Inverse Scattering Problem of a Two-Dimensional Dielectric Cylinder in Slab Medium

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Abstract — In this paper, we apply the dynamic differential evolution (DDE) algorithm to solve the inverse scattering problem for which a two-dimensional (2-D) dielectric cylinder is buried in a dielectric slab medium. Finite difference time domain method (FDTD) is used to solve the scattering electromagnetic wave of a dielectric cylinder. The inverse problem is resolved by an optimization approach and the global searching scheme DDE is then employed to search the parameter space. Numerical simulation shows the quality of the reconstructed result obtained by the DDE algorithm is very good.

Index Terms — Inverse Scattering, Time Domain, Dynamic Differential Evolution.

I. INTRODUCTION

The detection and reconstruction of buried and inaccessible scatterers by inverting microwave electromagnetic measurements is a research field of considerable interest because of numerous applications in geophysical prospecting, me, biomedical imaging and nondestructive testing. Numerical inverse scattering studies found in the literature are based on either frequency or time domain approaches [1]-[5].

Most of the proposed inversion techniques for the inverse problems are formulated in the frequency domain [6]-[7]. However, the time domain scheme is a potential alternative for the inverse problems because the time domain data contain more information about scatterer than those in the scattered data of single frequency. Therefore, various time domain inversion approaches are proposed intensively in recent decade years that could be briefly classified as the iterative approach: Distorted Born iterative method (DBIM) [8], and optimization approach [9]-[10].

In this paper, the computational methods combining the FDTD method and the DDE algorithm are presented. The forward problem is solved by the FDTD method, for which the subgridding technique [11] is implemented to closely describe the fine structure of the cylinder. The inverse problem is formulated into an optimization one, and then the global searching scheme DDE is used to search the parameter space. Cubic spline interpolation technique [12] is employed to reduce the number of parameters needed to closely describe a cylinder of arbitrary shape as compared to the Fourier series expansion.

II. DIRECT PROBLEM

Let us consider a two-dimensional three-layer structure with buried dielectric cylinder as shown in Figure. 1. The dielectric cylinder is illuminated by a line source with Gaussian pulse placed at two different positions sequentially denoted by Tx in the first layer, and then the scattered $E$ fields are recorded at those points denoted simultaneously by Rx in the same layer. The computational domain is discretized by using Yee cells. It should be mentioned that the computational domain is surrounded by optimized absorber of the perfect matching layer (PML) to reduce the reflection from the environment PML interface.

The direct scattering problem is to calculate the scattered electric fields while the shape and location of the scatterer is given. The shape function $F(\theta)$ of the scatterer is described by the trigonometric series in the direct scattering problem

$$F(\theta) = \sum_{n=0}^{N} B_n \cos(n \theta) + \sum_{n=1}^{N} C_n \sin(n \theta)$$

where $B_n$ and $C_n$ are real coefficients to expand the shape function.In order to closely describe the shape of the cylinder for both the forward and inverse scattering procedure, the subgridding technique is implemented in the FDTD code. For the time domain scattering and/or inverse scattering problem, the scatterers are assigned with the fine region such that the fine structure can be easily described. If higher resolution is needed, only the fine region needs to be rescaled using a higher ratio for subgridding. This can avoid griding the whole problem space using the finest resolution such that the computational resources are utilized in a more efficient way.
which is quite important for the computationally intensive inverse scattering problems. More detail on the FDTD-Subgridding scheme can be found in [2].

Fig. 1. Geometry of the problem in (x,y) plane

III. INVERSE PROBLEM

For the inverse scattering problem, the shape and location of the metallic cylinder are reconstructed by the given scattered electric field measured at the receivers. This problem is resolved by an optimization approach, for which the global searching scheme DDE is employed to minimize the following cost function (CF):

\[
CF = \frac{1}{N_c} \sum_{i=1}^{N_c} \left| E_{cal}^{i} (n, m, b \Delta t) - E_{exp}^{i} (n, m, b \Delta t) \right|
\]

where \( E_{cal}^{i} \) is the electric field data to mimic the measurement data in the forward procedure and \( E_{exp}^{i} \) is the calculated electric fields in the inversion procedure, respectively. The \( N_c \) and \( M \) are the total number of the transmitters and receivers, respectively. \( B \) is the total time step number of the recorded electric fields.

Dynamic differential evolution starts with an initial population of potential solutions that is composed by a group of randomly generated individuals which represents the center position \( Y, \) the radius \( r \) and the population size \( N_p \). The dielectric cylinder is buried in lossless slab medium (\( \sigma_1 = \sigma_2 = \sigma_3 = 0 \)). The transmitters and receivers are placed in free space above the homogeneous dielectric. The permittivities in region 1, region 2 and region 3 are characterized by \( \varepsilon_1 = \varepsilon_0 \), \( \varepsilon_2 = 8\varepsilon_0 \) and \( \varepsilon_3 = \varepsilon_0 \), respectively, while the permeability \( \mu_0 \) is used for each region, i.e., only non-magnetic media are concerned here. Two examples are investigated for the inverse scattering of the proposed structure by using the DDE algorithm. There are twelve unknown parameters to retrieve, which include the center position \( X_0, Y_0 \), the radius \( \rho_i \), \( i = 1, 2, \cdots, 8 \) of the shape function and the slope \( \rho_N^i \) plus the relative permittivity of the object, \( \varepsilon_i = \varepsilon_i / \varepsilon_0 \). Very wide searching ranges are set for the DDE to optimize the cost function given by (2). The parameters and the corresponding searching ranges are listed as follows: \( -119 \text{mm} \leq X_0 \leq 119 \text{mm} \), \( -47.6 \text{mm} \leq Y_0 \leq 47.6 \text{mm} \), \( 0 \text{mm} \leq \rho_i \leq 71.4 \text{mm} \), \( i = 1, 2, \cdots, 8 \), \( -1 \leq \rho_N^i \leq 1 \) and \( 1 \leq \varepsilon_i \leq 16 \). The operational coefficients for the DDE algorithm are set as below: The crossover rate \( CR=0.8 \). The scaling factors \( \xi = 0.8 \), \( \chi = 0.5 \) and the population size \( N_p = 110 \). Here, the r.m.s. error \( (\text{ERRs}) \) of the reconstructed shape \( F_{cal}^{cal}(\theta) \) and the relative error \( (\text{ERRe}) \) of \( E_{cal}^{cal} \) are defined as

\[
\text{ERRs} = \frac{1}{N_c} \sum_{i=1}^{N_c} \left[ \frac{F_{cal}^{cal}(\theta_i) - F(\theta_i)}{F(\theta_i)} \right]^2 \quad (4)
\]

\[
\text{ERRe} = \left| \frac{F_{cal}^{cal} - E_{cal}}{E_{cal}} \right| \quad (5)
\]

where the \( N^c \) is set to 720.

For the example, the shape function of this object is given by \( F(\theta) = 23.8 + 5.95 \cos(\theta) + 11.9 \cos(2\theta) \) mm and the relative permittivity of the object is \( \varepsilon_r = 3.0 \). The final reconstructed shape at the 300th generation is compared to the exact shape in Figure 2. The r.m.s. error \( (\text{ERRs}) \) of the reconstructed shape \( F_{cal}^{cal}(\theta) \) and the relative error \( (\text{ERRe}) \)

\( E_{cal}^{cal} \) with respect to the corresponding exact values versus generations are shown in Fig. 3. The results show that the
DDE scheme is able to achieve good convergences within 100 generations. The r.m.s. error ERRs is about 2.7% and $\varepsilon_r^{\text{ref}} = 2.9858$ (ERRe=0.473%) in the final generation.

![Diagram of reconstructed shape](image1)

Fig. 2. The reconstructed shape of the cylinder for example.

![Diagram of error versus generation](image2)

Fig. 3. Shape error (ERRs) and relative permittivity errors (ERRe) versus generation for example.

V. CONCLUSIONS

We have presented a study of applying the DDE algorithm to reconstruct the shape of a dielectric cylinder through knowledge of scattered fields under time domain. In order to describe the shape of the scatterer efficiently, the technique of cubic spline interpolation is utilized. The inverse scattering problem is formulated into an optimization problem, and then the global searching scheme DDE is employed to search the parameter space. Simulated result shows the good reconstruction via the use of FDTD method and the DDE algorithm can be obtained.

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