Reconstruction of Boundary Perturbations in a Waveguide

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Abstract—The inverse problem of reconstructing boundary deformations in a parallel plate waveguide is solved by using a first order perturbation method applied to the dominant TEM-mode. Regularization using the L-curve criterion is applied and investigated. Reconstruction results from using reflection data are presented and influences from higher order propagating and trapped modes are demonstrated.

I. INTRODUCTION

In power transformers, local deformations of the windings increase the risk of failures, which may cause serious electrical power outages. Hence, it is of interest to develop methods for detecting the onset of winding deformations at an early stage. When disconnected from the power grid, some features of power transformers can be diagnosed by frequency response analysis (FRA), and in e.g. [1] FRA has been applied for detecting winding deformations. However, to prevent power outages, there is a need for methods that can be applied online - while the transformer is in operation. In [2] we have proposed a method based on positioning microwave antennas inside the transformer tank and use the microwaves to reconstruct the locations of the winding conductors. In that work, we use an approximate model and apply a full wave method that takes into account the details of the geometry, and the locations of the individual conductors are reconstructed using an optimization method. However, for a more realistic case, we foresee that a detailed numerical model used in conjunction with optimization will be too massive computationally to be an efficient reconstruction method.

Since our prime interest is in detecting small deformations, we find it of interest to investigate whether inversion methods based on weak scattering, can be applied to a transformer problem, and in this paper we take the first step towards such a method. In order to reduce the computational complexity, the winding is not considered in detail, but instead modeled as an equivalent outer boundary surface, which shape is to be reconstructed.

II. PROBLEM FORMULATION AND SCATTERING ANALYSIS

We consider a two-dimensional scattering configuration with a parallel plate PEC waveguide oriented along the z-axis; see Fig. 1. The upper boundary plate is located at \( x = a \). The lower boundary plate is located at \( x = 0 \). In the context of a power transformer, the upper boundary models the wall of the transformer tank while the lower boundary models the outermost layer of the winding structure. Hence, we describe the winding as an equivalent PEC surface. In a more realistic treatment it can be described by e.g. an anisotropic boundary condition [3]. Although a realistic transformer is filled with oil, we here assume that the medium inside the waveguide is air (or vacuum).

Fig. 1: Parallel plate waveguide with local deformation in one plate.

At the lower boundary in the section \( z_1 < z < z_2 \) there is a local deformation described by the equation

\[
x = a g(z)
\]

where the dimensionless function \( g(z) \) fulfills

\[
\max |g(z)| \ll 1
\]

\[
g(z_1) = g(z_2) = 0
\]

The inverse problem is to reconstruct \( g(z) \) in the estimation region \( z_1 < z < z_2 \) using scattering data obtained when the waveguide is excited from one or both ends. In order to focus the present study on the primary scattering mechanism, due to the local deformation of the boundary, we assume that there are no reflections from the ends of the waveguide.

A. The direct scattering problem

In the present study, we restrict the analysis to the TM-modes:

\[
H_m \propto \cos \left( \frac{m \pi x}{a} \right) \hat{y}, \quad m = 0, 1, 2, \ldots
\]

and especially the dominant TEM-mode (TM0), which propagates at all frequencies.
1) Perturbation method: The inversion scheme (Sect. II-B) is based on solving the direct scattering problem by means of a boundary perturbation method for waveguides, similar to the one used in [4]. For the TEM-mode, the corrections to the transmission parameters are second order in the perturbation parameter $g(z)$:

\[
S_{12} = 1 + \mathcal{O}\left(\max|g|^2\right) \quad (5)
\]

\[
S_{21} = 1 + \mathcal{O}\left(\max|g|^2\right) \quad (6)
\]

Hence, we consider transmission data to be too vulnerable to measurement errors, wherefore it will not be included in the subsequent analysis. Locating the measurement planes to the boundaries of the estimation region, the first order approximation of the reflection parameters become

\[
S_{11} \approx jk e^{2kz_1} \int_{z_1}^{z_2} g(z) e^{-2kz} dz \quad (7)
\]

\[
S_{22} \approx -jk e^{-2kz_2} \int_{z_1}^{z_2} g(z) e^{2kz} dz \quad (8)
\]

where $k = \sqrt{\epsilon_0 \mu_0}$ is the vacuum wavenumber.

2) HFSS model: As generator of synthetic measurement data, we use a full-wave FEM model implemented in the commercial program HFSS. This model takes into account all TM modes in the waveguide.

3) Transmission Line (TL) model: For comparison purposes, we have also modeled the TEM-mode scattering with a transmission line model, in which the $L$ and $C$ parameters are computed from the local value $a(1 - g(z))$ of the waveguide width. From [5], it follows that e.g. $S_{11}$ obeys the Riccati equation

\[
\frac{dS_{11}}{dz} = 2 \left( \frac{1}{1 - g} + \frac{1}{1 - 1 + g} \right) S_{11}
\]

\[
S_{11}(z_2) = 0 \quad (9)
\]

Note that, like the perturbation method, this model does not take into account any higher order TM-modes. Approximating $S_{11}$ and (9) to the first order in $g$, it is easily verified that (9) and (10) imply (7). Hence, to the first order in $g$ the TL-model and the waveguide perturbation method are equivalent.

B. The inverse scattering problem

Approximating the continuous inverse problem with a discrete inverse problem, the deformation $g(z)$ is expanded into a finite set of functions, as follows:

\[
g(z) = \sum_{n=1}^{N} p_n \phi_n(z) \quad (11)
\]

\[
\phi_n(z) = \sin \left( \frac{n\pi z - z_1}{z_2 - z_1} \right) \quad (12)
\]

Note that \( \{\phi_n(z)\}_{n=1}^{N} \) is an orthogonal basis over the estimation region, $z_1 < z < z_2$, and that $\phi_n(z_1) = \phi_n(z_2) = 0$. Inserting (11) and (12) into (7) and (8), we obtain

\[
k \sum_{n=1}^{N} p_n \Phi_n^*(k) = jS_{11}(k) e^{-j2kz_1} \quad (13)
\]

\[
k \sum_{n=1}^{N} p_n \Phi_n(k) = jS_{22}(k) e^{j2kz_2} \quad (14)
\]

where

\[
\Phi_n(k) = \int_{z_1}^{z_2} \phi_n(z) e^{j2kz} dz = \frac{n\pi}{2} \left( z_2 - z_1 \right) \left( e^{2kz_1} - (-1)^n e^{2kz_2} \right) \frac{s_n^2}{4\pi^2 - 4k^2 (z_2 - z_1)^2} \quad (15)
\]

For the reconstruction of the expansion coefficients \( \{p_n\}_{n=1}^{N} \), we can use one of (13) and (14) (one-sided reflection data) or both (two-sided reflection data). Since \( \{p_n\}_{n=1}^{N} \) are real-valued, we treat the real and imaginary parts of (13) and (14) as separate equations. The coefficients \( \{p_n\}_{n=1}^{N} \) are collected into the vector $p$. From measurements of $S_{11}(k)$ and/or $S_{22}(k)$ at several values of $k$, i.e. frequencies, the right hand sides of (13) and (14) are collected into the vector $b$, while the evaluations of $\Phi_n(k)$ and $\Phi_n^*(k)$ are collected into the matrix $A$. Assuming more data points than expansion coefficients, the vector $p$ is determined as the solution of the least square problem

\[
\min \|Ap - b\|_2^2 \quad (16)
\]

C. Regularization

Due to the illposedness of the inverse problem, we invoke regularization by adding a penalty term to (16). For the $r$:th order derivative of the deformation $g(z)$, (11) and (12) imply

\[
\left\| \frac{d^r g}{dz^r} \right\|_2^2 = \int_{z_1}^{z_2} \left( \frac{d^r g}{dz^r} \right)^2 dz \propto \sum_{n=1}^{N} n^r \phi_n^2 = \|Lp\|_2^2 \quad (17)
\]

where the matrix $L = [\text{diag}\{1, 2, \ldots, N\}]^T$. We define $r$ as the order of the regularization, in the regularized problem

\[
\min \left\{ \|Ap - b\|_2^2 + \lambda^2 \|Lp\|_2^2 \right\} \quad (18)
\]

where $\lambda$ is the regularization parameter. The coefficient vector $p$ that solves (18) is obtained from

\[
(A^T A + \lambda^2 L^T L)^{-1} p = A^T b \quad (19)
\]

To find appropriate values of the regularization parameter $\lambda$, we use the so-called L-curve method [6], in which $\log \|Ap - b\|_2$ and $\log \|Lp\|_2$ are plotted against each other, when solving (19) parameterized by $\lambda$. The optimal $\lambda$ is estimated by visual inspection of where the L-curve has the maximum curvature.
III. RECONSTRUCTION RESULTS

In all reconstruction examples, we

- use \( N = 30 \) basis functions in the expansion (11) of the deformation \( g(z) \)
- use the width \( a = 1 \) m for the undistorted part of the waveguide. Hence, the higher order TM-modes have their cut-off frequencies at multiples of 150 MHz.
- use HFSS to simulate reflection data from the frequencies 2 MHz to 200 MHz in steps of 2 MHz.
- present results from using either of \( S_{11} \) and \( S_{22} \) and from using both.

A. Example 1: 10 % intrusion

First we consider an intrusion having the maximum 10 % of the undistorted waveguide width. The intrusion is located between \( 0 \) m \( \leq z \leq 4 \) m, and the estimation region is chosen somewhat larger, with \( z_1 = -1 \) m and \( z_2 = 5 \) m. We test with three different orders of regularization: \( r = \{0, 1, 2\} \), for which the results are depicted in Figures 2-4, respectively.

![Fig. 2: Reconstructions of a 10 % intrusion when order of regularization \( r = 0 \).](image)

![Fig. 3: Reconstructions of a 10 % intrusion when order of regularization \( r = 1 \).](image)

In our observations we find that double-sided reflection data seems to give a more “distinct” corner of the L-curve, than what is obtained when using one-sided data, and that the reconstructions from one-sided reflection data are deteriorated at the far side of the deformation. This is most clearly seen in Fig. 2, when using \( r = 0 \) i.e. suppression directly onto the function \( g(z) \); the reconstructions are generally under-smoothed, due to that the L-curve criterion underestimates the regularization parameter \( \lambda \).

B. Example 2: 20 % intrusion and 20 % extrusion

Next, we consider an intrusion and an extrusion of the same shape as the intrusion in Sect. III-A, but scaled down a factor 0.5 lengthwise and scaled up a factor 2 in the peak value. The intrusion/extrusion are located between \( 0 \) m \( \leq z \leq 2 \) m, and the estimation region is chosen between \( z_1 = -0.5 \) m and \( z_2 = 2.5 \) m. In this example, we only consider the regularization order \( r = 1 \). With 20 % maximum deformation, we expect that this example may reveal some limitations of the first order perturbation approach. The reconstruction results for the intrusion and the extrusion are shown in Figures 5 and 6, respectively.

![Fig. 4: Reconstructions of a 10 % intrusion when order of regularization \( r = 2 \).](image)

![Fig. 5: Reconstructions of a 20 % intrusion (\( r = 1 \)).](image)

Compared with the reconstructions in Sect. III-A, the present reconstructions exhibit lower quality, especially for the case with the extruding deformation. Using the first order perturbation theory (see (7) and (8)), the intrusion and the extrusion give apart from a change in sign the same reflection coefficients. In Fig. 5, we see that the reconstructed intrusion is over-estimated, while in Fig. 6, we see that the reconstructed...
extrusion is under-estimated. To understand this, we have in Fig. 7 plotted the magnitudes of the reflection coefficients, determined by the HFSS full wave solver, as functions of the frequency.

![Fig. 7: Magnitudes of \( S_{11} \) (solid lines) and \( S_{22} \) (dashed lines) for 20% intrusion (blue lines) and extrusion (green lines).](image)

Since we consider a reciprocal and lossless system, it follows that in the dominant mode interval below 150 MHz we have \(|S_{11}| = |S_{22}|\). The main contributions to the reconstructions are from data obtained below 100 MHz, where the reflection is stronger for the intrusion than for the extrusion. This explains the differences obtained when using the first order inversion algorithm. For frequencies above 150 MHz, different leakages into higher order propagating modes result in that \(|S_{11}| \neq |S_{22}|\). Note that part of the reflection data is taken from the interval 150 MHz to 200 MHz, which likely contribute to deteriorate the reconstructions, since higher order modes are not accounted for by the inversion algorithm. Finally, we observe for the extrusion case a strong peak in the reflection around 140 MHz (and a smaller peak around 270 MHz). With a maximal waveguide width of 1.2 m in the extruded part, the “local” cut-off frequency for the TM\(_1\)-mode varies between 125 MHz and 150 MHz. Hence, we conclude that the reflection peak is due to a higher order mode trapped inside the extrusion region. This likely explains the further deterioration in the reconstruction of the extrusion.

IV. CONCLUSIONS

We have presented a simple and efficient first order perturbation method to the inverse problem of reconstructing deformations in waveguide boundaries; a method that we currently develop for detecting deformations in power transformers. The L-curve criterion, for determination the appropriate regularization parameter, has been tested for different orders of regularization, and it seems that applying penalty on first or second order derivatives of the reconstructed profile is better than applying penalty on the profile itself. The reconstruction results indicate that the method has potential usefulness, although one must be cautious about the existence of higher order modes that can propagate freely or be trapped in the waveguiding structure.

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REFERENCES


