Effective Permittivity and Transmission of 2-Dimensional Straight Conductive Strip Array

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Abstract—We calculated scattered field of a strip conductor array on a dielectric sheet by using the electric field integral equation method. The transmission coefficients and the effective permittivities are calculated from the current distribution on the conductors in wide frequency range. The theoretical results are compared with the experimental results.

Key words: Artificial material, Electric field integral equation method, transmission coefficients, effective permittivity

I. INTRODUCTION

Since artificial dielectrics were proposed more than 50 years ago [1], many works have been reported using periodic structures of conductors. A typical application of artificial dielectrics in the past was a lens antenna, but it was used in the frequency range much lower than the resonant frequency determined by the physical dimension of the conductor. This is because the high value permittivity with low loss was desired for their purpose.

In an electromagnetic wave absorber design, the conductive wire dispersed composites, a kind of artificial dielectrics, were developed in the 1980s [2]. The wire length is the same order to the wavelength, and then the composite average permittivity shows the dispersion around the wire resonant frequency. Due to the dispersion, the composite becomes absorbing medium. After this work, the conductive wires have been widely used as one of the key elements that produces electric loss, and EM wave absorbers using metal wire composite have been developed [3].

As another use of the wire array structure providing the resonance, the authors are interested in the application for frequency selective shielding material. In this paper, transmission coefficients and effective permittivities of 2-dimensional periodic array of the conductive strip are numerically studied. The strips are placed on a dielectric sheet, which resembles the practical conductive array structure.

II. SIMULATION MODEL AND COMPUTATIONAL APPROACH

A. Conductive Strip Array on a Dielectric Sheet

Fig. 1 shows the strip conductor array placed periodically on a dielectric sheet. In ref.[4], characteristics of the conductive strip array, that are placed in free space, are numerically obtained. The structure in Fig. 1 resembles the practical structure, since the conductors are placed on a dielectric sheet. The dimensions of each strip conductor are denoted by L in length and W in width. The conductors are assumed to be perfect conductors whose thickness is infinitely thin. The

spacings of the conductor arrangement are denoted by Δx and Δy for each direction. The numbers of strip conductors are infinitive in both x- and y-direction. The thickness of the dielectric sheet is Δt, and the permittivity is εd. The incident waves are plane waves propagating from +z toward -z with electric fields parallel to x-axis.

B. Treatment of the Conductive Strip array Sheet

The Maxwell’s equation can be expressed as

\[ \nabla \times \mathbf{H} = j \omega \varepsilon \mathbf{E} + \sigma \mathbf{E} \]

Where ε and ε₀ are the permittivities of the dielectric and vacuum, respectively, and σ is the conductivity of the conductors. Since the dielectric sheet is very thin (Δt<<λ), the current direction in the dielectric sheet can be assumed to be parallel to the sheet.

We define new effective current J' as

\[ J' = (j \omega (\varepsilon - \varepsilon_0) + \sigma) \mathbf{E} \Delta t \]

and we obtain

\[ \nabla \times \mathbf{H} = j \omega \varepsilon_0 \mathbf{E} + J' / \Delta t \]

The electric field E in (3) is the summation of the incident electric field E' and the scattered electric field E'.

\[ J' = (j \omega (\varepsilon - \varepsilon_0) + \sigma)(\mathbf{E}' + \mathbf{E'}) \Delta t \]
The incident electric field $E'$ is expressed as

$$E' = \frac{j \omega (\varepsilon - \varepsilon_0) + \sigma}{j \omega (\varepsilon - \varepsilon_0) + \sigma} \Delta t \cdot J^*(r) - E' = \frac{j \omega (\varepsilon - \varepsilon_0) + \sigma}{j \omega (\varepsilon - \varepsilon_0) + \sigma} \Delta t$$

$$- \left( j \omega \mu_0 \int S \nabla \cdot J^*(r')G(r, r') ds' \right)$$

$$- \frac{1}{j \omega \varepsilon_0} \nabla \int S \nabla \cdot J^*(r')G(r, r') ds'$$

where $G(r, r')$ is the free space Green's function. As we consider the conductivity $\sigma$ as the complex permittivity $\varepsilon_r$, we obtain

$$E' = \frac{j \omega (\varepsilon_r - 1) \varepsilon_0 \Delta t}{j \omega (\varepsilon_r - 1) \varepsilon_0 \Delta t} \cdot J^*(r)$$

$$- \left( j \omega \mu_0 \int S \nabla \cdot J^*(r')G(r, r') ds' \right)$$

$$- \frac{1}{j \omega \varepsilon_0} \nabla \int S \nabla \cdot J^*(r')G(r, r') ds'$$

We define impedance $Z_s$ as

$$Z_s = \frac{1}{j \omega (\varepsilon_r - 1) \varepsilon_0 \Delta t},$$

where $\varepsilon_r$ is given by

$$\varepsilon_r = \varepsilon_r - j \varepsilon_r'' = \frac{\varepsilon}{\varepsilon_0} - j \frac{\sigma}{\omega \varepsilon_0}. \quad (8)$$

The electric field integral equation

$$E' = Z_s J^*(r) -$$

$$\left( j \omega \mu_0 \int S \nabla \cdot J^*(r')G(r, r') ds' \right)$$

$$- \frac{1}{j \omega \varepsilon_0} \nabla \int S \nabla \cdot J^*(r')G(r, r') ds'$$

which provides the relationships between the incident electric field $E'$ and current distribution $J(r)$.

The method of the calculation of infinite array is described in the former papers [4], [5].

C. Calculation

Calculations were carried out with the parameters shown in Table 1. The conductivity of the strip conductors is infinitive. The permittivity of the dielectric sheet is chosen as 3.2, which corresponds to Polyethylene Terephthalate (PET). The imaginary part of the permittivity is assumed to be zero.

In this calculation the size of unit cell is 25mm by 5mm, and is divided into 25 by 5 segments (1mm x 1mm). In each segment, the current flows for x- and y-directions. Figure 2 shows the division of the unit cell. A white square shows the dielectric part and a black square shows the conductor part. The point matching method is used to solve the electric field integral equation (9). The pulse functions are used as the weight function and the delta functions as the test function.

The calculation area is determined by [5]. The number of unit cells is 101 for x-direction, and 601 for y-direction, so the actual dimension is $2.5m \times 3m$. The incident electric field strength is $1V/m$.

### Table I

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L: length of a strip conductor</td>
<td>15mm</td>
</tr>
<tr>
<td>W: width of a strip conductor</td>
<td>1mm</td>
</tr>
<tr>
<td>$\Delta x$: strip spacing in x-direction</td>
<td>25mm</td>
</tr>
<tr>
<td>$\Delta y$: strip spacing in y-direction</td>
<td>5mm</td>
</tr>
<tr>
<td>$\Delta t$: dielectric sheet thickness</td>
<td>0.2mm</td>
</tr>
<tr>
<td>Permittivity of the dielectric sheet</td>
<td>3.2</td>
</tr>
</tbody>
</table>

D. Measurement

The specimen for the measurement is made of strip conductors sandwiched by two PET sheets whose thickness is 0.1mm. The strip conductors are made of copper, and their dimensions are 15mm in length, 0.3mm in width, and 0.03mm in thickness. Fig. 3 shows the photograph of conductive array that we measured.

The size of the specimen is 300mm x 300mm, and the measurement was carried out using the shielding wall with the aperture of $180mm \times 180mm$.

![Fig. 2 Division of unit cell](image)

![Fig. 3 Photo of the specimen of conductive strip array](image)
III. RESULTS OF SIMULATION AND EXPERIMENT

A. Current Distribution

The current distribution on every strip conductor is substantially sinusoidal, and the maximum value is given at the center of the strip conductor. Fig. 4 shows the frequency characteristic of the maximum currents induced on strip conductors. The solid line shows the maximum current on the strip conductors with the dielectric sheet, and the dotted line shows that on the strip conductors without the dielectric sheet (the conductors are placed in a free space). The effect of the dielectric sheet is to make the resonant frequency slightly low.

Fig. 4 Frequency characteristics of the maximum currents
Induced on the strip conductors

Fig. 5 shows the magnitude and the direction of the current on the dielectric sheet. At the lower frequency, in (a), the currents flow from the adjacent cell toward the left side of the conductive strip of the figure, and they flow out from the right half of the conductive strip toward the unit cell next to the right side. Above the resonant frequency, in (b), the currents flow from one end of the conductor to the other end.

B. Scattered and Total Electric Field

Scattered electric fields are calculated using the current distribution on the strip conductors and the dielectric sheet. Total electric fields are the summation of the scattered electric fields and the incident electric fields. Figures 6(a), (b), and (c) show the scattered electric fields and the total electric fields along z-axis at f=4, 11, and 12GHz, respectively. The solid and dotted lines are the scattered and total electric field, respectively. Due to the finite calculation area, the fluctuation of the scattered electric fields is observed. The incident electric fields whose amplitude is 1V/m propagate from +z to –z in these figures. In z>0, the scattered waves going backward are
reflections. The summation of the incident and reflected waves form standing waves shown by dotted lines. In z<0, the summation of the incident and reflected waves become the transmitted waves shown by solid lines.

As shown in Fig. 6(a), at 4GHz, the reflection is very small and almost all the incident waves are transmitted, while in (b) the transmission is very small and almost all the incident waves are reflected at f=11GHz, the resonant frequency. At f=12GHz in (c), the transmitted waves become large again. These results show the reflection and the transmission are largely depend on the resonance of the strip.

C. Transmission Coefficients and Effective Permittivity

The complex transmission coefficients can be calculated from the total electric fields. Fig. 7 shows the transmission coefficients on a complex plane, and the dotted curve shows the interpolated value. Fig. 8 shows the magnitude of the transmission coefficients. In both figures, the dotted lines with markers show the calculated values, and the solid lines measured values of the specimen mentioned above.

The measured values agree well to the calculated values, however, the frequency at which the calculated value becomes minimum is lower than that of the measured one by 0.5GHz.

The effective, or average, permittivity of the strip array structure is calculated from the complex transmission on the assumption that the structure be sufficiently thin compared to the wavelength. Fig. 9 shows the calculated and measured effective permittivities. The solid and dotted lines are the real and imaginary parts, respectively, and the lines with and without markers are the results of the measurement and the simulation, respectively.

The permittivity shows the resonant type dispersion. At the resonant frequency, around 11GHz, the imaginary part becomes maximum, which makes the transmission minimum.

IV. CONCLUSION

The transmission coefficients and the effective permittivity of the conductive strip array on a dielectric sheet are numerically obtained, and are compared with the experimental results. Though these results agree each other with some error, we confirmed that the simulation procedure including a dielectric sheet is effective for analyze the conductor array.

We are now trying to get more accurate results of the artificial materials and to apply them to the next studies.

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REFERENCES