A New Method of Field to Two-Wire Transmission Line Coupling Calculation

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Abstract: This paper using a new method calculates the response of a two-wire transmission line excited by an external electromagnetic field. The two-wire transmission line is regarded as a receiving antenna. By using the reciprocity theorem, we get the analytic solution of the voltage at the terminal of the two-wire transmission line induced by the external electromagnetic field. The terms of effective length and effective aperture are used to describe the field-to-wire coupling capability. In some calculated instances, it is studied that how different parameters affect the effective length and effective aperture.

Key words: Two-wire transmission line, reciprocity theorem, effective length, effective aperture

1. Introduction

When incident electromagnetic waves couple to a pair of parallel wires, the current on the wires is induced, as well as the induced voltage is generated between the terminals of the wires. Generally, it is very difficult to get a precise solution of the induced current, which is a scatter issue. Therefore, the transmission line model is formerly used in field-to-wire coupling calculations [1][2][3]. The model is based on the following assumptions. The transmission line is uniform. The effect of the dielectric around the parallel line is ignored. The transverse dimension of the wires is short in comparison to the wavelength of electromagnetic field. High order modes are not considered.

This paper using a new method calculates the response of a two-wire transmission line excited by an external electromagnetic field. The two-wire line is regarded as a receiving antenna. The issue is simplified by using the reciprocity theorem in transforming the scattering problem into a radiation problem of the antenna. The paper has deduced an analytic solution of the induced voltage at the terminal of the parallel line, which changes with the frequency of the incident wave.

The terms of effective length and effective aperture is defined. By these terms, the induced voltage and energy at the terminal of the parallel line or the input port of the communication equipment can be calculated as long as the direction and the magnitude of the electromagnetic field are known.

2. Physical Model and Mathematic Model

![Figure 1: The physical model of the two-wire transmission line](image)

The physical model of the two-wire transmission line is shown in figure 1. The wires are in xz plane. The wires parallel with z axis. The terminals parallel with x axis. The wires' length is l. The distance between the two wires is b. The conductor's diameter is a. The impedances at each terminal are $Z_1$ and $Z_2$, respectively.

When uniform plane waves couple in the isolated two-wire transmission line, the following assumptions are made. $\vec{E}_i(x,y,z)$ is the electric field of the incident wave. $\vec{H}_i(x,y,z)$ is the magnetic field of the incident wave. Propagation vector $\vec{k}$ make the angle $\theta$ with z axis. The projection of $\vec{k}$ in xy plane makes the angle $\phi$ with x axis. The electric field $\vec{E}_i$ is in the plane determined by propagation vector $\vec{k}$ and z axis.

The two-wire transmission line is regarded as a receiving antenna. According to the reciprocity theorem, the open voltage at the terminal of the parallel line induced by external electric field $\vec{E}_i$ is

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\[ V_0 = \frac{1}{T(0)} \int |E| \cdot d\vec{l} \]  \hspace{1cm} (1)

where \( I \) is the current on the wires and \( I(0) \) is the input current of the wires when the parallel line is used as a transmitting antenna.

The current on the wires used as a transmitting antenna can be calculated with numerical methods (e.g. the moment method). When the incident electromagnetic waves couple to a pair of parallel wires, differential mode (DM) and common mode (CM) current are induced on the wires. The CM current means the current on the wires with the same amplitude and phase. The DM current means the current on the wires when the parallel line is opposite phase. So the total current on each wire is not equal. This paper mainly discusses the terminal voltage and doesn't concern the current on the wires. Moreover, the CM current on the terminal load is zero. So the sine current distribution is assumed which can reach the requirement of engineering.

Consider that the wires don't match to the terminal loads \( Z_1 \) and \( Z_2 \). Therefore, the current on the wires is a standing wave. The expression of the current is (see Fig. 1)

\[ I_1(z) = I_0 \left[ e^{-\gamma z} - \Gamma_2 e^{-2\gamma l} \right] \sum_{n=0}^{\infty} (\Gamma_1 \Gamma_2)^n e^{-2n\gamma l} \]  \hspace{1cm} (2)

\[ I_2(z) = -I_0 \left[ e^{-\gamma z} - \Gamma_2 e^{-2\gamma l} \right] \sum_{n=0}^{\infty} (\Gamma_1 \Gamma_2)^n e^{-2n\gamma l} \]

The current at the input port \((z=0)\) is

\[ I(0) = I_0 \left[ 1 - \Gamma_2 e^{-2\gamma l} \right] \sum_{n=0}^{\infty} (\Gamma_1 \Gamma_2)^n e^{-2n\gamma l} \]  \hspace{1cm} (3)

where \( \Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0} \), \( \Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} \).

\( \Gamma_1 \) is the reflection coefficient at \( z=0 \). \( \Gamma_2 \) is the reflection coefficient at \( z=l \). \( Z_0 \) is the characteristic impedance of the parallel line. Propagation constant \( \gamma = \alpha + j\beta \), where \( \alpha \) is the attenuation constant and \( \beta \) is the phase constant. \( \beta = k \) in free space.

2.1 The Calculation of Terminal Open Voltage

According to (1), when uniform plane waves couple in the isolated two-wire transmission line (see Fig. 1), the terminal open voltage of the two-wire line (i.e. the voltage at \( z=0 \) when \( Z_1 = \infty \)) is

\[ V_0 = V_1 + V_2 \]

\[ = \left[ \frac{|\vec{E}|}{1 - \Gamma_2 e^{-2\gamma l}} \right] \left[ 1 - e^{i\theta} \cos \phi \right] \]

where \( \vec{E} \) is the magnitude of the external electric field. Then define \( h_e \) as the effective length of the parallel line. From (4), the effective length of the parallel line is

\[ h_e(\omega, \phi, \theta) = \frac{|\vec{E}|}{|\vec{E}_0|} \left[ \frac{\sin \theta}{1 - \Gamma_2 e^{-2\gamma l}} \right] \left[ 1 - e^{i\theta} \cos \phi \right] \]

(8) shows that the terminal open voltage is able to be calculated as long as the magnitude of the external electric field is known.

2.3 The Effective Aperture of The Parallel Line

When the incident wave is a broadband pulse (e.g. a trapezoidal pulse or a single period sine pulse), the effective length can't show the field-to-wire
The coupling capability of the parallel line, because the waveform of the induced voltage is changed with the incident direction. Therefore, the effective aperture is defined.

\[ A_e = \frac{\text{Energy received}}{\text{Incident energy density at the wires}} \]

The incident energy density is

\[ W_i = \frac{1}{\eta_0} \int_0^\infty |E_r(t)|^2 dt = \frac{1}{\eta_0} \left[ 1 \frac{1}{2\pi} \int_{-\infty}^{+\infty} |E_r(\omega)|^2 d\omega \right] \]

and the energy received is

\[ W = \frac{1}{Z_0} \int_0^\infty |V_r(t)|^2 dt = \frac{1}{Z_0} \left[ 1 \frac{1}{2\pi} \int_{-\infty}^{+\infty} |V_r(\omega)|^2 d\omega \right] \]

where \( E_r(t) \) is the incident electric field of time domain and \( E_r(\omega) \) is the spectrum function of \( E_r(t) \) and \( V_r(t) \) is the induced voltage of time domain and \( V_r(\omega) \) is the spectrum function of \( V_r(t) \). \( \eta_0 \) is the free space wave impedance.

Therefore, the effective aperture is

\[ A_e = \frac{\eta_0}{Z_0} \int_0^\infty |V_r(t)|^2 dt = \frac{\eta_0}{Z_0} \int_{-\infty}^{+\infty} |V_r(\omega)|^2 d\omega \]

3. Calculated Sample of Effective Length and Effective Aperture

When the other parameters are fixed, we select the wires' length \( l \) as 0.4m (short line) and 20m (long line) to calculate the figures of the effective length changing with the frequency and the effective aperture changing with the breadth of the pulse.

Assume that the wires are made of copper. The copper's resistivity \( \rho = 1.7 \times 10^{-8} \Omega \cdot \text{m} \). The diameter of the wires \( a = 2 \times 10^{-4} \text{m} \). The distance between the two wires \( b = 6.4 \times 10^{-4} \text{m} \). Then the characteristic impedance of the parallel line \( Z_p = 219.6 \Omega \). Assume that the incident angle \( \theta = \pi/2 \) and \( \phi = 0 \).

3.1 Effective Length

(1) Short line \((l=0.4m)\)

The curves of the short line's effective length changing with the incident wave's frequency (300MHz-3GHz) are shown in figure 2. They are calculated by (8) with the matched load \((Z_2 = Z_0 \text{ and } \Gamma_2 = 0)\) and a greater load \((Z_2 = 10k\Omega \text{ and } \Gamma_2 = 0.957)\), respectively. The figure shows that the load greatly affects the effective length when the wires are short and the effective length is relatively short when the wires match to the load.

\[ Z_2^{10k\Omega} \]

Figure 2: Short line's effective length at 300MHz-3GHz with \( Z_2=Z_0 \)

Figure 3: Long line's effective length at 300MHz-2GHz with \( Z_2=Z_0 \)

Figure 4: Long line's effective length at 300MHz-2GHz with \( Z_2=10k\Omega \)

(2) Long line \((l=20m)\)

Figure 3 and figure 4 are the curves of the long line's effective length changing with the incident wave's frequency (300MHz-2GHz). They are calculated by (8) with the matched load \((Z_2 = Z_0 \text{ and } \Gamma_2 = 0)\) and a greater load \((Z_2 = 10k\Omega \text{ and } \Gamma_2 = 0.957)\), respectively. The figures show that
the effect of the load to the long line's effective length is less than the short.

3.2 Effective Aperture

(1) The incident wave is a single period sine pulse. The equation of a single period sine pulse is

\[ E(t) = \begin{cases} \frac{2 \pi E_0}{T} \sin \frac{2 \pi t}{T} & (|t| < \frac{T}{2}) \\ 0 & (|t| \geq \frac{T}{2}) \end{cases} \]

From (11), the curves are shown in Figure 5, which are the short line's and the long line's effective apertures changing with the breadth of the sine pulse (0.1ns-5ns).

(2) The incident wave is a trapezoidal pulse. The equation of a trapezoidal pulse is

\[ E(t) = \begin{cases} \frac{2E_0}{T_2 - T_1} \left( t + \frac{T_1}{2} \right) & \left( -\frac{T_1}{2} < t < \frac{T_1}{2} \right) \\ \frac{E_0}{T_2 - T_1} \left( \frac{T_1}{2} < t < \frac{T_1}{2} \right) & \left( \frac{T_1}{2} < t < \frac{T_1}{2} \right) \\ 0 & \text{others} \end{cases} \]

The curves in figure 6 are calculated by (11), when \( \frac{T_2 - T_1}{2} = 0.03\text{ns} \). They are the short line's and the long line's effective apertures that changing with the breadth of the trapezoidal pulse (0.1ns-5ns).

The comparison between figure 5 and figure 6 shows that the effective aperture of the parallel line is different with different incident pulse.

4. Conclusion

This paper regards the two-wire transmission line induced by the external electromagnetic field is got by the use of the reciprocity theorem. The equations (8) and (11) define the effective length and the effective aperture, respectively. They are used to describe the field-to-wire coupling capability.

With the calculated instances, we draw the following conclusion. The effective length of the parallel line is changed with the incident wave's frequency. The effect of the load \( Z_2 \) to the short line's effective length is greater than the long. If the other parameters are fixed, the effective length is relatively short when the wires match to the load. Namely, the induced open voltage with matched load is relatively small. The waveform and the breadth of incident pulse and the wires' length all affect the effective aperture. With the selected parameters and the matched load, the effective length of the parallel line is about the magnitude of \( 2 \times 10^{-3} \text{m} \) and the effective aperture is about the magnitude of \( 6 \times 10^{-6} \text{m}^2 \).

References


