Analysis of Resonant Characteristics of Conducting Polygons

Shinichiro Ohnuki, Ryuichi Ohsawa
Department of Electrical Engineering, College of Science and Technology, Nihon University
1-8-14 Surugadai, Kanda, Chiyoda-ku, Tokyo 101-8308, JAPAN
1ohnuki@ele.cst.nihon-u.ac.jp
2csry08007@g.nihon-u.ac.jp

Abstract—Radar cross sections of various polygonal cylinders are investigated by using a kind of mode matching techniques. Applying a novel field-decomposition method, electromagnetic scattering analysis can be performed very precisely. We will discuss frequency responses of conducting cylinders with cavities and bumps.

I. INTRODUCTION
Electromagnetic scattering analysis is important for target recognition and reduction of the RCS (Radar Cross Section) [1, 2]. When a target has an aperture, the scattering phenomena become more complicated due to the multiple scattering and resonance in the interior region of the aperture. Hence, development of a highly reliable computational technique is important [3, 4, 5, 6]. The authors have proposed the PMM (Point Matching Method) taking account of the edge conditions and reported that electromagnetic scattering problems can be analyzed with high accuracy [6, 7, 8, 9].

In this paper, two types of field decomposition techniques are proposed for the PMM analysis of scattering problems involving various polygonal cylinders. The numerical analysis is performed for the cases of a rectangular cylinder, one with a cavity, and one with a bump. We will discuss resonant characteristics of the RCS in terms of the size and wedge angle.

II. FORMULATION
A scatterer shown in Figure 1 is assumed to be uniform along the z-axis. The original structure is the rectangle cylinder whose cross section is \(2a \times 2b\). We can obtain the polygonal geometry to shift the midpoint \(O_1\) of the side of a rectangle along the x-axis. Wedge cavities can be expressed for changing the placement of the midpoint. The proposed formulation can be applied to solving scattering problems for polygonal geometries as shown in Figure 2.

The incident plane wave for the \(H\)-polarized case in the cylindrical coordinate system \(O(r, \theta)\) can be written as

\[
H_z^{(i)}[p] = H_z \exp[jkr \cos(\theta - \phi_s)]
\]

where \(k\) is the wave number in free space and \(\phi_s\) is the angle of incidence. The time dependence is assumed to be \(e^{j\omega t}\) and suppressed throughout the paper.

We impose the symmetry due to the x-axis on the scatterer. Hence, the incident wave can be decomposed into the two components as

\[
H_z^{(i)}[p] = \frac{H_z}{2} \left[ \exp[jkr \cos(\theta - \phi_s)] + (-1)^p \exp[jkr \cos(\theta + \phi_s)] \right],
\]

where the even-phase component is represented by \(p = 0\) and the odd-phase component is represented by \(p = 1\).

In our PMM, the whole physical space is divided into a finite number of sub-domains in which electromagnetic fields can be expanded by a sum of solutions to the Helmholtz equation. Considering the symmetry along the x-axis, we introduce the following two field decompositions in the upper half-space \((y \geq 0)\).
A. Field Decomposition A

The decomposition A is proposed to solve scattering problems for a conducting cylinder with a shallow cavity or a bump. The upper half-space can be decomposed into the six regions as shown in Figure 3(a). The electromagnetic field in each region can be expanded using the modes which satisfy the Helmholtz equation in the local coordinate system. We define all the separated regions and the electromagnetic fields as follows;

\[ \text{Region } S_i: \text{Outside the circle } C_i \text{ [origin } O_i \text{, radius } \rho_i \text{]} \]

The scattered field in this region satisfies the radiation condition. Therefore, it can be approximated using a finite sum of modes in the coordinates

\[ H_z^{(s)}[p] = \sum_{\nu=1}^{m} B_{\nu}^{(s)}[p] J_{\nu}(kr) \cos(n\theta), \]

(3)

where \( \rho_s = (1+\delta)\sqrt{a^2 + b^2} \), \( H_z^{(s)}(\cdot) \) is the \( \nu \)-th order of the second kind of the Bessel function, and \( N \) is the truncation number.

\[ \text{Region } S_5: \text{Inside the circle } C_i \text{ [origin } O_i \text{, radius } \rho_i \text{]} \]

To satisfy the edge condition, the magnetic field can be written in the local coordinate system as

\[ H_z^{(s)}[p] = \sum_{\nu=1}^{m} B_{\nu}^{(s)}[p] J_{\nu}(kr) \cos(n\theta), \]

(4)

where \( J_{\nu}(\cdot) \) is the \( \nu \)-th order of the Bessel function, \( M \) is the truncation mode number for each separated region, and \( \nu = \pi/\alpha \).

B. Field Decomposition B

The decomposition B is introduced to solve scattering problems for a conducting cylinder with a deep cavity. The upper half-space can be decomposed into the seven regions as shown in Figure 3(b). Among these regions, \( S_1 \) and \( S_4 \) are the same as those for the field decomposition A. Here, we discuss the other two regions \( S_i \) and \( S_j \).

\[ \text{Region } S_i: \text{Inside the circle } C_i \text{ [origin } O_i \text{, radius } \rho_i \text{]} \]

This region is adjacent to the open-ended of the cavity. The field can be expanded by using the combination of trigonometric functions. The magnetic field can be written in the local coordinate system as

\[ H_z^{(s)}[p] = \sum_{\nu=1}^{m} B_{\nu}^{(s)}[p] J_{\nu}(kr) \cos(n\theta) + p B_{\nu}^{(s)}[p] \sin(n\theta), \]

(5)

where \( M \) is the truncation mode number.

\[ \text{Region } S_j: \text{Inside the circle } C_i \text{ [origin } O_i \text{, radius } \rho_i \text{]} \]

If the \( x \)-coordinate of \( O_i \) is larger than that of \( O_j \), the field can be expanded using the \( \nu \)-th order of the Bessel functions as follows;

\[ H_z^{(s)}[p] = \sum_{\nu=1}^{m} B_{\nu}^{(s)}[p] J_{\nu}(kr) \cos(n\theta), \]

(6)

where \( M \) is the truncation mode number.

If the \( x \)-coordinate of \( O_i \) is smaller than that of \( O_j \) such as Figure 2(d), the field can be expanded using the combination of the \( \nu \)-th order of the Bessel and Neumann functions as follows;

\[ H_z^{(s)}[p] = \sum_{\nu=1}^{m} B_{\nu}^{(s)}[p] J_{\nu}(kr) + D_{\nu}(kr) N_{\nu}(kr), \]

(7)

where \( D_{\nu}(kr) \) is the distance \( O_i P \).

The expansion coefficients \( \{ A_{\nu}^{(s)} \} \) and \( \{ B_{\nu}^{(s)} \} \) are unknown. They are determined to satisfy the continuity conditions at the sampling points. The sampling points are placed at the almost same interval on the boundaries \( B_i, B_i', B_i'', B_i''' \), and \( B_i'''' \) defined as

\[ B_i: C_i \cap S_i \ (i = 1 \sim 5), \]

(8)

\[ B_i': C_i \cap S_i \ (i = 1 \sim 3), \]

(9)

\[ B_i'': C_i \cap S_i \ (j = 3 \sim 5), \]

(10)

\[ B_i''' : C_i \cap S_1 \]

(11)

\[ B_i'''' : C_i \cap S_4. \]

(12)
The numbers of sampling points \( L - L_0 \) on the boundaries \( B_i - B_0 \) are determined by the following rules:
\[
L_i = \text{int}[|B_i|/|C_x| \times N] + \gamma, \\
L_0 = \text{int}[|B_0|/|C_x| \times N] + \gamma, \\
E_i = \text{int}[|B_i|/|C_x| \times N], \\
E_0 = \text{int}[|B_0|/|C_x| \times N],
\]
where \( \text{int}[\xi] \) is the integer part of \( \xi \) and \( \gamma = \rho(l=1), 0(l=2-5) \). The relationships between sampling points and truncation mode numbers are as follows;
\[
M_i = L_i + L_0, \\
M_i = L_i + L_0 + L_0, \\
M_i = L_i + L_0, \\
M_i = L_i + L_0, \\
M_i = L_i, \\
M_i = L_0 + L_0.
\]

III. Computational Results

To verify the computational accuracy, the convergence tests for varying the truncation mode number \( N \) are performed. Figure 4 shows the convergence process of the RCS at \( \theta = 0^\circ \) when the incident \( H \)-polarized wave impinges from \( \phi = 0^\circ \). The geometrical parameters are \( ka = 2\pi, a/b = 1, \alpha_1 = 10^\circ \) and \( \alpha_2 = 355^\circ \). In this case, we can obtain 9-digit accuracy when \( N > 89 \) using the field decomposition A. In Figure 5, the same convergence test is performed for the case of a deep cavity, \( \alpha_1 = 170^\circ \) and \( \alpha_2 = 275^\circ \). We can obtain 9-digit accuracy when \( N > 69 \) using the field decomposition B. From these figures, the extrapolated true values can be estimated for \( N \) to infinity.

Using the extrapolated true value and the computational results, the absolute relative error for varying \( \alpha_1 \) is plotted in Figure 6. The field decomposition A is suitable when \( \alpha_1 = 240^\circ - 290^\circ \) and the field decomposition B is suitable for \( \alpha_1 = 290^\circ - 360^\circ \). Switching the field decompositions A to B around \( \alpha_1 = 290^\circ \), we can make the relative error less than \( 10^{-6} \) for all the values of \( \alpha_1 \).

Figure 7 shows the monostatic RCSs for varying \( ka \) when \( \alpha_1 = 356^\circ, \alpha_2 = 358^\circ, \) and \( \alpha_1 = 360^\circ \). They are almost identical expect some resonant peaks.

Figure 8 shows the resonant characteristic for \( ka = 3.1 - 3.7 \) to investigate a resonant peak for varying \( \alpha_1 \). The peak for \( \alpha_1 = 360^\circ \) is observed at \( ka = 3.23 \). Decreasing the wedge angle \( \alpha_1 = 360^\circ - 354^\circ \) (1) the peak shifts to a higher frequency, and (2) the bandwidth becomes wider.

The low frequency analysis for cavity structures is very important for the \( H \)-polarized case, since the specific phenomenon called the Helmholtz resonance exists[7]. The resonant frequency is much smaller than those for cavity resonances and its characteristics cannot be explained in terms of waveguide modes[9]. Figure 9 shows the resonant characteristic for \( ka = 0.2 - 1.0 \) to investigate the Helmholtz resonance. The resonant peak for \( \alpha_1 = 360^\circ \) is observed at \( ka = 0.52 \). Decreasing the wedge angle \( \alpha_1 = 360^\circ - 330^\circ \), (1) the peak value is almost the same, (2) the peak shifts to a higher frequency, and (3) the bandwidth becomes wider.

Figure 10 shows the monostatic RCS for varying \( ka \) when the wedge angle \( \alpha_1 = 270^\circ - 280^\circ \). The RCSs are almost identical when \( ka < 4 \), however, the value for larger \( \alpha_1 \) becomes smaller when \( ka > 4 \). Increasing \( \alpha_1 \), the specular direction shifts from the backscattered angle.

IV. Conclusion

In this paper, we introduce two types of field decompositions for our point matching method to investigate the RCSs of various polygonal cylinders. Using the proposed field decomposition, the relative error can be controlled under \( 10^{-6} \) for any types of polygonal cylinders with wedge cavities. The numerical analysis is performed for the cases of a rectangular cylinder, one with a cavity, and one with a bump. The resonant characteristics of RCSs in terms of the size and wedge angle are clarified.
REFERENCES