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A Piecewise-Linear Particle Swarm Optimizer with Locally-Coupled Topology

Tomoyuki Sasaki† and Hidehiro Nakano‡

†Shonan Institute of Technology
1–1–25 Tsujido-nishikaigan, Fujisawa-shi, Kanagawa 251-8511, Japan
‡Tokyo City University
1–28–1 Tamazutumi, Setagaya-ku, Tokyo 158-8557, Japan
Email: sasaki@info.shonan-it.ac.jp, nakano@ic.cs.tcu.ac.jp

Abstract—We proposed a deterministic particle swarm optimizer, called piecewise-linear particle swarm optimizer (PPSO). In PPSO, each particle has two search modes which are a convergence mode and a divergence mode, and switches both search modes irregularly. PPSO is effective to solve non-separable problems, however, it has not been clarified how dynamics of particles contribute to search performances for the non-separable problems. Here, we focused on and investigated a search direction of PPSO particles. In addition, in order to improve search performances of PPSO, a neighborhood topology between particles is introduced to PPSO. We further compared the search performances of PPSO with those of deterministic PSO, and classic PSO in the numerical simulations.

1. Introduction

Here, in order to solve a real-parameter optimization problem, various optimization algorithms have been proposed. The optimization problem for minimizing its evaluation value is defined by the following equation:

$$\text{minimize} f(\boldsymbol{x}), \boldsymbol{x} = \{x_1, x_2, \ldots, x_D\}^\top \in \mathcal{F} \subseteq \mathbb{R}^n \quad (1)$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is an objective function, n is the number of dimensions, and $\mathcal{F}$ is a search space. Although there are many real-world optimization problems, it is difficult to know gradient information of the problem, whether a landscape of the problem is unimodal or multimodal, or whether the problem is separable or non-separable. These optimization problems are called black-box optimization problems in which we can use only evaluation value of $f(\boldsymbol{x})$. Various metaheuristics which can solve black-box optimization problems effectively, have been proposed. However, because a search space of such black-box optimization problem is large-scale and complicated, it is difficult to solve the problem. As such, powerful metaheuristics is required to search for a good quality of solution effectively.

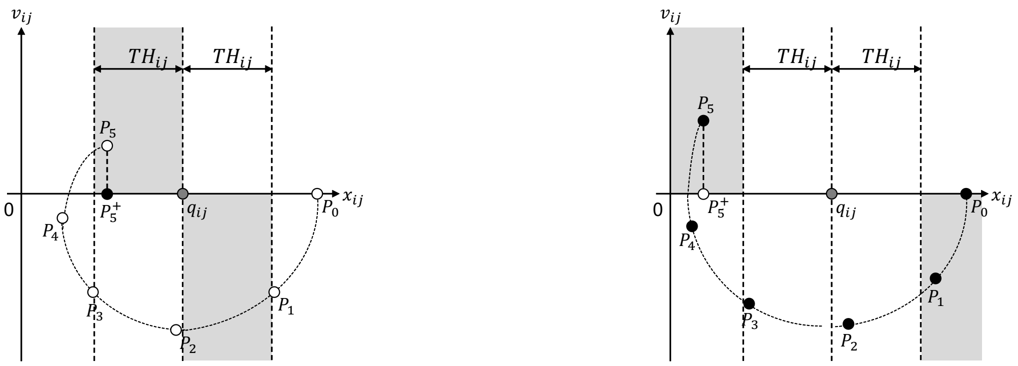
Particle swarm optimization (PSO) is one of the stochastic population-based metaheuristics developed by J. Kennedy and R. C. Eberhart [1]. PSO algorithm simulates social behaviors of creatures such as fish schooling or bird flocking. The creatures are represented by particles as solution candidates, which fly a search space by sharing the best solution information in a swarm. PSO algorithm is very simple and is easy to implement to applications. PSO can search the search space without analytical information such as gradient of objective functions. As such, PSO algorithm has been applied to various applications. However, search performances of PSO are worse in solving non-separable problems. Because, PSO algorithm does not have rotational invariant and the search direction of particle tends to be biased in parallel to the coordinate axes [3].

In our previous research, piecewise-linear particle swarm optimizer (PPSO) which has a convergence mode and a divergence mode was proposed [4], which is one of the deterministic PSO (DPSO). A particle of PPSO switches both search modes dynamically, and PPSO is effective for solving non-separable problems. We considered that the search direction of PPSO particle does not tend to be biased to the coordinate axes, and then particles of PPSO can move in a search space freely more than those of PSO.

Here, in order to improve search performances of PPSO, a neighborhood topology between particles is introduced to PPSO algorithm. PSO with the neighborhood topology (N-PSO) is effective for solving multimodal problems [2]. In N-PSO, particles can only exchange good solution information between specific particles, and "Degree" is the total number of particles which can exchange good solution. When Degree is small, particles tend to move in a search space independently, and this behavior can prevent particle to converge to a local optimum solution prematurely. Therefore, it is considered that PPSO with the neighborhood topology (N-PPSO) is more effective in solving multimodal problems than the classic PPSO. In order to clarify that N-PPSO is more effective than PPSO in solving multimodal problems, we evaluated search performances through numerical experiments.

2. Piecewise-linear particle swarm optimizer

The basic idea of PPSO is explained bellow. The ith particle of PPSO has velocity vector $\boldsymbol{v}_i^t = (v_{i1}^t, v_{i2}^t, \ldots, v_{iD}^t)$, position vector $\boldsymbol{x}_i^t = (x_{i1}^t, x_{i2}^t, \ldots, x_{iD}^t)$, and the personal best vector $\boldsymbol{pbest}_i^t = (pb_{i1}^t, pb_{i2}^t, \ldots, pb_{iD}^t)$, and has the convergence and divergence modes. Each particle shares the global best vector $\boldsymbol{gbest}^t = (gb_1^t, gb_2^t, \ldots, gb_D^t)$ in a swarm. D denotes the number of design variables, and t denotes



(a) The trajectory of PPSO for the convergence mode (b) The trajectory of PPSO for the divergence mode

Figure 1: Particle trajectory of PPSO

current iteration.

The updating rules of the jth component of the ith particle in the swarm are given by the following equations.

$$q_{ij}^t = (1-\gamma)pb_{ij}^t + \gamma gb_j^t \tag{2}$$

$$y_{ij}^t = x_{ij}^t - q_{ij}^t \tag{3}$$

$$\begin{bmatrix} v_{ij}^{t+1} \\ y_{ij}^{t+1} \end{bmatrix} = \delta_{ij}^t \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} v_{ij}^t \\ y_{ij}^t \end{bmatrix} \tag{4}$$

where q_{ij}^t denotes an equilibrium point, γ $(0 \le \gamma \le 1)$ denotes a connection parameter between pb_{ij}^t and gb_j^t. When $\gamma = 0.5$, the impact of pb_{ij} and gb_j is the same, and we set $\gamma = 0.5$ in this paper. y_{ij}^t denotes a relative position from the equilibrium point q_{ij}^t to the ith particle's position x_{ij}^t. δ_{ij}^t denotes a damping factor, and θ $(0 < \theta < \frac{\pi}{2})$ denotes a rotational angle. When $0 < \delta_c < 1$ (i.e., $\delta_{ij}^t = \delta_c$; the convergence mode), a particle converges to the equilibrium point q_{ij}^t gradually. While, when $\delta_d > 1$ (i.e., $\delta_{ij}^t = \delta_d$; the divergence mode), the particle leaves from the equilibrium point gradually. Equation (4) is a discrete-time linear system if δ_{ij}^t and q_{ij}^t are static. Hence, PPSO is regarded as a kind of piecewise-linear systems. The switching rule from the convergence mode to the divergence mode is given by the following equation.

$$\begin{cases} v_{ij}^t \cdot v_{ij}^{t+1} < 0 \\ |y_{ij}^{t+1}| < TH_{ij}^t \end{cases} \tag{5}$$

where TH_{ij}^t denotes a switching threshold in each search mode. If the search mode of a particle is switched to the divergence mode, the particle updates its own information by Eq. (6).

$$\begin{cases} \delta_{ij}^{t+1} = \delta_d \\ v_{ij}^{t+1} = 0 \\ TH_{ij}^{t+1} = \alpha TH_{ij}^t \end{cases} \tag{6}$$

where α $(0 < \alpha < 1)$ denotes a scale parameter of TH_{ij}^t. TH_{ij}^t is only updated at the end of the convergence mode.

On the other hand, the switching rule from the divergence mode to the convergence mode is given by the following equation.

$$\begin{cases} v_{ij}^t \cdot v_{ij}^{t+1} < 0 \\ |y_{ij}^{t+1}| > TH_{ij}^t \end{cases} \tag{7}$$

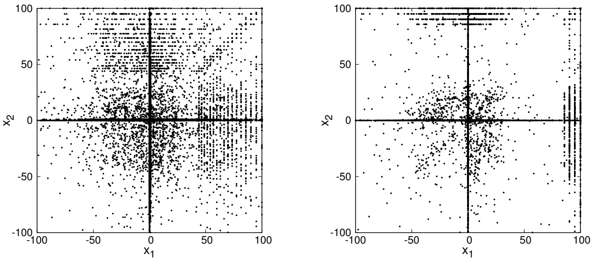
If the search mode of a particle is switched to the convergence mode, the particle updates its own information by Eq. (8).

$$\begin{cases} \delta_{ij}^{t+1} = \delta_c \\ v_{ij}^{t+1} = 0 \end{cases} \tag{8}$$

Figure 1 shows an example of a particle trajectory of PPSO. As shown in Fig. 1 (a), the particle in the convergence mode (i.e., $\delta_{ij}^t = \delta_c$) searches around the equilibrium point and converges to the equilibrium point ($P_0 \sim P_4$). Gray regions denote switching conditions for the divergence mode. In case of P_5, the conditions $v_{ij}^4 \cdot v_{ij}^5 < 0$ and $|y_{ij}^5| < TH_{ij}^4$ are satisfied, then the particle search mode is switched from the convergence mode to the divergence mode. Furthermore, the particle velocity is set to 0 (P_5^+), and then TH_{ij}^5 is updated by Eq. (6).

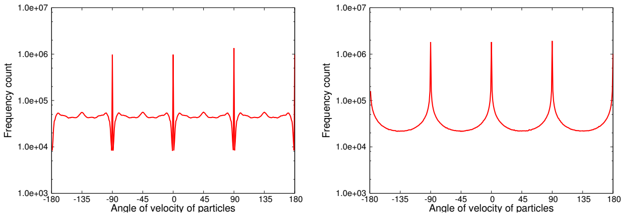
On the other hand, as shown in Fig. 1 (b), the particle in the divergence mode (i.e., $\delta_{ij}^t = \delta_d$) leaves from the equilibrium point ($P_0 \sim P_4$). Gray regions denote switching conditions for the convergence mode. In case of P_5, the conditions $v_{ij}^4 \cdot v_{ij}^5 < 0$ and $|y_{ij}^5| > TH_{ij}^4$ are satisfied, and then the particle search mode is switched from the divergence mode to the convergence mode. Furthermore, the particle velocity is set to 0 (P_5^+).

In the initial search stage, since each particle should search a search space globally, the switching threshold TH_{ij}^0 should be a large value and the particle can search the search space globally. Hence, each particle searches the search space away from the equilibrium point in the initial search stage, and moves to the equilibrium point gradually. In the final search stage, each particle searches around the equilibrium point intensively.



(1) $\theta = 55°$ (2) $\theta = 5°$

Figure 2: The trajectories of PPSO particles (δ_c = 0.6, δ_d = 1.2, α = 0.95)



(1) $\theta = 55°$ (2) $\theta = 5°$

Figure 3: The angles of velocity of PPSO particles (δ_c = 0.6, δ_d = 1.2, α = 0.95)

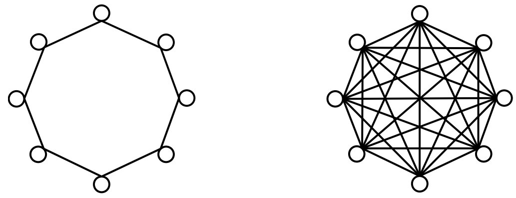
3. Behaviors of PPSO particles

In this section, the behaviors of PPSO particles are explained. In PPSO, there are parameters which control a particle trajectories and biases of movement toward coordinate axes. Figure 2 shows the trajectories of PPSO particles in solving 2 dimensional rotated rastrigin's function, the search range of which is $[-100, 100]$. As shown in Fig. 2, particles with $\theta = 55°$ can search various regions of a search space more than those with $\theta = 5°$.

Figure 3 shows the frequency of angles of velocity vectors in solving 20 dimensional rotated rastrigin's function. The angles of velocity denotes a search direction, which is computed using two different axes selected randomly. We counted the number of angles of all particles. When the frequency count becomes large at the angles of 0°, ±90°, and ±180°, while being small at the angles of ±45° and ±135°, as shown in Fig. 3 (2), the search direction of particles tends to be biased in parallel to the coordinate axes (see Fig. 2 (2)). In contrast, when the frequency count in Fig. 3 (1) becomes larger at the angle of ±45° and ±135° than that in Fig. 3 (2), the search direction of particles is not biased in parallel to the coordinate axes (see Fig. 2 (1)). Thus, the particles of (1) can move in a search space without depending on the directions in parallel to the coordinate axes more than those of (2).

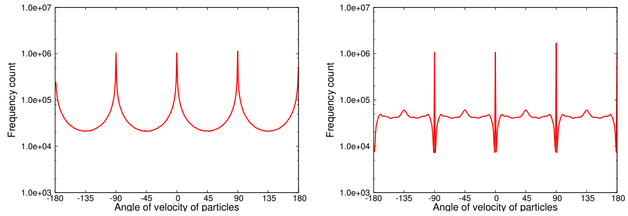
4. PPSO with neighborhood topology

In this section, PSO with a neighborhood topology (N-PSO) is explained. In the classic PSO, all particles share information of the global best solution (*gbest*) in a swarm, the topology of which is called gbest topology. On the other hand, each particle shares information of good solution (*lbest*) between specific particles, the topology of which is called lbest topology. In lbest topology, "*Degree*" denotes the total number of particles which can exchange information of good solution between neighborhood particles. Figure 4 shows neighborhood topology of particles, and the number of particles is 8. As shown in Fig. 4 (1), when *Degree* is small, information of *lbest* is propagated to all particles slowly, and all particles can not converge to *lbest* prematurely. This behavior is effective in solving multimodal problems. As shown in Fig. 4 (2), when *Degree* is large, information of *lbest* is propagated to all particles quickly, and then all particles can converge to a local optimum solution prematurely. Therefore, *Degree* controls a convergence performance of particles.



(1) *Degree* = 2 (2) *Degree* = 7

Figure 4: Neighborhood topology



(1) N-PSO (2) N-PPSO

Figure 5: The angle of velocity of particles (*Degree* = 2)

Slow convergence characteristic of N-PSO (i.e., *Degree* is small) is effective in solving multimodal problems, because each particle tends to search its own search region, and then, particles can search a search space globally. However, in N-PSO, the search direction of particles tends to be biased in parallel to the coordinate axes (see Fig. 5 (1)), it is not effective in solving non-separable problems.

In PPSO, particles can move in a search space freely without depending on the coordinate axes of a problem. This characteristic is also the same when the neighborhood topology is introduced to PPSO (see Fig. 5 (2)). As such, in order to improve search performances of PPSO in solving non-separable multimodal problems, we considered that it is effective to introduce the neighborhood topology into PPSO (N-PPSO).

5. Numerical experiments

In this section, search performances of N-PPSO were evaluated compared with N-DPSO and N-PSO. The number of particles (N) is 20, the maximum iteration (t_{max}) is

Table 1: Comparison results

		N-PPSO		N-PSO		N-DPSO	
f	*Degree*	2	19	2	19	2	19
f_2^U	Mean	2.05E+06	1.56E+06	2.84E+06	**7.78E+05**	7.45E+07	7.35E+07
	SD	8.63E+05	7.24E+05	1.57E+06	4.31E+05	4.64E+07	5.10E+07
f_3^U	Mean	**7.79E+06**	4.36E+07	1.52E+08	1.20E+08	4.92E+16	7.42E+16
	SD	9.91E+06	6.99E+07	2.54E+08	2.16E+08	1.74E+17	4.63E+17
f_4^U	Mean	2.77E+04	**1.54E+04**	4.12E+04	2.09E+04	3.69E+04	5.79E+04
	SD	6.25E+03	6.15E+03	1.11E+04	1.01E+04	1.01E+04	1.62E+04
f_6	Mean	**1.69E+00**	1.02E+01	6.27E+00	5.05E+00	4.32E+03	3.48E+03
	SD	6.72E+00	2.25E+01	1.63E+01	1.48E+01	2.42E+03	1.93E+03
f_7	Mean	**9.40E+00**	2.04E+01	4.58E+01	9.76E+01	2.83E+05	1.84E+05
	SD	5.57E+00	1.64E+01	2.06E+01	1.38E+02	6.03E+05	4.26E+05
f_8	Mean	2.09E+01	2.09E+01	**2.08E+01**	**2.08E+01**	2.09E+01	2.09E+01
	SD	6.18E-02	7.69E-02	8.09E-02	8.50E-02	9.28E-02	7.59E-02
f_9	Mean	1.07E+01	**1.04E+01**	1.87E+01	1.82E+01	2.50E+01	2.49E+01
	SD	2.36E+00	2.46E+00	2.27E+00	2.88E+00	2.28E+00	2.60E+00
f_{10}	Mean	1.33E+00	1.13E+00	3.81E-01	**2.67E-01**	2.14E+03	1.71E+03
	SD	1.87E-01	6.30E-02	2.38E-01	2.22E-01	6.71E+02	6.98E+02
f_{12}	Mean	**2.52E+01**	3.33E+01	6.78E+01	9.93E+01	4.73E+02	4.42E+02
	SD	6.72E+00	1.12E+01	2.53E+01	3.89E+01	1.48E+02	1.34E+02
f_{13}	Mean	**5.40E+01**	7.15E+01	1.01E+02	1.33E+02	4.68E+02	4.75E+02
	SD	1.67E+01	2.18E+01	2.02E+01	3.61E+01	1.19E+02	1.54E+02
f_{15}	Mean	**1.76E+03**	1.91E+03	2.65E+03	2.48E+03	4.62E+03	4.17E+03
	SD	4.29E+02	5.38E+02	6.60E+02	6.06E+02	4.83E+02	6.34E+02
f_{16}	Mean	**1.35E+00**	1.80E+00	1.54E+00	1.63E+00	2.66E+00	2.63E+00
	SD	2.79E-01	4.61E-01	3.99E-01	5.00E-01	5.05E-01	6.75E-01
f_{18}	Mean	**1.00E+02**	1.07E+02	1.17E+02	1.05E+02	4.31E+02	4.31E+02
	SD	1.06E+01	1.77E+01	1.68E+01	3.08E+01	9.23E+01	1.07E+02
f_{19}	Mean	4.39E+00	**3.93E+00**	4.11E+00	4.48E+00	5.04E+04	5.26E+04
	SD	1.03E+00	1.47E+00	1.60E+00	2.20E+00	5.96E+04	8.15E+04
f_{20}	Mean	**9.89E+00**	9.91E+00	1.00E+01	1.00E+01	1.00E+01	9.99E+00
	SD	6.43E-01	4.94E-01	4.88E-02	0.00E+00	0.00E+00	8.36E-02
f_{21}	Mean	3.62E+02	3.38E+02	**2.76E+02**	3.24E+02	1.40E+03	1.30E+03
	SD	7.30E+01	7.59E+01	9.90E+01	8.85E+01	1.53E+02	1.57E+02
f_{23}	Mean	2.37E+03	**2.35E+03**	3.55E+03	3.45E+03	5.62E+03	5.18E+03
	SD	5.05E+02	6.87E+02	6.81E+02	6.51E+02	4.38E+02	6.39E+02
f_{24}	Mean	**2.23E+02**	2.34E+02	2.55E+02	2.60E+02	3.23E+02	3.18E+02
	SD	1.17E+01	1.51E+01	7.70E+00	8.98E+00	3.25E+01	2.88E+01
f_{25}	Mean	**2.44E+02**	2.49E+02	2.72E+02	2.76E+02	3.27E+02	3.33E+02
	SD	1.50E+01	1.28E+01	7.14E+00	9.67E+00	1.75E+01	2.21E+01
f_{26}	Mean	**2.07E+02**	2.40E+02	2.10E+02	2.90E+02	2.83E+02	3.21E+02
	SD	2.88E+01	5.91E+01	3.66E+01	7.24E+01	6.93E+01	7.11E+01
f_{27}	Mean	**5.14E+02**	6.04E+02	8.02E+02	8.31E+02	1.14E+03	1.14E+03
	SD	9.71E+01	8.56E+01	8.23E+01	6.58E+01	1.14E+02	1.16E+02
f_{28}	Mean	**4.78E+02**	5.90E+02	1.37E+03	2.07E+03	4.43E+03	4.35E+03
	SD	4.12E+02	5.06E+02	6.48E+02	5.77E+02	7.00E+02	7.77E+02

Bolds: Best result
U: Unimodal problems

2000, and the number of dimensions (D) is 20. The benchmark functions are used cited from CEC'13 benchmark functions [5], and are 22 non-separable problems. The optimum fitness value of each benchmark function is corrected to 0. The simulation results were evaluated by average value "Mean" and standard deviation "SD" with different initial conditions for 100 trials. In N-PSO and N-DPSO, we selected $w = 0.729$ and $c_1 = c_2 = 1.4955$. In N-PPSO, we selected $\delta_c = 0.6$, $\delta_d = 1.2$, $\theta = 55°$, and $\alpha = 0.95$. *Degree* = 2 (ring topology) and *Degree* = 19 (gbest topology) are selected. When *Degree* = 19, N-PPSO, N-PSO and N-DPSO are the standard PPSO, PSO and DPSO, respectively.

Table 1 shows the comparison results. As shown in Table 1, N-PPSO (ring topology) has the best search performance for 14 out of 22 non-separable problems, and further, N-PSO (ring topology) has better search performance for 13 out of 19 non-separable multimodal problems than PSO (gbest topology). N-DPSO is worse search performance than N-PPSO and N-PSO, because particles of N-DPSO may converge to a local optimum solution. When ring topology, N-PPSO is effective in solving non-separable multimodal problems. Therefore, PPSO with neighborhood topology has good search performances in solving non-separable multimodal problems.

6. Conclusion

Here, we proposed PPSO with locally-coupled topology (N-PPSO), and clarified effectiveness of N-PPSO compared with N-PSO and N-DPSO. PSO with the neighborhood topology (N-PSO) is effective in solving multimodal problems when *Degree* is small. As such, we introduced the neighborhood topology to PPSO. The search direction of PPSO particles tends to be biased in parallel to the coordinate axes, which can be controlled by parameters. Furthermore, this characteristic is the same regardless of the neighborhood topology between particles. In order to clarify the effectiveness of N-PPSO, N-PPSO was compared with N-PSO and N-DPSO through numerical experiments. As the results, search performances of N-PPSO are good in solving non-separable multimodal problems when neighborhood topology is ring topology (i.e., *Degree* = 2). Therefore, we concluded that N-PPSO particles do not converge to a local optimum solution prematurely when the neighborhood topology is ring topology. In our future works, we will analyze relationship between the neighborhood topology and search performances of N-PPSO in more detail. Furthermore, we will apply N-PPSO to real-world applications.

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