Numerical Integration of Physical Optics for 2-Dimensional Scatterer using Fresnel Zone Numbers with Frequency-Independent Computation Time

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Abstract—Many techniques to reduce the computational cost of the physical optics (PO) have been developed. In this paper, we introduce the technique for evaluation of radiation integrals in the PO, whose computational loads become independent of the frequency. Fresnel zone number is used for the criterion to localize and divide the radiation integrals. The proposed method was applied to the PO computation of the monostatic radar cross section (RCS) for 2-dimensional conducting strip and rectangular cylinder. We found the good balance between the computational time and accuracy.

I. INTRODUCTION

The physical optics (PO) approximation [1] is the well-known technique which is widely used to analyse scattering fields from the electrically large conducting scatterers such as the reflector antennas. This is also being applied for radio propagation or wireless communication environmental analysis in high frequency such the millimeter wave. The PO assumes the induced currents on the illuminated region and the radiation integral is conducted for the surface of scatterer. Therefore, the computational cost of the PO is usually proportional to the electrical size of scatterer and often becomes extremely heavy. Many techniques to reduce the computation loads have been developed, such as the stationary phase method [2] and adaptive sampling method [3]. Fast physical optics and multilevel physical optics proposed by A. Boag in [4], [5] also reduces the computation cost not for the size of scatterer but for the number of observation and frequency sampling points.

The authors have proposed the localization method of radiation integrals using the locality of scattering phenomena in [6]. The locality is visualized in figure 1 [7]. Although a dipole source illuminates the whole surface of scatterer, the strong contributions appear in only the area around the scattering centers, such as reflection and edge diffraction points, which are the important points in the geometrical theory of diffraction (GTD) [8]. The localization method suggests that the scattering field can be reproduced by integrating the currents only in the important and effective areas with adequate weighting functions. In order to extract the locality and truncate the effective areas appropriately, Fresnel zone number is used for criterion. Moreover, the authors confirmed that computation cost becomes independent of the frequency when Fresnel zone number is imported to the division law of numerical integrals [9].

In this paper, the localization and divisions method for radiation integrals using Fresnel zone numbers is implements to the PO computation of the radar cross section (RCS) for 2-dimensional structure. We discuss the computational accuracy and time.

II. LOCALIZATION AND EFFICIENT SEGMENTATION OF RADIATION INTEGRALS

A. Procedure for Localization of Radiation Integral

The authors have proposed that radiation integrals should be conducted not for the whole surface but for the local areas around the scattering centers [6]. In this paper, only the key points of procedure to determine the local areas are explained.

Fig. 2 shows the detailed decision law of the local areas. At each point, the Fresnel zone number \( n \) is defined as \( n=(2L)/\lambda \), where \( L \) is the path difference between two paths; directed path from source (S) to observer (O) and diverted path from source (S) to observer (O) via the point on the scatterer (P), and \( \lambda \) is the wavelength. The local areas around some scattering centers, (R) in the figure 2, are specified by the difference of the Fresnel zone numbers as in Fig. 2; \( |n - n_R| = \Delta n \leq \Delta n_B \), where \( n \) and \( n_R \) are the Fresnel zone number at the point of integration on the scattering surface and at (R), respectively. The parameter \( \Delta n_B \) is the number to determine the size of the local area and corresponds to the number of peaks in the integrand. In the computations, \( \Delta n_B = 3 \) in this paper.
When the localization method is adopted, the number of peaks in the integrand for given integration area is unchanged for any frequencies in terms of $\Delta n_{B}\neq3$, and the proposed division law makes the division number per one oscillation constant. Therefore, the total number of divisions and computation time would become the constant value for any frequencies.

### III. Numerical Discussion

Here, we discuss the computation accuracy and time of the proposed PO computation in the radar cross section (RCS) analysis. The 2-dimensional RCS $\sigma_{2D}$ is expressed by (2). $E_{i}$ is the incident electric field and $E_{s}$ is the scattering electric field radiated by the PO currents $J_{PO}$ defined by $2\hat{n}\times H_{i}$, where $\hat{n}$ is the unit vector normal to the scatterer surface and $H_{i}$ is the incident magnetic field.

$$\sigma_{2D} = \lim_{r \to \infty} \left( \frac{2\pi r}{|E_{s}|^{2}} \right) \text{[m]}$$

When the localization method is adopted, the number of peaks in the integrand for given integration area is unchanged for any frequencies in terms of $\Delta n_{B}\neq3$, and the proposed division law makes the division number per one oscillation constant. Therefore, the total number of divisions and computation time would become the constant value for any frequencies.

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In the usual PO computation, the numerical radiation integral is conducted for the whole surface by dividing the areas equally. On the other hand, it is conducted only for the local areas with EYE functions by dividing the areas based on the phase changes in the proposed PO computation.

We consider the monostatic RCS patterns for the conducting 2-dimensional strip shown in figure 4. The two patterns by each computation method are compared in figure 5. These two results are in good agreement. The RCS patterns for 2-D rectangular cylinder of figure 6 also are computed in figure 7. The validity of the proposed localization and segmentation method is confirmed.

![Fig. 2. Localization for the scattering center in terms of Fresnel zone](image)

After localized, fictitious edges which do not exist in the original problem appear and would produce undesired edge waves. A weighting smoothly vanishing at the boundary is introduced to suppress these, while it should take the values 1 at the scattering center so that the result of radiation integral remains unchanged due to localization. In this paper, we use EYE function, defined by (1), as the weighting function.

$$EYE\left( \frac{\Delta n}{\Delta n_{B}} \right) = \begin{cases} \frac{1}{2} \cos \left( \frac{\Delta n}{\Delta n_{B}} \theta \right) + 1 & (\Delta n \leq \Delta n_{B}) \\ 0 & (\Delta n > \Delta n_{B}) \end{cases}$$

### B. Efficient Segmentation of Radiation Integral Areas

When the radiation integral are evaluated numerically, integration areas must be divided into small segments. The number per one oscillation or per one phase change, for any cases.

![Fig. 3. Segmentation method](image)

When the localization method is adopted, the number of peaks in the integrand for given integration area is unchanged for any frequencies in terms of $\Delta n_{B}\neq3$, and the proposed division law makes the division number per one oscillation constant. Therefore, the total number of divisions and computation time would become the constant value for any frequencies.

![Fig. 4. Monostatic RCS of 2-D conducting strip for TM waves](image)

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![Fig. 5. Comparison between the usual PO (segment size is $\lambda/40$) and proposed PO ($\pi/12$ phase changes) in the case of RCS computation of 2-D strip](image)
is the result to be compared and \( \theta \) is the reference value computed by usual PO with \( N_{\theta} = 1801 \) here.

\[
\Delta_{\text{rms}} = \sqrt{\frac{1}{N_{\theta}} \sum_{i=1}^{N_{\theta}} \left[ E_{\text{interest}}^i(\theta) - E_{\text{reference}}^i(\theta) \right]^2} \times 100[\%] \tag{3}
\]

The proposed PO computation realizes the comparable accuracy with the usual one despite the small number of divisions. The result of the figure 8 shows that the proposed method keeps the computational time for higher frequencies without losing the computational accuracy.

Figure 8 shows the frequency dependence of the computational time for calculating the monostatic RCS of the conducting strip in figure 4. The CPU time by usual PO is proportional to the frequency while one by proposed PO is always almost constant and independent of the frequency.

The normalized root-mean-square (RMS) error defined by (3) is also indicated with the figure 8, where \( E_{\text{interest}}^i(\theta) \) is the reference value computed by usual PO with \( \lambda/10000 \) interval, \( E_{\text{interest}}^i(\theta) \) is the result to be compared and \( N_{\theta} \) is the number of observation sampling points. We use \( N_{\theta} = 1801 \) here.

\[
\Delta_{\text{rms}} = \sqrt{\frac{1}{N_{\theta}} \sum_{i=1}^{N_{\theta}} \left[ E_{\text{interest}}^i(\theta) - E_{\text{reference}}^i(\theta) \right]^2} \times 100[\%] \tag{3}
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The proposed PO computation realizes the comparable accuracy with the usual one despite the small number of divisions. The result of the figure 8 shows that the proposed method keeps the computational time for higher frequencies without losing the computational accuracy.

### IV. CONCLUSION

The technique, which the computational cost of PO radiation integrals becomes independent of the frequency, was introduced. Fresnel zone numbers are used for the criterion to localized and divide the radiation integrals. This technique was applied to the monostatic RCS analysis of 2-dimensional structure and the good balance between the computational time and accuracy was found. The applications to scattering analysis of 3-dimensional structure and current analysis by the MoM are left for the future work.

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### REFERENCES


