FDTD-SPICE Direct Linking Simulation of Transient Fields Caused by Electrostatic Discharge

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Abstract—This paper presents FDTD-SPICE direct linking simulations of transient electromagnetic fields caused by electrostatic discharges (ESD) in general structures. Especially, the air-discharge ESD simulation is performed by simultaneously combining a full-wave model for structures of analytical objects and SPICE equivalent circuit models based on nonlinear spark resistance formulae. Unlike existing numerical methods, this approach has several advantages: accurate modelling of arbitrary structures, flexible treatment of various spark resistance equations such as Rompe-Weizel’s and Toepler’s formulae and no extraction of impedance for the full-wave model. The advantages of our approach are verified by means of several canonical ESD problems. The FDTD-SPICE direct linking simulation results are compared with those of an analytical approach and two numerical methods, and measurement data from a reference.

I. INTRODUCTION

In recent designs of advanced electronic equipment, the electrostatic discharge (ESD) events are one of the most important issues [1]. A practical way of calculating transient electromagnetic (EM) fields caused by ESD in an electronic system is to use numerical methods. Simulation techniques of ESD-induced EM field can roughly be categorized as follows:

- Equivalent circuit analysis [2]
- Full-wave analysis [3]-[9]
- Sequential linking analysis [13]

The equivalent circuit analysis [2] is performed by Simulation Program with Integrated Circuit Emphasis (SPICE) and discharge current is calculated from combined equivalent circuit of discharging objects with a nonlinear spark model. Transient EM fields in far region are then estimated by using a current dipole model having the obtained discharge current as a source. The conventional finite-difference time-domain (FDTD) method [10] has been successfully applied to full-wave simulations of an ESD generator in the contact-discharge mode [3]-[6] and also to system-level contact-mode ESD simulations [7]. Full-wave schemes [8],[9] specialized for the Rompe-Weizel nonlinear spark resistance formula [11] were presented for transient analysis of EM fields due to air-discharge ESD. However, no full-wave schemes specialized for other resistance formulae (e.g. Toepler’s formula [12]) are yet published. Recently, a sequential linking analysis [13] based on using full-wave model and equivalent circuit model was presented as an extension of the equivalent circuit approach [2]. Although full-wave simulations of air-discharge mode are demonstrated for the ESD generator and a small product, this approach requires complicated procedures: extraction of impedance at the position of a spark channel placed in a full-wave model, reimport of discharge current obtained in SPICE simulation and so on.

The main purpose of the paper is to apply a FDTD-SPICE direct linking method [10],[14] into transient analysis of EM fields caused by air-discharges in general structures. Unlike existing numerical methods, our approach has several advantages in the ESD-field analysis: accurate modelling of arbitrary structures, flexible treatment of various spark resistance equations such as Rompe-Weizel’s and Toepler’s formulae and no extraction of impedance for the full-wave model. The proposed approach is verified for some canonical ESD-field problems.

II. METHODOLOGY

In order to describe the dynamics of EM fields caused by air-discharges, an analytical model is separated into three models: a linear full-wave model including the ESD generator and the electronic equipment (e.g., printed circuit board with passive circuit elements and the enclosure parts), a nonlinear spark model and an complicated circuit model of nonlinear lumped network such as varistor and the LSI circuit. A FDTD-based full-wave solver computes the linear model and a SPICE-like solver calculates the nonlinear parts. The nonlinear spark model is then expressed as an equivalent circuit within the SPICE netlist. The above models can be simultaneously simulated in time domain with the FDTD-SPICE direct linking method [14]. The used method is briefly introduced here.

In this work, most simulation results are obtained with the commercial software Pointrying for Microwave [15] based on the FDTD-SPICE direct linking method. The software has been developed by our R&D group.

A. FDTD-SPICE Direct Linking Method

A numerical scheme of the FDTD-SPICE direct linking method is illustrated in Fig.1. The two-terminal port of an embedded circuit model is allocated on the edge (length D) of a single Yee cell in the discrete space as shown in the upper side of Fig.1. In the standard leapfrog scheme, the electric and
magnetic field components \((E, H)\) are computed alternately at every half time step on the time axis. One cycle procedure from magnetic field updating to electric field updating is explained below.

In this work, coupling of SPICE with FDTD is based on the use of the integral form of Ampere’s law as follows:

\[
\frac{\partial}{\partial t} \int_A \mathbf{E} \cdot dA + \int_A \mathbf{J}(\mathbf{E}) \cdot dA = \oint_C \mathbf{H} \cdot d\Gamma
\]

(1)

where the contour \(\Gamma\) bounds the cross section \(A\) of the cell where the port is embedded. In order to solve (1) in a SPICE solver, (1) can be rewritten in a simpler description as

\[
C_0 \frac{\partial V}{\partial t} + I(V) = I_N,
\]

(2)

where \(C_0 = \varepsilon A / d\) is the so-called FDTD cell capacitance, \(V\) is the voltage calculated from the line integral of \(E\) on the cell edge, the second term \(I(V)\) in the left-hand side of (2) represents the current flowing through the embedded circuit model and the right-hand side can be calculated from

\[
I_N(t) = \oint_A H \cdot d\Gamma.
\]

Note that (2) can be illustrated as in the lower side of Fig.1 and the entire circuit model can be solved by the SPICE.

The numerical procedure is summarized in the following: First, all magnetic field components in the computational domain are calculated using the conventional FDTD scheme. Then, the current \(I\) flowing on the objective circuit element is calculated from the four adjacent magnetic field components which loop around the electric field component \(E\) on the edge. The calculated current is set up as a current source connecting with the cell capacitance and the circuit model. Next, a circuit simulation is performed and the electric field value \(E\) allocated on the cell edge is calculated from the obtained terminal voltage \(V\) divided by the edge length \(d\). Finally, all other components of electric field in the domain are calculated according to the conventional FDTD scheme.

**B. Modelling of Spark Channel**

**FDTD Solver**

\[
I = \Delta A, \quad E\quad H, \quad d \quad \downarrow \\
I = \oint A H \cdot d\Gamma, \quad E = V / d
\]

**SPICE Solver**

\[
I_N = \oint A H \cdot d\Gamma, \quad E = V / d
\]

Fig. 1. FDTD–SPICE direct linking method

In order to avoid the detailed analysis of the physical process of air-discharge and to characterize the relation between discharge voltage and current, the spark channel is modelled by nonlinear time-varying resistor. This can be regarded as a phenomenological model.

So far, many kinds of spark resistance formulae derived theoretically or experimentally were proposed [16]. For example, Toeper’s (TP) and Rompe-Weizel’s (RW) formulae for the spark resistance are often used in a lot of numerical modelling [8]-[9,13] and experimental studies [17]-[19] of ESD events. The Rompe-Weizel’s formula is expressed as follows:

\[
r_{RW}(t) = \frac{l}{\sqrt{(2\alpha_R/p)}} \int_{-\infty}^{0} i^2(t')dt',
\]

(1)

where \(r(t)\) is the spark resistance at time \(t\), \(l\) is the spark length, \(a_R\) is a parameter derived from basic ionization processes, \(i\) is the spark current and \(p\) is the gas pressure. The Toeper’s formula is expressed as

\[
r_{TP}(t) = \frac{K_T I}{\int_{-\infty}^{0} i(t')dt'},
\]

(2)

where \(K_T\) is the Toeper’s constant.

Note that (1) and (2) can be expressed as an equivalent circuit consisting of voltage-controlled current source, integrator, current-controlled current source and so on. Therefore, for full-wave simulation of transient EM fields caused by air discharges, the FDTD-SPICE direct linking method allows us to treat a lot of spark resistance formulae as equivalent circuits.

**C. Charging and Discharging Processes**

After setting up the linear and nonlinear models, we can perform a time domain full-wave simulation. In our approach, the time domain simulation is separated into charging and discharging processes. The aim of the charging process is to know the electrostatic field distribution in the computational domain as initial condition [8],[9] of discharging process.

**III. NUMERICAL EXAMPLES**

In order to validate the proposed method, we show several application examples for canonical ESD problems. Numerical results of our approach are compared with those of analytical and other numerical methods, and also a measurement data [9].

**A. Comparison with Analytical Solutions**

In order to validate the FDTD-SPICE direct linking simulations, we consider a simple current dipole as shown in Fig.2. The analytical solutions of EM fields caused by the dipole are well-known for different spark resistance formulae [20]. The dipole model [18] is commonly used to estimate the fundamental properties of transient ESD fields. Here we treat the Rompe-Weizel’s (RW) and Toeper’s (TP) formulae. The model parameters are same as those described in [21]. Fig. 3, 4 shows comparison of the discharge current and transient magnetic fields calculated by two approaches. We can find
good agreements between the FDTD-SPICE and the analytical results for the different resistance formulae.

B. Comparison with a Specialized FDTD Scheme

Here we deal with a more realistic model including a spark channel and metal objects as in Fig. 5. Fig. 6 shows transient magnetic field caused by air discharge generated in vicinity of two metal spheres (radius $R=25\text{mm}$) of perfectly electric conductor (PEC) calculated with the FDTD-SPICE direct linking, and also with the single FDTD algorithm specialized for the Rompe-Weizel formula for comparison. A spark channel is built between the two metal spheres (spark voltage $V_s=13.6\text{kV}$, gap length $g=2\text{mm}$, $a_R=1.1 \times 10^{-4}\text{m}^2/V^2$). The calculations are performed with 2mm cell size and the number of cells is $14,070,000$ ($=350 \times 200 \times 201$). The numerical example shows excellent agreement between the FDTD-SPICE and the RW-specialized FDTD results.

The above calculations are done on a PC workstation (Intel Core™ i7 2920XM 2.5GHz, 16GB memory and Windows 7 64bit). The required memories are about 2GB. The calculation times are 26m3s for the FDTD-SPICE and 25m58s for the RW-specialized FDTD. We can see that the increase of calculation time due to use of SPICE solver is small.

C. Comparison with Sequential Linking Method
Fig. 7 shows the comparison of transient magnetic fields calculated with two different FDTD-SPICE linking methods: the direct linking method (simultaneous method) proposed in this work and the sequential linking method presented in [13]. The model geometry and parameters are same as those of the previous section. We can find very good agreement between both linking methods. In this simulation, the sequential method requires the extraction of impedance at the position of spark channel placed in the full-wave model, the SPICE simulation for discharge current and the EM field calculation from a full-wave model with the reimported discharge current. By contrast, the proposed simultaneous method does not need their complicated procedures and only a time domain simulation is required.

D. Comparison with Measurement Data

Finally, we compare the FDTD-SPICE result with the measurement data given in [9]. Fig. 8(a) shows a numerical model consisting of a spheroid (PEC), a spark channel characterized by the Rompe-Weizel formula and a ground plane (PEC). The calculation parameters are: \( V_i = 5 \text{KV}, a = 5 \text{cm}, b = 31 \text{cm}, h = 0.323 \text{mm}, \alpha = 2.5 \times 10^{-4} \text{m}^{2}/\text{V}s \). The number of cells is 6,183,056 (=172x172x209). Fig. 8(b) shows the comparison of the FDTD-SPICE direct linking result with the measurement data. We can see good agreement of the FDTD-SPICE result with the measurement data and the validity of the FDTD-SPICE direct linking method is confirmed.

IV. CONCLUSION

Numerical simulations of transient electromagnetic field caused by air discharge with the FDTD-SPICE direct linking method have been presented. Application of the presented direct linking method to air-discharge ESD-field problems has been in detail discussed. Comparison of the presented method with the analytical solutions of current dipole model, other numerical methods and measured data has been demonstrated for several canonical problems. Good agreements have been demonstrated in the numerical examples. The numerical results show that the proposed method allows for simple and efficient simulations of transient field caused by spark channel characterized by different spark resistance formulae in arbitrary structures.

Fig. 8. Comparison of FDTD-SPICE result and measurement data for spark discharge model of metal spheroid and ground plane.

REFERENCES