Application of Fresnel Zone Numbers Localization for Equivalent Edge Currents Line Integration

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Abstract—Equivalent edge currents (EECs) for high frequency diffraction analysis are the intermediate concepts between Ray theory and Physical Optics (Wave optics). EEC eliminates the caustic singularities of the ray theory at the cost of additional line integration. The high frequency localization by using the Fresnel zone numbers in stead of the stationary phase method could be applied to EEC and the uniform convergence to Ray theory is expected. Modified Edge Representation (MER) as one of the concepts for complete definition of EEC is jointly used to demonstrate the recovery of uniform type of GTD from Keller-type EEC. The RB/SB singularities and the ambiguity of current definition are removed. The dipole wave scattering from a rectangular and circular plate will be demonstrated in the talk.

I. INTRODUCTION

Equivalent edge currents (EECs) for high frequency diffraction analysis [1] are the intermediate concepts between Ray theory and Physical Optics (Wave optics). EEC eliminates the caustic singularities of the ray theory at the cost of additional line integration. The high frequency localization by using the Fresnel zone numbers in stead of the stationary phase method has been studied by the authors[2] and could be applied to EEC. In this paper, the numerical recovery of uniform type GTD patterns from Keller-type EEC is tested for the dipole wave scattering from a rectangular/circular plate. Modified Edge Representation (MER) [3][4] as one of the concept for complete definition of EEC is jointly used with the Fresnel zone number localization to eliminate the ambiguity in practical line integration and to demonstrate the uniform convergence to the quasi-ray theory. The concepts may be extended to Fringe wave -EEC to recover PTD.

II. ASYMPTOTIC REDUCTION OF WAVE OPTICS INTO EQUIVALENT EDGE CURRENTS AND FURTHER LOCALIZATION FOR RAY THEORY

Fig.1 illustrates the conceptual relation between the wave theory and the ray theory. The scattering is generally expressed in terms of surface radiation integrals of currents induced on the scatterer. The surface radiation integral is asymptotically reduced to the line radiation integral of EEC by applying the method of stationary phase (SP), which is expressed
as method of EEC(MEC) in the figure. EEC is not real but fictitious currents asymptotically representing the diffraction due to edge truncation. If SP is applied again for EEC line integration, one could obtain the ray theory such as GTD which dispenses with integration. Singularities associated with the each use of SP in surface-line or line-to-point integral reduction, are regarded as RB/SB singularities and caustics, respectively. In addition, the EEC is not unique at every point on the periphery except at the stationary phase points or the diffraction points since these are inversely derived from GTD in most cases. On the other hand, if the rigorous surface diffraction points since these are inversely derived from GTD on the periphery except at the stationary phase points or the stationary phase points respectively. In addition, the EEC is not unique at every point on the surface and the line integration correspond to the group of approximations named PO, PO-MEC and PO-SP.

III. MODIFIED EDGE REPRESENTATION (MER)

Modified edge representation (MER) is a unique concept for generalizing the definition of EEC at diffraction points to that at arbitrary points on the periphery. Instead of modifying the diffraction coefficients, MER introduced a fictitious edge which satisfies the diffraction law at the point for given direction of the incidence and the observer. [4]-[6] as in Fig. 2. In MER, EEC could be defined not only along the periphery but also everywhere on the surface and the line integration give approximation for the diffraction and the Geometrical optics component (SGO), respectively.

The fields are expressed by the line integrals of Keller-type EECs clearly defined everywhere since the modified edge satisfies the diffraction law.

\[
(\hat{r} + \hat{n}) \cdot \hat{t} = 0 ; \hat{n} \cdot \hat{t} = 0
\]  

\[
E_{\text{MER}}^{\text{SP}} = \lim_{\rho' \to 0} \left\{ \frac{k}{4\pi} \hat{r}_o \times \left[ \hat{r}_o \times \eta J_{\text{MER}} + M_{\text{MER}} \right] e^{i \rho'} r_o^{\rho_o} \right\} dl
\]

\[
E_{\text{MER}}^{\text{periphery}} = \frac{k}{4\pi} \hat{r}_o \times \left[ \hat{r}_o \times \eta J_{\text{MER}} + M_{\text{MER}} \right] e^{-i \rho_o} r_o^{\rho_o} \]

IV. LOCALIZATION IN TERMS OF FRESNEL ZONE NUMBER

Authors have been developing the localization procedure of surface radiation integrals[Kohama]. It extracts the scattering centers from whole integration area over the scatterer by utilizing the Fresnel zone numbers. It embodies the local feature of high frequency diffraction and provides the uniform convergence to the ray theory. The concept is depicted in Fig. 3.

The area of radiation integral is reduced to that adjacent to the stationary phase points by the formula in the figure without affecting the value [2]. In other words the currents on the scatterer will be weighted by the function defined by the Fresnel zone number at the point of integration together with the parameter of the size of localized area.

\[
E(Y) = \begin{cases} 
\frac{1}{2} \cos \left( \frac{\Delta n}{\Delta n_B} \pi \right) + 1 & (\Delta n \leq \Delta n_B) \\
0 & (\Delta n > \Delta n_B)
\end{cases}
\]

V. LOCALIZED LINE INTEGRATION OF MER-EEC

This paper proposes the inclusion of Fresnel-type weighting functions into the line integration of MER-EEC. It is expected that the integration would be localized as the frequency becomes high and approaches uniformly to the Ray theory. A unique advantage of MER and EEC would be maintained and the well known difficulties would be eliminated.

1) RB/SB singularities
2) Caustics
3) Ambiguity in the definition of EEC.

The weighting by EYE above is frequency dependent and is different from the weighting proposed in [5] which is frequency independent and depends only upon the geometry. The localization in terms of the latter weighting is illustrated in Fig. 4 where the hatching area indicate the weighting with the maximum near the diffraction points D1 and D2, where
the weighting is defined using the angle $\Phi$ between the real and the modified edge as

$$\cos^2 \Phi$$

Numerical comparison as for the various kinds of localization effects would be conducted for

$$\{J_{\text{MER}^*}, M_{\text{MER}^*}\} \rightarrow \{J_{\text{MER}^*} \times \text{EYE}, M_{\text{MER}^*} \times \text{EYE}\}$$

$$\{J_{\text{MER}^*}, M_{\text{MER}^*}\} \rightarrow \{J_{\text{MER}^*}^\text{PO} + J_{\text{MER}^*}^\text{FW} \times \text{EYE}, M_{\text{MER}^*}^\text{PO} + M_{\text{MER}^*}^\text{FW} \times \text{EYE}\}$$

$$\{J_{\text{MER}^*}, M_{\text{MER}^*}\} \rightarrow \{J_{\text{MER}^*} \times \cos^2 \Phi, M_{\text{MER}^*} \times \cos^2 \Phi\}$$

$$\{J_{\text{MER}^*}, M_{\text{MER}^*}\} \rightarrow \{J_{\text{MER}^*}^\text{PO} + J_{\text{MER}^*}^\text{FW} \times \cos^2 \Phi, M_{\text{MER}^*}^\text{PO} + M_{\text{MER}^*}^\text{FW} \times \cos^2 \Phi\}$$

Numerical data of the dipole wave scattering from various shapes of scatterers will be demonstrated in the talk.

Fig. 4 Localization by weighting $\cos^2 \Phi$ in [5].

References