Complex Permittivity Measurement Method of High Loss Materials Using Cylindrical Cavity Resonator in Millimeter-wave Band

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Abstract—We proposed a new method to measure the complex permittivity of small biological tissue in millimeter-wave band using the TE\textsubscript{01n} mode of a cylindrical cavity resonator. We examined the validity of the proposed measurement technique using an agar sample. The agar sample was mixed with pure water 98 % and agar powder 2 %. We prepared agar samples of different thickness ranging from 0.3 to 0.8 mm, and measured the complex permittivity between the 32 GHz – 35 GHz band for each sample. It was assumed that the agar samples have approximately the same complex permittivity as pure water. We compared the measured value of agar sample with the theoretical values of water. As a result, when the thickness of the agar sample of 0.57 mm or less, the measured values approximately agreed with the theoretical values of water.

Keywords—Complex permittivity; Cavity resonator; Millimeter wave; biological tissue.

I. INTRODUCTION

In recent years, the use of millimeter waves (30 GHz – 300 GHz) is increasing. In millimeter-wave band, the majority of energy is absorbed at the surface of the human body. Therefore, there are concerns about the influence of millimeter waves on the eye or skin. Numerical dosimetry is often used as a mean to study the effects of electromagnetic waves on human body [1]. Since there are few reports about the permittivity of biological tissues in millimeter wave band, the extrapolated values of permittivity from values at lower frequencies reported by Gabriel et al. [2] are often used for numerical calculation in millimeter-wave band. Since the relaxation frequency of water is higher than 20 GHz, extrapolated values may not necessarily provide relevant estimation. The complex permittivity of biological tissues has been measured with various methods such as the coaxial probe method and the free space method. However, it is difficult to measure the complex permittivity of some biological tissues using the existing methods. One example is the cornea covering the eye. The coaxial probe method [3], [4], the free-space method [5]–[8], and the waveguide penetration method [9] have been used for measurement of complex permittivity of high loss materials including biological tissues. The cornea has a diameter about 10 mm with a thickness of several hundred \( \mu \text{m} \). It is difficult to measure its complex permittivity. The coaxial probe method usually that the material is semi-infinite homogeneous medium.

So it is difficult for this method to measure thin materials such as cornea. In the free space method, measurement of small sample is difficult. In the waveguide penetration method, the measurement is performed by inserting the sample into small tube. Therefore, this method is suitable for the measurement of liquids, but it is not suitable for the measurement of solid samples because it is difficult to insert the material into the thin tube.

In order to measure the complex permittivity of small and thin tissues with high loss such as the cornea, we propose a new measurement method that uses TE\textsubscript{01n} mode cylindrical cavity resonator. Cavity resonator is suitable for measurement of solid with small volume. Originally, resonator methods are used for measuring complex permittivity measurement of low loss materials [10], [11]. In this study, in order to measure the complex permittivity of small size and high loss samples we place the sample at the end face of the resonator where the electric field is small. We will examine the validity of the proposed method by using a solid agar sample.

II. \( \text{TE}_{01n} \) MODE CYLINDRICAL CAVITY RESONATOR

A. Theory

We insert a disc-shaped sample of thickness \( t \) on the end face of a cylindrical cavity resonator with length \( L \) and radius \( a \) as shown in Fig. 1.

In this case, the following equation is satisfied from the boundary conditions of the \( \text{TE}_{01n} \) mode \((n \text{ represents the number of resonant modes in the } z\text{-axis direction})\) electromagnetic field at boundary of Region 1 (air) and Region 2 (sample) in the resonator.

\[
\frac{\tanh \gamma_{z2} t}{\gamma_{z2}} + \frac{\tanh \gamma_{z1}(L-t)}{\gamma_{z1}} = 0.
\]

\( \gamma_{z1} \) and \( \gamma_{z2} \) are the propagation constants in the \( z \)-axis direction in the region \( 1 \) and the region \( 2 \). They are represented by the following equations.

\[
\begin{align*}
\gamma_{z1}^2 &= -\left\{ \omega^2 \varepsilon_0 \mu_0 - k_c^2 \right\} \\
\gamma_{z2}^2 &= -\left\{ \omega^2 \varepsilon_0 \varepsilon_r \mu_0 - k_c^2 \right\},
\end{align*}
\]

where \( \varepsilon_0 \) and \( \mu_0 \) represent the permittivity and permeability of vacuum respectively. \( \varepsilon_r = \varepsilon_r' - j\varepsilon_r'' \) represents the relative
complex permittivity of the sample in the region 2. $\hat{\omega}$ is the complex resonance angular frequency, and is expressed in (4).

$$\hat{\omega} = 2\pi f \left\{ 1 + \frac{j}{2} \left( \frac{1}{Q} - \frac{1}{Q_0} \right) \right\}$$  \hspace{1cm} (4)

Here, $Q_0$ is the quality factor of the empty resonator. $f$ and $Q$ are resonant frequency and quality factor respectively, when the sample is inserted in the resonator. $k_c$ represents the wave number in the radial direction of the cylindrical surface of the resonator. $k_c$ is represented by (5).

$$k_c = \frac{\rho_{01}'}{\rho} = \frac{3.832}{a}$$  \hspace{1cm} (5)

In (5), $\rho_{01}'$ represents the first zero of the derivative of the bessel function $J_0(\rho')$.

**B. Measurement system and the cavity resonator**

We measured the complex permittivity using the TE$_{01n}$ mode of a cylindrical cavity resonator. Figure 2 is the block diagram of the measurement system. Figure 3 is a photograph of the resonator. Figure 4 shows the sample insertion holder which is placed on the bottom surface of the resonator. The resonator is as long as about 496 mm in length to obtain high resolution of frequency. As a result the resonant frequencies appear at the interval of about 200 MHz as shown in Fig. 5 owing to the large length of the resonator. The complex permittivity is obtained at this frequency interval.

TM$_{11}$ mode and TE$_{01}$ mode are degenerate with each other. In order to attenuate the TM$_{11}$ mode, we used the resonator of spiral structure on the sidewall. This resonator method was originally developed as a method for measuring the complex permittivity of low-loss materials by placing the sample in the center of resonator where the electric field is maximal [10], [11]. In this study, we place the sample at the end face of the resonator, where the electric field is minimal, to allow measurement of high-loss material.

The cavity resonator method has an advantage in the small sample volume required to measure the complex permittivity. This merit is desirable for measurement of cornea of rabbit, whose volume is very small.
C. Experimental Method

First, we measure the resonance frequency $f_0$ and quality factor $Q_0$ of the empty resonator. Then the sample of disc shape is inserted with the sample holder as shown in Fig. 4. Then we measure the resonant frequency $f$ and quality factor $Q$ with the sample inserted. Finally, we substitute the obtained values in the expressions (2) – (4), and calculate the complex permittivity using equation (1).

D. Determination of the dimensions of the resonator

The resonator used in the experiments is designed to be 14 mm inner diameter and 496 mm in length. However, since the cavity inside has a spiral structure, 14 mm inner diameter is not a correct value. Therefore, we determined the resonator length $L$ and its effective inner diameter $a$.

The resonant frequency of the empty resonator $f_0$ satisfies the following equation.

$$4\pi^2\varepsilon_0\mu_0 \cdot f_0^2 = \left(\frac{\rho_0}{a}\right)^2 + \left(\frac{n\pi}{L}\right)^2$$

(6)

We measured the resonance frequency for several resonance modes $n$. Then we determined $a$ and $L$ by using the least squares method, with $a$ and $L$ as unknowns in equation (6). As a result, the values $a = 14.083$ mm and $L = 496.56$ mm were obtained.

III. INVESTIGATION OF THE VALIDITY OF THE MEASUREMENT METHOD

A. Experimental sample

We investigated the validity of the measurement method by measuring a sample with known permittivity. A good sample for use in investigation is a sample having a complex permittivity close to that of the biological tissues and can be freely adjusted in thickness. Water is a sample that satisfies these requirements. However, there are some problems in the experiment because water is a liquid. The first problem is that the resonator must be kept in a vertical position. The second problem is that the meniscus effect is being formed. The thickness of the sample becomes not uniform due to meniscus formation. Therefore we used a sample of agar which is solidified by mixing the agar powder with pure water (98% pure water, 2% agar powder) instead of water.

We prepared agar samples of different thickness $t$ ranging from 0.3 mm to 0.8 mm, and measured the complex permittivity for the 32 GHz – 35 GHz band for each sample. The thickness of the agar sample was measured using a micrometer. The measured complex permittivity of agar was compared with the literature values of the complex permittivity of pure water [12]. Since 98% of the sample are composed of pure water, agar was assumed to have approximately the same complex permittivity as pure water.

B. Experimental results and discussion

Figures 6 and 7 show the measurement results for the resonant frequency $f$ and quality factor $Q$ of agar. The room temperature during measurement was 25.6 °C. Measured resonant frequency and quality factor were compared with the theoretical value for pure water. The theoretical values were calculated by reference to the values of the complex permittivity of water by Kaaze et al. [12] and (1) – (4).

When the thickness of the agar sample of 0.57 mm or less, measured values approximately agreed with the theoretical values. Figures 8 and 9 show the measurement results for the relative permittivity of agar (the average of the results in the sample of 0.57 mm or less). Solid line is the complex permittivity of water at the same temperature. Difference between measured value and theoretical value were up to 8% in the real part and up to 15% in the imaginary part. The cause of the difference could be attributed to the errors in the measurement of sample thickness, the inaccurate levelness of the sample face and the difference of the complex permittivity between the agar and pure water.

The result of the experiment shows that the complex permittivity of agar sample of 0.57 mm or less in thickness can be measured. However, measurement is difficult for the samples with a thickness of 0.68 mm or more. As sample becomes thicker, electromagnetic waves do not penetrate into the end of the sample where the wall of the resonator lies. In this case the resonance due to reflection at the surface of the sample could prevail over the resonance due to the reflection at the wall of the resonator. Different formula of estimating complex permittivity should be derived in order to measure the complex dielectric constant of a sample of 0.57 mm or more in thickness.
In order to measure the complex permittivity of cornea of rabbit eye, we proposed a measurement method that employs a cylindrical cavity resonator. We examined the validity of the proposed method by using agar sample. When the thickness of the agar sample is 0.57 mm or less, we found that the complex permittivity can be measured. We will measure the complex permittivity of the cornea of the rabbit eye with this method for the next step. It should be noted that the thickness of the cornea of the rabbit eye, the final goal of measurement, is about 0.40 mm. We consider that the proposed method is applicable to the measurement of the complex permittivity of the cornea of rabbit eye.

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