Electromagnetic Field Distribution in Areas surrounded by Many Wires

Hiroshi ECHIGO † and Kazuo AIZAWA
†Tohoku Gakuin University
echigo@tjcc.tohoku-gakuin.ac.jp

Abstract - In this paper, shielding effects of some enclosures composed with thin wires (wire arrays) are studied. For examples, the thin wire arrays lined on a circle making a tube are analyzed to evaluate the shielding effectiveness as well as the field distributions in and around the structure. The near fields formed by the wire system that includes an active wire or a wire driven by an external power source are shown.

Keywords - EM Scattering; Shielding Effectiveness; Conductive Wire Array

I. INTRODUCTION

Electromagnetic shielding is one of the important subjects in EMC technologies. Authors have tried to make shielding effect using many thin metal wires. To analyze the shielding effectiveness, they applied scattering theories referring to the results shown in some articles. [1][2][3]

Especially, authors focused their interests to the near fields formed by the scattering from the metal wires and showed some results that the desired fields were achieved by the metal wires aligned in proper positions.

Ordinarily, solid metal boxes are used for the electronic circuits to be kept out the EM (electromagnetic) fields. The other hand, our proposed way using many wires has advantages as air conditioning or cooling and transparency as well as the lightweight. Some parameters as like the total wire number, the separation between wires, and the diameter of wires can be used to tune the shielding effectiveness selecting the performances for any cases.

To confirm their proposals, numerical calculations were achieved for some cases and the results were estimated. The results are shown as the field patterns to give the insights through visualization for the shielding effectiveness.

In this paper, scattering of TM electromagnetic plane waves is analyzed and the electromagnetic coupling between electric currents on the many thin wires is formulated to lead the impedance matrix governing the system. By using this matrix, electromagnetic fields surrounding the wire system can be estimated when specific wires are selected as the driving source-currents or the active wires.

II. SCATTERING BY A CYLINDRICAL CONDUCTOR

Suppose that the longitudinal axis of the conductive cylinder is coincident with z-axis of Cartesian coordinate as shown in

Fig. 1. When the incident TM (Transverse Magnetic) wave propagating along the x-axis is given as,

\[ E_x = E_0 e^{-j k x} \]  

where \( E_0 \) is the amplitude of the incident wave, \( k \) is the propagation constant, \( J_n \) is the Bessel function of order \( n \), \( a \) is the radius of the conductor and \( H_n^{(2)}(ka) \) is the 2nd kind of Hankel function of order \( n \). Under the assumption that the wire diameter is sufficiently small, the formula is reduce to

\[ E_x = -\frac{k Z_0}{4} I_n H_n^{(2)}(k \rho) \]  

where \( Z_0 \) is the wave impedance of the free space and \( \rho \) is the distance between the conductor axis and the observation point.

Because the boundary condition on the surface of the
cylinder forces \( E_z^i + E_z^s = 0 \), the equation relating the incident wave and the induced current is given as (3).

\[
E_z^i = \frac{kZ_0}{4} I_z H_0^{(2)}(ka) \tag{3}
\]

This formula gives the way to estimate the current \( I_z \) on a thin conductive wire when illuminated by TM plane wave of \( E_z \).

\[
I_z = \frac{4}{kZ_0} \frac{E_0 e^{-ikz}}{H_0^{(2)}(ka)} \tag{4}
\]

III. FORMULATION FOR THE MULTI-WIRE SYSTEM

Considering the scattering by multi-thin-conductor system of \( N \) wires with the same radius \( a \) as in Fig. 2, the induced current on the \( n \)-th wire \( I_n \) causes the scattering as (5) according to (2),

\[
E_n^s = \frac{-kZ_0}{4} I_n H_0^{(2)}(\rho_n) \tag{5}
\]

where \( \rho_n \) is the distance from the \( n \)-th conductor to the observation point.

All \( N \) wires form the scattering EM wave \( E_z^s \) as (6).

\[
E_z^s = \frac{-kZ_0}{4} \sum_{n=1}^{N} I_n H_0^{(2)}(\rho_n) \tag{6}
\]

Since the electric field is zero on the surface of \( m \)-th conductor, the induced currents on the wires \( I_n \) and incident electric field \( E_m \) at the position of \( m \)-th wire are related as (7).

\[
\frac{kZ_0}{4} \sum_{n=1}^{N} I_n H_0^{(2)}(\rho_m) = E_m \tag{7}
\]

where \( \rho_m \) is separation between the \( m \)-th and the \( n \)-th wires when \( m \neq n \), is taken as the wire radius \( a \) when \( m = n \).

This relation gives the linear system as described by the following matrix expression of (8).

\[
\{E_m\} = [Z] \{I_n\} \tag{8}
\]

where

\[
Z_{mn} = \frac{kZ_0}{4} H_0^{(2)}(k\rho_m) \]

When the electric fields at each wire position are given, the induced currents are determined using the inverse matrix \([Y]\) of \([Z]\). As each current forms the scattering field, the scattering field is determined by summing up these fields. Then the total field is the superposition of the fields of the incident and the scattering waves.

Even if only one wire is illuminated by the incident wave, the currents on the wires are determined by the same way by setting the incident fields on wires to be zeros and nonzero for the wire selected as a source line. Consequently, currents on the other wires except the source one are all induced ones. In this way, the induction currents on the wires are determined when only one wire has the driving or source current. If the \( q \)-th wire is the source current, the situation is accomplished by setting only the \( q \)-th element of vector \( \{ V \} \) to be non-zero and others zeros.

After the calculation of all currents \( \{ I_n \} \), the fields are derived by using (9).

\[
E_z^s = -\frac{kZ_0}{4} \sum_{n=1}^{N} I_n H_0^{(2)}(\rho_n) \tag{9}
\]

IV. E-FIELD DISTRIBUTION CAUSED BY A SINGLE DRIVEN WIRE SURROUNDED BY MULTI-WIRES

In this analysis, the wire system has multiple wires those are parallel each other, perpendicular to the x-y plane and extending from minus infinity to plus infinity in the z direction. Therefore the problems treated here are all 2-dimensional problems. For the example shown in Fig.2, the driven wire (or active wire) is located on the origin i.e. just along the z-axis. Some wires with their same radius \( a \) located on a circle of its radius \( r \) surround the active line as shown in the figure showing the case of the 12 surrounding wires. For this calculation, the radius of the circle \( r \) was set to be 0.01[m] while the wire radius \( a \) is \( r/100 \). The driven current on the source or active line is assumed to be 1[A].
The calculated E-field distributions are shown by the blue trace in Fig.3 under the frequency condition of $k \cdot d = \pi$, where $d=2$ r is the diameter of the array circle, $k$ is wave number given as $\frac{2\pi}{\lambda}$, and $\lambda$ is the wave length. For the reference, the E-field around only a wire at the origin with $I[A]$ is also shown by the red trace in the figure.

The results together with for other parameters give us some information about this structure as follows,

1. the strength of the induced electric fields differs each other depending on the parameters as wavelengths and the radii of the array circle,
2. the leakage or outer fields are less than the inner field about by 20dB~30dB for the lower frequency,
3. radiation from this system would decrease rapidly in distance if the wavelength increases i.e. the frequency becomes lower.

\[
Z_{wire} = \frac{k Z_{wire}}{4} H_{1}^{(2)}(k a) \tag{10}
\]

In the case of wire material with finite conductivities, if $Z_{wire}$ is given by other routines \[7\], the multi wire system would be calculated in the same way by replacing the diagonal component (10) of $Z_{nm}$ by this $Z_{wire}$.

One example is given in Fig.5 where 101 wires with conductivity:5.0x10\(^{7}\) [S/m], relative permittivity:10, radius:0.0001[m] are located on the half circle in positive x region. A single source line is set at the origin of x-y plane.

\[\begin{align*}
\frac{\text{Reduction of E-Field in dB}}{\text{Distance [m] on x-axis from the current source}}
\end{align*}\]

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\end{align*}\]

**Fig. 5 Shielding effects by conductive wires on the half circle. The trace shows the reduction (dB) of E-field strength on x axis.**

\[\begin{align*}
\frac{\text{Reduction of E-Field in dB}}{\text{Distance [m] on x-axis from the current source}}
\end{align*}\]

**Fig. 4 Electric field strength curves on the x-axis in the range of x:-2 r ~ +2 r.
(r: radius of array circle, frequency:7.4948[GHz] <k r = Pi/2>)**

\[\begin{align*}
\frac{\text{Reduction of E-Field in dB}}{\text{Distance [m] on x-axis from the current source}}
\end{align*}\]

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\frac{\text{Reduction of E-Field in dB}}{\text{Distance [m] on x-axis from the current source}}
\end{align*}\]

V. **WIRE SYSTEM WITH FINITE CONDUCTIVITY**

The induced current $I$ on a wire of perfect conductor is given by (4) when illuminated by TM plane wave of $E_{r}$. The relation is described by using “Wire Impedance” that appeared on the diagonal of the matrix $Z_{nm}$ in (8).

\[\begin{align*}
Z_{wire} = \frac{k Z_{wire}}{4} H_{1}^{(2)}(k a)
\end{align*}\]

VI. **DISPERSED WIRE SYSTEM**

In some cases of the multi layered PCB (printed circuit boards), the upper ground plate and the lower one have to be the same potential. To accomplish this requirement, many thru-Vias are used to connect the layers. To keep the uniformity of the potential level, the Vias are dispersed on the whole board. To analyze the field distribution between the layers or plates in this kind of systems, the calculation procedure developed here as mentioned above can be used.

To have the insights of the field distributions, a special structure was assumed, in which the wires or Vias were positioned at the nodes of the hexagonal array pattern as shown in Fig.6.

50 wires were positioned at nodes of hexagonal array pattern. The spacing of wires is $d=0.01$m. The active wire is located at the origin. When this active wire was driven to flow the current to simulate a certain signal Via to connect between layers. This current can induce the EM field between board plates. The calculated result is given in Fig.6. As predictions, strong field appeared around the driven wire. It is interesting that another concentrated fields appeared in the some regions of the hexagonal pattern far from the origin. This phenomenon may come from the resonance of the space surrounded by 6 wires at the node of a hexagon. In the real PCB with many...
Vias diversely on the board, it would be suspicious to make this kind of resonance. This might be another cause of unwanted interferences.

**VII. CONCLUSIVE REMARKS**

The authors have studied the scattering problems of TM plane waves from the conducting wires arrayed in parallel to z-axis. This problem is equivalent to 2-dimensional EM-field analyses as like in the layers of PCB(printed circuit board) with Vias.

In this paper, authors tried to analyze the shielding effectiveness of the surrounding wire system for an active wire (externally driven one) to flow currents. Only 12 wires can achieve about 20dB reduction of E-field in the lower frequencies. The effectiveness increases depending on the wavelength.

However, for the higher frequency, some resonance might happen in the space surrounded by wires. Actually, the resonance emerged especially when the number of the surrounding wires was increased to make a compact shielding wall.

When the surrounding wires or Vias were dispersed on the board to connect between two ground planes of a multi-layered PCB, the E-field in the layer was analyzed. The result showed that another resonance would occur in the spaces between Vias if the wavelength of the signal becomes to have some relations to the structure parameters. Attentions have to be paid to this phenomenon.

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The application software (Mathematica of Wolfram Research) was used to conduct all calculations and generate drawings of the results.

**REFERENCES**


