{q,12} G_{21}^{(q)} D^{(q)} S_{q-1,21}
\]
(45)
\[
S_{q,22} = W_{q,0} W_{q-1,q}^{-1}
\]
(46)
\[
S_{q,21} = G_{21}^{(q)} - S_{q,22} G_{21}^{(q)} D^{(q)} S_{q-1,21}
\]
(47)
\[
W_{q,21} = G_{21}^{(q)} + G_{21}^{(q)} D^{(q)} S_{q-1,22} D^{(q)}
\]
(48)
\[
W_{q,21} = G_{12}^{(q)} + G_{12}^{(q)} D^{(q)} S_{q-1,22} D^{(q)}
\]
(49)

Consequently, S-matrices \(S_{Q,11}, S_{Q,12}, S_{Q,21},\) and \(S_{Q,22}\) for the entire region are obtained by the initial matrices Eqs. (40)–(43) and the recursive relations Eqs. (44)–(49).
IV. Conclusion

This paper has provided the spectral-domain formulation of the pillar-type PCWD based on the RTMA with the use of the PPFT. The formulation decomposes the device structure into straight waveguide sections, and the input/output properties are derived by the recursive calculation for each straight waveguide section. The periodicity cell that makes up the PCWD has imperfect periodicity in the direction perpendicular to wave propagation. The field transformed by PPFT has a periodic property in terms of the transform parameter. To solve the integral equation, we introduced a discretization in the transform parameter. Then the Floquet-modes can be obtained by the eigenvalue calculations of the transfer matrix without using periodic boundary conditions. The present formulation can calculate not only the guided-modes but also the evanescent-modes.

REFERENCES