Scattering of Electromagnetic Waves by Inhomogeneous Dielectric Gratings with Parallel Perfectly Conducting Strips
–Matrix Formulation of Point Matching Method–

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Abstract—In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings loaded with parallel perfectly conducting strips using the combination of improved Fourier series expansion method and point matching method. This approach can treat periodic configurations having arbitrary combinations of dielectric, metallic, and perfectly conducting components.

I. INTRODUCTION

Recently, the refractive index can easily be controlled to make the periodic structures such as optoelectronic devices, photonic bandgap crystals, frequency selective devices, and other applications by the development of manufacturing technology of optical devices. Thus, the scattering and guiding problems of the inhomogeneous gratings have been considerable interest, and many analytical and numerical methods which are applicable to the dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials[1]-[4].

In this paper, we proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings with parallel perfectly conducting strips[9]-[11] using the combination of improved Fourier series expansion method[5]-[6] and the matrix formulation of point matching method[7]-[8].

Numerical results are given for the transmitted scattered characteristics for the case of frequency loaded with parallel perfectly conducting strips for TE and TM cases. The effects of the inhomogeneous dielectric gratings comparison with that of the slanted angle of the perfectly conducting strips on the transmitted power are discussed. Our approach also can treat periodic configurations having arbitrary combinations of dielectric, metallic, and perfectly conducting components.

II. METHOD OF ANALYSIS

We consider inhomogeneous dielectric gratings loaded with parallel perfectly conducting strips shown in Fig.1. The grating is uniform in the y-direction and the permittivity \( \varepsilon(x,z) \) is an arbitrary periodic function of \( z \) with period \( p \). The permittivity is assumed to be \( \mu_0 \). The time dependence is \( \exp(-i\omega t) \) and suppressed throughout.

In the formulation, the TM wave is discussed. When the TM wave (the magnetic field has only the \( y \)-component) is assumed to be incident from \( x>0 \) at the angle \( \theta_0 \),

\[
H_y^{(i)} = \exp(ik_y(z \sin \theta_0 - x \cos \theta_0)) \quad k_y = \omega \sqrt{\varepsilon_0 \mu_0} \quad \text{(1)}
\]

the electric fields in the regions \( S_1(x \leq 0) \), and \( S_1(x \geq D) \) are expressed[10] as

\[
S_1(x \leq 0) : \quad H_y^{(i)} = H_y^{(1)} + e^{i\beta z \sin \theta_0} \sum_{n=-N}^{N} b_n e^{i\left[k_y(x+2\pi n \alpha)/p\right]} \quad \text{(2)}
\]

\[
S_1(x \geq D) : \quad H_y^{(3)} = e^{i\beta z \sin \theta_0} \sum_{n=-N}^{N} a_n e^{i\left[k_y(x-D+2\pi n \beta)/p\right]} \quad \text{(3)}
\]

\[
E_z^{(i)} = \{i\omega \varepsilon_0 \}^{-1} \partial H_y^{(i)}/\partial x, \quad (j = 1, 3)
\]

The inhomogeneous layer is approximated by a stratified layers of modulated index profile with \( d_\lambda \) shown in Fig.2 and taking each layer as a modulated dielectric grating, the elec-
tromagnetic fields are expanded appropriately by a finite Fourier series as follows:

\[ S_j(0 < x < D) = \sum_{m=1}^{2N+1} \left[ A_m^{(j)} e^{i \nu_m x} + B_m^{(j)} e^{-i \nu_m x} \right] \phi_m(z) \]  

(4)

\[ f_r^{(j)}(z) = e^{i \nu_m z} \sum_{n=0}^{N} u_n \phi_n(z) e^{i \nu_n z}, \quad l = 1 - M \]

where \( A_m^{(j)}, B_m^{(j)}, A_n^{(j)}, \) and \( u_n^{(j)} \) are unknown coefficients to be determined from boundary conditions. \( k_n^{(j)} (j = 1, 3) \) is propagation constants in the \( x \) direction, and \( k_n^{(j)}, u_n^{(j)}(l = 1 - M), \) the propagation constant and eigenvectors, are satisfy the following eigenvalue equation in regard to \( h_0^{(j)} \)

\[ A \mathbf{U} = h \mathbf{U}, \]

(5)

where

\[ a^{(j)}(x_0) = \left[ A_m^{(j)} \ldots A_n^{(j)} \ldots u_n^{(j)} \ldots \right] \quad \mathbf{A} = \left[ \begin{bmatrix} e^{i \nu_m} \end{bmatrix} \right] \]

and

\[ b^{(j)}(x_0) = \left[ B_m^{(j)} \ldots B_n^{(j)} \ldots \right] \quad \mathbf{B} = \left[ \begin{bmatrix} e^{-i \nu_m} \end{bmatrix} \right] \]

We obtain the matrix form combination of metallic region \( C \) and the dielectric region \( \overline{C} \) using boundary condition \( Z_x = (j - 1) p / (2N + 1) \); \( j = 1 - (2N + 1) \) at the matching points on \( x = 0 \), and \( x = D \) Boundary condition using point matching are as follows:

\[ Z_x \in C_1 ; \left[ E_x^{(1)} = 0, \quad E_x^{(2,1)} = 0 \right] \]

(6)

\[ Z_x \in \overline{C}_1 ; \left[ H_y^{(1)} = H_y^{(2,1)} \right] \quad \left[ E_y^{(1)} = E_y^{(2,1)} \right] \]

(7)

In the boundary condition at Eq.(6), and Eq.(7), it is satisfied in all matching points by using the orthogonality properties of \( \{ e^{i \nu_n z} \} \), we get following equation(8) in regard to \( A_n^{(j)}, B_n^{(j)}, A_n^{(j)}, \) and \( B_n^{(j)} \):

\[ Q_1 A^{(j)} + Q_2 B^{(j)} = F, \]

(8)

\[ Q_1 A^{(j)} + Q_2 B^{(j)} = 0 \]

(9)

where \( F = \left[ 0(Z_x \in C_1), 1 + X G E^{(j)}(Z_x \in \overline{C}_1) \right] \)

\( A^{(j)} = \left[ A_1^{(j)}, A_2^{(j)}, \ldots, A_{2N+1}^{(j)} \right]^T, \quad k = 1, M, \)

\( B^{(j)} = \left[ B_1^{(j)}, B_2^{(j)}, \ldots, B_{2N+1}^{(j)} \right]^T, \quad k = 1, M, \)

\( Q_1 = X^{(j)} B^{(j)} \quad (X^{(j)} = X)^{0}, \)

\( Q_2 = -X^{(j)}B^{(j)} \)

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\( D^{(j)} = \left[ h^{(j)} \right], \quad \mathbf{H} = \left[ h^{(j)} \right], \quad l = 1, M, \)

\( \nu = 1 - (2N + 1) \)

\( \nu = 1 - (2N + 1) \)
We obtain the relationship between $A^{(1)}$, $B^{(1)}$ in the first layer and $A^{(M)}$, $B^{(M)}$ in the end of unit layer using boundary condition at $x = -l d_{n}$ ($n = 1 \sim M - 1$) \[ \begin{align*} H_{r}^{(2,j)} &= H_{r}^{(2,j)}_{n = 1} \quad (\text{for } j = 1, 2, \ldots, M) \] \[ E_{z}^{(2,j)} = E_{z}^{(2,j)}_{n = 1} \quad (\text{for } j = 1, 2, \ldots, M) \]}

\[ \left( \begin{array}{c} A^{(1)} \\ B^{(1)} \end{array} \right) = \left( \begin{array}{cc} S_{1}^{(1)} & S_{2}^{(1)} \\ S_{3}^{(1)} & S_{4}^{(1)} \end{array} \right) \cdots \left( \begin{array}{cc} S_{1}^{(M)} & S_{2}^{(M)} \\ S_{3}^{(M)} & S_{4}^{(M)} \end{array} \right) \left( \begin{array}{c} A^{(M)} \\ B^{(M)} \end{array} \right) \] \[ \left( \begin{array}{c} A^{(1)} \\ B^{(1)} \end{array} \right) = \left( \begin{array}{c} S_{1} \\ S_{2} \end{array} \right) \left( \begin{array}{c} A^{(M)} \\ B^{(M)} \end{array} \right) \]
where $S^{(1)} \triangleq \left[ \begin{array}{c} (i)^{(a)} \\ (b) \end{array} \right]$, $k = 1 \sim 4, l = 1 \sim M$

$S^{(2)} \triangleq \frac{1}{2} \left[ (i)^{(a)} + (b)^{(a)} \right], (i)^{(a)} = (i)^{(a)} - (i)^{(b)} e^{-i(\zeta_{x} \cdot \xi_{x} + \zeta_{y} \cdot \xi_{y})/2}, (b)^{(a)} = (b)^{(a)} - (b)^{(b)} e^{-i(\zeta_{x} \cdot \xi_{x} + \zeta_{y} \cdot \xi_{y})/2}$,

$S^{(2)} \triangleq \frac{1}{2} \left[ (i)^{(a)} + (b)^{(a)} \right] / 2, (i)^{(b)} \triangleq \left[ (i)^{(a)} + (b)^{(a)} \right] e^{-i(\zeta_{x} \cdot \xi_{x} + \zeta_{y} \cdot \xi_{y})/2}$,

$\mathbf{V} \triangleq [\psi_{w}^{
u}] = [\mathbf{U}_{w}^{\nu}] [\mathbf{U}^{\nu}]$, $\mathbf{K} \triangleq [\psi_{w}^{
u}] = [\mathbf{U}_{w}^{\nu}] [\mathbf{U}^{\nu}]$

$\phi^{(a)} \triangleq [\phi^{(a)}]_{w}, \phi^{(b)} \triangleq \sum_{n=1}^{N} \phi^{(a)}_{n}, \Psi^{(a)} \triangleq [\psi^{(a)}]_{w}$

By using matrix relationship between Eq.(8), Eq.(9) and Figures 4(a) and 4(b) show $\mathbf{V}$ for various values of normalized frequency ($\nu$) with the same parameters as in Fig.3. We note that the characteristic tendencies of coupling resonance for the TE and TM waves are significant the Wood’s anomaly at $\nu = 1.2$ and 3.

IV. CONCLUSION

In this paper, we have proposed a new method for the scattering of electromagnetic waves by inhomogeneous dielectric gratings loaded with perfectly conducting strips using the combination of improved Fourier series expansion method and the matrix formulation of point matching method.

Numerical results are given for the transmitted scattered characteristics for the case of frequency loaded with the parallel perfectly conducting strips for TE and TM cases. The effects of the inhomogeneous dielectric gratings comparison with that of the slant angle on the transmitted power are discussed. This method also can be applied to the inhomogeneous dielectric gratings having an arbitrarily periodic structures combination of dielectric and metallic materials.

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