Scattering Analysis of the Microstrip Array Antenna by Using the PMCHWT-CBFM

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Abstract—The PMCHWT-CBFM (Characteristic Basis Function Method) is one of the effective numerical method for radar cross section (RCS) analysis of the electric large scatterer in terms of memory consumption. We analyzed and measured the monostatic RCS of 64-element microstrip array antenna considered as the terminal condition by using the above method. By comparing the conventional PMCHWT and experimental results with the PMCHWT-CBFM results, they agreed well with each other.

Keywords—characteristic basis function method; PMCHWT; radar cross section; array antenna

I. INTRODUCTION

In recent years, the characteristic basis function method (CBFM) is proposed to overcome the computer memory burden problem of the method of moments (MoM) [1-3]. In the CBFM, the scatterer is discretized into multiple spatial region (block). The characteristic basis function (CBF) is the macro basis function which represents the current distribution in each block. The impedance matrix size by using the macro basis function becomes smaller than the matrix of the principle basis function; hence the usage of the memory can be drastically reduced.

In addition, the CBFM is based on the direct method (e.g., LU decomposition). It indicates that the computational time is unchanged by values of the matrix elements in contrast to the iterative method.

In this paper, we verified the CBFM by applying PMCHWT (Poggio, Miller, Chang, Harrington, Wu and Tsai)[4], i.e., PMCHWT-CBFM[5] for analyzing the radar cross section (RCS) of the array antenna. In section II, we describe of the PMCHWT-CBFM formulation. In section III, we present the results of the RCS analysis of a 64-element microstrip array antenna. Finally, we conclude this paper in Section IV.

II. PMCHWT-CBFM FORMULATION

A. Overview[1-3]

The CBFM involves following 3 processes; the calculation of the primary and the secondary CBF, orthogonalization of the CBFs, and compression of the impedance matrix. In subsection B and C, we express the processes, and we represent the algorithm of PMCHWT-CBFM in subsection D.

B. Calculation of Primary and Secondary CBF

Firstly, the primary CBF $\mathbf{J}_p$ in $i$th block is calculated. The primary CBF is the macro basis function for expressing the current generated by the self-interaction in each block. Here, it is important that the secondary CBF is calculated from the primary CBF. Therefore, the secondary CBF also becomes inappropriate if the primary CBF has no appropriate value. From the above reason, the correct calculation of primary CBF is very important in this method.

Fig. 1. Definition of Block and Extended Block

$\mathbf{J}_p$ in $i$th block is calculated independently of other blocks. However, in actual physical structure, the current does not...
break off at end of the block. This breaking off will generate unnecessary edge effects. To remove that and to obtain accurate \( J_i \), we define the extended block whose length on a side is extended \( \Delta r \) from the original block length (Fig. 1). A number of the segments in \( i \)th extended block is \( N_{ie} \). In hereafter, the super script \( eb \) represents the variable which is for the extended block. The primary CBF \( J_{ii}^{eb} \) in \( i \)th extended block is obtained by solving the equation as follows:

\[
Z_{ii}^{eb} J_{ii}^{eb} = V_{i}^{eb} \quad (i = 1, 2, \cdots, M)
\]

Here, \( Z_{ii}^{eb} \) is the impedance matrix with the size \( N_{i}^{eb} \times N_{i}^{eb} \). \( M \) is the number of block. \( V_{i}^{eb} \) is the incident voltage matrix consisting of two orthogonal polarization wave vectors. If the voltage is restricted to a specific vector of a incident angle and a polarization, the impedance matrix made from CBF will be dependent on the direction and polarization. A number of \( \theta \) and \( \phi \) direction vectors is defined by \( N_{\theta}, N_{\phi} \), respectively. They are located in a line along the column direction; hence, the size of the matrix \( V_{i}^{eb} \) becomes \( N_{i}^{eb} \times 2N_{\theta}N_{\phi} \).

The \( J_{ii}^{eb} \) becomes the matrix of the size \( N_{i}^{eb} \times 2N_{\theta}N_{\phi} \). It requires stored memory more than that of a incidence. However, it is not necessary to calculate for every direction; hence, the total computation time is reduced than the calculation with each change in incidence and polarization. \( J_{ii}^{eb} \) is the matrix excluded the segment values which belong only to \( i \)th extended block from \( J_{ii}^{eb} \). After the calculation of primary CBFs, the secondary CBF \( J_{ik}^{eb} \) \((i \neq k)\) is calculated. \( J_{ik}^{eb} \) represents the current generated by the mutual coupling between the blocks. Secondary CBF \( J_{ik}^{eb} \) in \( i \)th extended block is obtained by solving the equation as follows:

\[
Z_{ik}^{eb} J_{ik}^{eb} = V_{ik}^{eb}
\]

\[
(k = i, \cdots, i-1, i+1, \cdots, M)
\]

where

\[
V_{ik}^{eb} = -Z_{ik}^{eb} J_{kk}^{eb}.
\]

As same as primary CBF calculation, \( J_{ik}^{eb} \) becomes the matrix excluding the elements in connection with the segments which belong only to the extended block from \( J_{ik}^{eb} \).

C. Orthogonalization of the CBFs

The set of CBFs \( J_i \) in \( i \)th block is defined the matrix that the primary and the secondary CBF are aligned in the direction of column. Therefore, the size of \( J_i \) is \( N_i \times 2MgN_{\beta} \). This set includes the CBF vector having only small influence to the calculation of the block current distribution. It becomes the redundancy for the CBF calculation. To remove them and orthogonalize other CBFs, special value decomposition (SVD) is used. The SVD of \( J_i \) represents as follows:

\[
J_i = U \sum V_i^H
\]

where \( U \) and \( V_i^H \) are the unitary matrix \( (U^H U = \mathbf{E}, \mathbf{E} \) is the identity matrix) with the size of \( N_i \times N_i \) and \( 2MN_{\theta}N_{\phi} \times 2MN_{\theta}N_{\phi} \), respectively. They are the set of the orthonormal vectors. \( \Sigma \) is the rectangular diagonal matrix having the singular value of rank \( r_i \). The order of a row of the singular value is

\[
\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{r_i} > 0.
\]

We defined the threshold \( \sigma_i \). If the number of \( \sigma, \) which is larger than \( \sigma_i \), are \( K_i, \) row vectors with the number of \( K_i \) are extracted from the left column of \( U \). We defined that this new set is \( u_{ik} \), \( u_{ik} \) is final macro basis function of each block. Thus, the total number \( K \) of the CBFs in all blocks is

\[
K = \sum_{i=1}^{M} K_i.
\]

The impedance matrix \( Z \) to micro basis function with the size of \( N \times N \) is compressed into \( Z \) of the size of \( \sum K_i \times \sum K_i \). By using the Galerkin method [6], the final matrix equation becomes following equation.

\[
\langle J_{pq} *, Z_{qy}, J_{ij} \rangle = \langle J_{pq} *, V_y, J_{ij} \rangle
\]

If we choose small number for \( M \) compared with \( N \), the size of \( Z \) becomes smaller than that of \( Z \). It enables us to reduce the memory usage.

D. PMCHWT-CBFM[5]

In the PMCHWT, the electric current \( J \) and magnetic current \( M \) of the segments of the number \( N \) are expanded by following sum of products [4].

\[
J(r') = \sum_{n=1}^{N} \alpha_n f_n(r')
\]

\[
M(r') = \sum_{n=1}^{N} \beta_n f_n(r')
\]

Here, \( \alpha_n \) and \( \beta_n \) are the expansion coefficient, \( f_n(r') \) is the basis function (e.g., RWG basis function[7]).
The matrix equation of the PMCHWT represents as follows:

\[
\begin{bmatrix}
Z_{dd}^{EJ} & Z_{dd}^{EM} & Z_{dd}^{Ec} \\
Z_{dd}^{HM} & Z_{dd}^{HJ} & Z_{dd}^{Hc} \\
Z_{cd}^{EJ} & Z_{cd}^{EM} & Z_{cd}^{Ec}
\end{bmatrix}
\begin{bmatrix}
I_d^j \\
I_d^M \\
I_c^j
\end{bmatrix}
= 
\begin{bmatrix}
V_d^j \\
V_d^M \\
V_c^j
\end{bmatrix}
\tag{9}
\]

where subscripts \(d\) and \(c\) indicates that the subset represents the interaction for dielectrics and conductor.

The PMCHWT-CBFM is the numerical method by solving the (9) in each block to obtain the CBF. The block impedance matrix for calculating the CBF is the subset shown in (9). When the CBF in a block is calculated, required memory for impedance matrix is only for the elements in a block as same as conventional CBFM. Therefore, PMCHWT-CBFM also enables us to reduce the memory usage compared with the conventional PMCHWT.

### III. RCS ANALYSIS OF MICRO STRIP ARRAY ANTENNA

To verify the PMCHWT-CBFM, we analyzed the monostatic RCS of 64-element microstrip array antenna by using the method. Furthermore, we compared above the results with conventional PMCHWT and experimental results. The frequency and polarization is 5GHz and \(\phi\), respectively. Figure 2 shows the analysis antenna and definition of coordinate system. The relative permittivity \(\varepsilon_r\) of the substrate is 2.17-j0.001. The average segment length of the radiation conductor is 1.5 mm and the other parts are 5mm.

The analysis frequency is resonance frequency of the antenna. In the operating band, the RCS of the antenna mode changes depended on the terminal conditions\[8, 9\]. We defined the two types of the terminal, 50Ω connection and short connection. The values of the terminals of each connector are added in the diagonal matrix elements of the feeding segments as the lamped parameters.

| Number of blocks: \(M\) | 16 |
| Size of block: \(x\times y\times z\) (mm) | 60×60×20 |
| Extended length: \(\Delta r\) (mm) | 5 |
| Interval of incidence for \(\theta\) and \(\phi\) directions (°) | 10, 20 |
| SVD Threshold: \(\sigma_t\) | 0.0001 |

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<th>Item</th>
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![Fig. 2. Analysis model of a 64-element Microstrip Array Antenna and Coordinate Definition](image)

The analysis frequency is resonance frequency of the antenna. In the operating band, the RCS of the antenna mode changes depended on the terminal conditions\[8, 9\]. We defined the two types of the terminal, 50Ω connection and short connection. The values of the terminals of each connector are

![Fig. 3. Analysis and Experimental Results of Monostatic RCS of 64-Elements Microstrip Array Antenna](image)

(a) 50Ω Termination

(b) Short Termination
Table I shows required parameters for the PMCHWT-CBFM. Incident angle is defined that the interval for $\theta$ angle is $10^\circ$ ($0^\circ \leq \theta \leq 180^\circ$), that for $\phi$ angle is $10^\circ$ ($0^\circ \leq \phi \leq 360^\circ$). Thus, the total number of the incidence direction is 342.

The results of the analysis are shown in Fig. 3. All results are totally in good agreement. Even if the terminal conditions are changed, the calculated RCS patterns can reappear the measurement pattern well.

| TABLE II. NUMBER OF UNKNOWNS AND REQUIRED MAXIMUM MEMORY |
|---------------------------------|----------------|----------------|
| Item                            | Element number | Memory (GB)   |
| PMCHWT                          | $1.45 \times 10^9$ | 232           |
| PMCHWT-CBFM (50 $\Omega$)      | $1.95 \times 10^6$ | 10.5          |
| PMCHWT-CBFM (short)            | $1.70 \times 10^5$ | 10.5          |

Next, we evaluated the usage memory of the PMCHWT-CBFM.

Table II shows the element number and required memory usage of the conventional PMCHWT and the PMCHWT-CBFM. In the PMCHWT, the element numbers of the impedance matrix is independent of the terminal condition. On the other hand, the element numbers are changed in PMCHWT-CBFM, because the singular values changes depended on the terminal condition.

The average number of the matrix elements is $1.83 \times 10^6$. The size of the matrix is suppressed to 1/7500 compared with that for the conventional PMCHWT. Furthermore, the maximum usage memory became 1/22. The stored CBF value is mostly occupied in the memory. The required memory for storing the suppressed impedance matrix is only tens of Mega Bytes. The CBFs in all blocks are not simultaneously needed; hence, if the CBF of the required block is saved on in-core memory region and others are written out to out-of-core region, the memory usage can be further reduced.

IV. CONCLUSION

We verified the PMCHWT-CBFM by analyzing the microstrip array antenna and comparing the conventional PMCHWT and the experimental results. All results are in good agreement.

Furthermore, it's shown that the memory amount of the PMCHWT-CBFM can be reduced to 1/22 as compared to that for the conventional PMCHWT. This result indicates that the PMCHWT-CBFM is effective for analyzing the RCS of the array antenna.

REFERENCES