Electromagnetic Fields Measurement of a TEM Cell Using a Loop Scatterer

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Abstract—By loading a passive scatterer in a TEM cell, the reflection coefficient \( S_{11} \) deviates from that of the empty cell. Since the deviation of \( S_{11} \) can be related to the field strength at the scatterer location by using reaction theorem, a map of the field strength can be obtained. By expanding the current on a loop scatterer and the incident field into Fourier series, the electric and magnetic-field effects on the deviation of \( S_{11} \) of the loaded cell can be separated. In the present paper, measured and calculated results of the deviation of \( S_{11} \) are shown to validate the proposed method for the simultaneous measurements of the electric and magnetic-field.

Key words: TEM cell, Electromagnetic field strength, Reaction theorem, Loop scatterer, Reflection coefficient

I. INTRODUCTION

A resonant or half-wavelength dipole antenna is often used as an electric field sensor. Since a straight dipole antenna has the sensitivity only to the electric field component in the dipole axis, it has an advantage in the precision measurements. However, such a dipole antenna cannot measure a rapid change of the field strength since the element length of a common dipole antenna is about a half wavelength of the measurement frequency. In addition to this, measurements in a small anechoic chamber are difficult and it is impossible to put the element in a small space, such as a G-TEM cell due to space limitation. A small probe compared with the operating frequency has advantages of high spatial resolution and compactness.

A probe is often calibrated in the known electromagnetic fields generated in an anechoic chamber [1]. However, when the probe is small, a transformed coaxial transmission line is useful. A TEM cell is one type of such transmission line [2]. Cross section of a coaxial transmission line is transformed to be rectangular by maintaining the characteristic impedance of 50 \( \Omega \). Since the space inside the cell is small, smaller power is necessary to generate a field strength compared with free space and the cost of introducing a calibration system can be reduced. In addition to this, it has an advantage in terms of spurious radiations.

It is important to study field distribution of the cell. However, the electric field strength of the cell is often assumed to be ideally uniform by representing the field strength at the mid-point of an infinite parallel-plate transmission line. By using a numerical method, the field distribution in the cell can be calculated [3]. However, a real cell is not ideal as a numerical model. In the present paper, a method to measure the electromagnetic field strength in a cell by using a passive loop is proposed.

II. THEORY AND MEASUREMENTS

A. Theory of Measurement

Reaction is one of the important concepts in the electromagnetic theory and some applications of this concept are described in [4]. Using this concept, the electric field distribution in free space can be characterized by measuring the echo voltage induced at the port of the antenna [5]. A study applying this concept to the TEM cell measurement has been published [6]. Since this study focuses mainly on the printed circuit board (PCB) measurements, unlike this study, the ideal field uniformity is assumed in the cell by representing the electric field strength at the center position between the septum and the outer conductor. The co-conical field generation system [7] has been developed at National Institute of Standards and Technology (NIST) and the electric field distribution in the closed environment using this concept was studied. The co-conical cell is well designed to avoid field perturbation due to the higher-order modes and the measured field distribution shows this advantage. However, an ordinary TEM cell generates higher-order modes and no papers applying this concept to such a cell have been published.

Obviously, it has a clear advantage of the spatial resolution to use a small scatterer compared with the operating wavelength. However, if the small signal from the scatterer is to be detected, there are very tight requirements on the measurement system, such as a highly-isolated, stable source and sensitive receiver. One instrument that meets these requirements is a vector network analyzer (VNA) with a good dynamic range and this is widely available. A VNA accurately measures the reflection coefficient \( S_{11} \). An HP8753C VNA was employed to evaluate the electric field uniformity by locating a passive scatterer in the cell. A two-step measurement process was used to isolate the echo field from the scatterer. Figure 1 shows a schematic view of this process: an \( S_{11} \) measurement is made with and then without a passive scatterer. An initial condition is shown in Fig. 1 (a). An empty cell is connected to a signal source with an output wave amplitude of \( a_{in} \). The reflection coefficient of the signal
source is assumed to be zero and this is approximately true since the calibration procedure of the VNA corrects the source match at the cable port. By introducing a scatter inside the cell the reflected wave amplitude from the scatterer-loaded cell will be changed and it is denoted by \( V' \) as shown in Fig. 1 (b). The re-radiated electromagnetic fields from the induced current on the scatterer are the sources of the deviation of the reflected wave amplitude from the empty cell. Thus, the scatterer can be replaced with an equivalent source having the wave amplitude of \( a_s \). Finally the equivalent model of the scatterer-loaded cell can be represented as shown in Fig. 1 (c).

To isolate the effect of the loaded scatterer the subtracted reflection coefficient \( S_{11s} \) is defined as the difference of the loaded and empty cell:

\[
S_{11s} = S_{11}^{\text{loaded}} - S_{11}^{\text{empty}} = a_s / a_G
\]  

(1)

where \( S_{11}^{\text{loaded}} \) and \( S_{11}^{\text{empty}} \) are the reflection coefficients with and without a scatterer inside the cell. When a wire scatterer is located in the cell, the echo voltage at the port of the cell \( V_e \) is given by:

\[
V_e = V_{\text{loaded}} - V_{\text{empty}} = C \int \mathbf{I} \cdot \mathbf{E} \, dl
\]  

(2)

where \( C \) is a constant. \( V_{\text{loaded}} \) and \( V_{\text{empty}} \) are the port voltages with and without a scatterer. \( \mathbf{I} \) and \( \mathbf{E} \) are the current and incident field vectors on the wire, respectively. Accordingly, \( S_{11s} \) can be related to the electric field by using the characteristic impedance of the transmission line, \( Z_0 \).

A loop scatterer shown in Fig. 2 is one basic element. The induced current on the loop \( I(\phi) \) should satisfy the following integral equation when a plane-wave electric field \( E' \) impinges on the element:

\[
b E'_z(b, \phi) = \frac{i \eta}{4 \pi} \int_0^{2\pi} I(\phi - \phi') I(\phi') \, d\phi'
\]  

(3)

When \( I(\phi - \phi') \) is expanded into the Fourier series, it is given by:

\[
I(\phi - \phi') = \sum_{-n}^{n} a_n e^{i n (\phi - \phi')}
\]  

(4)

Finally, \( I(\phi) \) is obtained as follows [8].

\[
I(\phi) = 2 \pi \eta E'_z a(\phi)
\]  

(5)

where

\[
a(\phi) = - \frac{1}{\pi \eta} \left( \frac{f_n + 2 f_n \cos \phi}{a_i} \right)
\]  

(6)

\( f_n \) is a coefficient of the Fourier series expansion of the incident electric field and given by:

\[
f_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} [\cos \theta \cos(\phi - \phi_0) - \sin \theta \sin(\phi - \phi_0) \cos \phi_0] e^{-i n \phi} \, d\phi
\]  

(7)

\( a_i \) is a coefficient of the Fourier series expansion and given by:

\[
a_i = \frac{k b}{2} \left( N_{n+1} + N_{n+1} \right) - \frac{n^2}{kb} N_n
\]  

(8)

where,

\[
N_n = \frac{b}{4 \pi^2} \int_0^{2\pi} \int_{-\infty}^{\infty} e^{in(\theta - \theta')} e^{-\frac{r^2}{\alpha^2}} \, dr \, d\phi
\]  

(9)

\[
r = \sqrt{b^2 \sin^2(\phi - \phi')^2 + 4 a^2 \sin^2(\alpha/2)}
\]  

(10)

Since the first and second terms in (6) are respectively caused by the magnetic and electric-field, their effects can be separated.

B. Measurement

The TEM cell used in the measurement is shown in Fig. 3. \( S_{11s} \) measurements were carried out in two planes. However, measured results in one plane are shown due to space limitation. Measurement plane is at \( z = -30 \) mm. To make the measurement possible at transverse positions of the cell (in \( x \)-direction) five small holes are drilled on the ceiling. The diameters of the hole are approximately 3 mm except at the transverse center. The diameter of the hole at the transverse center is about 8 mm. The perturbation due to these holes was found to be negligible by the measurements. To specify the location of the scatterer easily, the transverse positions are named from T1 to T5 in the direction of \( x \) separated by 35 mm. T3 locates in the \( y-z \) plane. T4 and T5 are at the symmetrical positions of T2 and T1 with respect to the \( y-z \) plane. With respect to the vertical position, measurements at four distances

(c) An equivalent model of the scatterer-loaded cell

Fig. 1 Network models of the measurement.

(a) An empty cell
(b) A scatterer-loaded cell

Fig. 2 Coordinate system on a loop scatterer.
from the septum with the increment of 10 mm were carried out. Vertical positions are named from V1 to V4 and the loop center is at 15 mm from the septum surface when the loop is located at V1.

The vector-subtraction technique is extremely sensitive to the change of the background situation, such as the temperature drift of the equipment and the deformation of the cell due to a small stress. Accordingly, $S_{\text{11}}$ and $S_{\text{11}}^\text{echo}$ should be measured in a short time and the scatterer should be removed or inserted without changing the cell conditions. To locate the straight wire at a position the wire was suspended by a thin thread through the hole on the ceiling since a small object to locate the scatterer may degrade reliability of the results significantly by the vector-subtraction technique between two measurements. Suspending the scatterer by the thin thread produces the good repeatability of the scatterer location and has little influence on the field distribution in the cell [9]. In addition to these advantages, the scatterer can be removed from the cell in a short time without any stress on the cell. These are quite important to avoid degradation of the measurement. However, the loop is laid on the center of the septum to measure $S_{\text{11}}^\text{echo}$ since a loop scatterer cannot pass through such a small hole on the outer conductor. The effects of the laid loop on $S_{\text{11}}^\text{echo}$ measurement were carefully inspected. It became apparent that the laid loop degrades $S_{\text{11}}^\text{echo}$ considerably when the loop is located in the vicinity of the edges of the septum due to the capacitance between the septum and the loop. However, the effect can be sufficiently reduced by locating the loop at the center of the septum. Accordingly, the loop is laid at the center of the septum for $S_{\text{11}}^\text{echo}$ measurements.

III. RESULTS AND DISCUSSION

Since the current on a loop scatterer is induced not only by the magnetic-field but also by the electric-field, the simultaneous measurement of the electromagnetic fields might be possible. Measurements using a loop with the diameter of 24 mm were carried out for two orientations. The wire diameter is 1 mm, approximately. A loop orientation when the loop is in the measurement plane (parallel to x-y plane) is named case 1 and the orthogonal orientation to case 1 (the loop is orientated parallel to y-z plane) is named case 2, respectively. If an ideal TEM mode is generated in the cell, there is no magnetic field component across the loop plane for case 1. Accordingly, $S_{\text{11}}$ for case 1 contains only the electric field response. On the other hand, $S_{\text{11}}$ for case 2 should contain the superposed responses of the electric and magnetic fields.

Figure 4 shows $S_{\text{11}}$ using a loop scatterer with respect to the loop location. Paired reflection coefficient measurements were repeated five times at each position and orientation. The measurement repeatability is very good and the results shown in this paper are the averaged results of the repeated measurements. Since the uniformity of $S_{\text{11}}$ by the loop is almost the same over the frequency range, maps of the measured $S_{\text{11}}$ for two loop orientations at 500 MHz are shown. Since no magnetic field passing through the area surrounded by the periphery of the loop is supposed to exist, the response shown in Fig. 4 (a) is thought to be caused by the electric field. $S_{\text{11}}$ around the center of the measured plane is 2.6 dB smaller compared with case 2. On the other hand, the larger $S_{\text{11}}$ were obtained over the measurement plane for case 2 since the induced current on the loop is the superposed current by the electric and magnetic fields as given by (5). $S_{\text{11}}$ uniformities in both planes are within ±1 dB in the vicinity of the plane center.
With respect to case 1 the angles for the incident electric field in Fig. 2 are given by \( \theta = \pi \), \( \varphi = \pi / 2 \) and \( \phi_0 = -\pi / 2 \), respectively. The echo voltage for case 1 \( V_{e1} \) is obtained by using (2), (5) and (6) as;

\[
V_{e1} = -j \frac{2\pi \omega CE_0^2}{\eta} \frac{1}{a_0}.
\]

Similarly, the echo voltage for case 2 is obtained as;

\[
V_{e2} = -j \frac{2\pi \omega CE_0^2}{\eta} \left( \frac{f_0}{a_0} \int_{-\pi}^{\pi} \cos \phi \exp(-jb_0 \cos \phi) \, d\phi \right) + \frac{2f_1}{a_1} \int_{-\pi}^{\pi} \cos \phi \exp(-jb_1 \cos \phi) \, d\phi.
\]

The difference of \( S_{11s} \) between cases 1 and 2 at 500 MHz is 3.5 dB by using (11) and (12), approximately.

Measured frequency characteristics of \( S_{11s} \) for case 1 and 2 are shown in Fig. 5. Calculated frequency characteristics of \( S_{11s} \) by using (11) and (12) are also shown in the same figure. To compare the calculated \( S_{11s} \) with the measured one it is necessary to measure the input wave amplitude \( a_s \) precisely. In the present paper calculated results are simply corrected by referring to the measured \( S_{11s} \) of case 1 at 500 MHz. Measured and calculated results are in good agreement in the entire frequency band except above 750 MHz for both cases. The differences between measured and calculated results are thought to be caused by the resonance of the cell and an assumption of \( b >> a \) in the calculation.

IV. CONCLUSIONS

In the present paper, a method to measure the electromagnetic fields of a cell by locating a loop scatterer was proposed. Differences of the reflection coefficients with and without a scatterer are related to the electromagnetic fields when the scatterer is absent by the reaction theorem. By using a loop scatterer the electric and magnetic field responses were separated. The measured and theoretical frequency responses were shown and they agreed well. Input power measurements to estimate field strength inside the cell are planned to be carried out.

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