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Nonlinear Dynamics and Control of Walking Robot

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Abstract– Passive dynamic walking robot can walk on a slight slope without actuators. It is interesting that the walking is a stable phenomenon and is the most natural motion. The authors claim that a control law in walking robot should be constructed based on the passive dynamic walking. In this paper, we introduce a stability analysis of passive dynamic walking and show a kind of simple control law based on delayed feedback control.

1. Introduction

Robot, which has no actuators and walks down a slight slope, is called 'passive dynamic walking robot'. It is well known that the passive dynamic walking robot is important and interesting target [1]. In the past papers, for example [2] [3], a stable steady periodic motion and a chaotic behavior of passive walking robot were analyzed by numerical simulations. To justify the numerical analysis, we showed the occurrence through experiments [4].

It is interesting that the passive dynamic walking is a stable phenomenon and is the most natural motion. Therefore, we have claimed that a control law in walking robot should be constructed based on the passive dynamic walking. In this paper, we introduce a stability analysis of passive dynamic walking and show a kind of simple control law based on delayed feedback control.

The construction of this paper is as follows. In Chapter 2, we introduce a passive dynamic walking, and show the stability of the walking through experimental results. Then, we analyze the stability based on Poincare Map. In Chapter 3, we show a design method of a kind of simple control law for Quasi-Passive Dynamic Walking Robot. And also show the effectiveness of the control law through experiments. Finally, we conclude in Chapter 4.

2. Passive Dynamic Walking

In this chapter, at first, we introduce that the passive dynamic walking is a stable phenomenon [4]. Next we carry out the stability analysis of the passive dynamic walking.

In Fig.1, we show a passive dynamic walking robot "QUARTET II' developed by us. The robot has straight legs of the same length and four legs are connected via bridges and connecting links. Other four legs are also connected in the same manner. As the result, the robot can be regarded as a two-legged robot. This robot has no actuator but has potentiometers for measuring the angles of the legs.

Fig.1 QUARTET II

Fig.2 shows the walking experimental results. In the figure, the graph of step number vs. step period is presented. As you can see in this figure, the passive dynamic walking is a stable phenomenon.

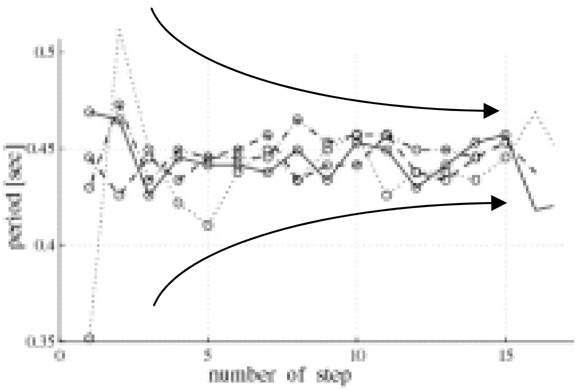

Fig.2 Stability of the walking

Through some experiments, we also showed that a kind of bifurcation phenomenon can be observed in passive dynamic walking. Since, we are very interested in this stability property, we focus on the stability of the walking in the following sections.

2.1 Model

We introduced our passive dynamic walking robot named QUARTET II in Fig.1. Although this robot has eight legs, because of its special link structure and symmetry, we can analysis it's motion by using a very simple biped walking robot model such as Fig.3. Then, hereafter, we use the biped walking robot model for analysis.

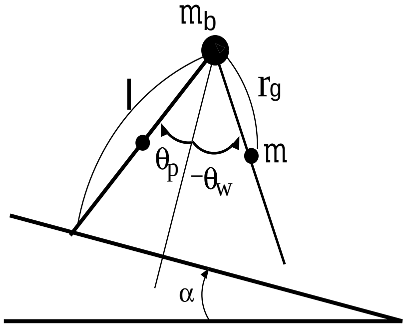

Fig.3 Model

Let the support leg angle be θ_p , swing (non-supported) leg angle be θ_w , a slope angle be parameter α . And β is the support leg angle at the collision of the swing leg with the ground.

The dynamic equation of the biped walking robot model in Fig.3 can be derived using the well known Euler-Lagrange approach:

$$M(\theta)\ddot{\theta}+N(\theta,\dot{\theta})\dot{\theta}+g(\theta,\alpha)=\tau . \tag{1}$$

Where, $\theta=[\theta_p,\theta_w]^T$, $M(\theta)$ is inertia matrix, $N(\theta,\dot{\theta})\dot{\theta}$ is Colioris and centrifugal term, $g(\theta,\alpha)$ is gravitational term, τ is input torque at the waist joints of the robot. In passive dynamic walking, τ =0.

Here, we have a linear approximated dynamical equation of Eq.(1) at an equilibrium point as follows:

$$M_0\ddot{\theta}+G_0\theta+b=\tau . \tag{2}$$

Here, setting $x=[\theta,\dot{\theta}]^T$, we have

$$\dot{x}=Ax+b+B\tau . \tag{3}$$

Next, if we assume that a transition of the support leg and the swing leg occurs instantaneously and the impact of the swing leg with the ground is inelastic and occurs without sliding, the equation of transition at the collision can be derived by using the conditions of conservation of angular momentum:

$$P_b(\beta)\dot{\theta}^-=P_a(\beta)\dot{\theta}^+ . \tag{4}$$

Where, $P_b(\beta)$, $P_a(\beta)$ are the matrices determined from the physical information of the robot and the angle β . In addition, using the matrices

$$T_r=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, T_a=\begin{bmatrix}-1 & 0\\ 1 & 0\end{bmatrix}, \tag{5}$$

we have the following state transformational equation.

$$x^+(t)=R(x^-(t))=\begin{bmatrix}T_a & 0\\ 0 & T_rP_a^{-1}P_b\end{bmatrix}x^-(t) . \tag{6}$$

Furthermore, we can regard that the collision of the swing leg to the ground occurs at timing when the state of the robot reaches a constraint plane. Therefore, using a constant matrix C=[1 1 0 0], this condition can be written as the following.

$$Cx^-=0 . \tag{7}$$

In the next section, the stability analysis of the passive dynamic walking is done by using the linearized state equation (2), the collision equation (4), and the jump condition (7). The state of the robot immediately before the swing leg lands on the ground is defined as "impact point". The point can be written as

$$p(k)=(\beta_k,\dot{\theta}_{p,k}^-,\dot{\theta}_{w,k}^-)^T . \tag{8}$$

2.2 Stability[5]

Let x_*^- be the steady impact point during passive dynamic walking, and let τ_* be the steady walking period during the walking. Let

$$\Delta x_k=x_k^--x_*^- \tag{9}$$

be the state error at the k-th impact. Then we have

$$\Delta x_{k+1}=\vec{P}(\Delta x_k) . \tag{10}$$

Where $\vec{P}$ is so called Poincare map.

Here, referring the method introduced in [5], we have

$$P_k=\left.\frac{\partial\vec{P}_k}{\partial x}\right|_{x=x_*^-}=Se^{A\tau_*}R_d . \tag{11}$$

Where,

$$S=I-\frac{v_*C}{Cv_*},\ R_d=\left.\frac{\partial R(x)}{\partial x}\right|_{x=x_*^-},v_*=Ax_*^-+b . \tag{12}$$

This map (11)(12) can be obtained as the following. At first, let us see that the relationship between the two impact points x_{k-1}^-,x_k^- as,

$$\begin{aligned}\Delta x_k &= x_k^-(x_*^-+\Delta x_{k-1},\tau_*+\Delta\tau_k)-x_*^-\\ &= e^{A\tau_*}R_d\Delta x_{k-1}+v_*\Delta\tau_k+o(\Delta\tau_k) .\end{aligned} \tag{13}$$

Also see that, since the states x_k^-, x_*^- satisfy the jump condition, we have $Cx_k^-=0,\ Cx_*^-=0$. Then using these equations, substituting the equation

$$\Delta\tau_k=-\frac{1}{Cv_*}Ce^{A\tau_*}R_d\Delta x_{k-1}+o(\Delta\tau_k) \tag{14}$$

into Eq.(12), then have Eqs. (11) and (12).

Then, notice that Eqs.(13) and (14) can be rewritten as

$$\begin{cases}\Delta x_k=Z_A\Delta x_{k-1}+Z_B\Delta\tau_k\\ \Delta\tau_k=-Z_K\Delta x_{k-1}\end{cases} . \tag{15}$$

Where,

$$Z_A=e^{A\tau_*}R_d,\ Z_B=v_* ,$$

$$Z_K=\frac{1}{Cv_*}Ce^{A\tau_*}R_d,\ P_k=Z_A-Z_BZ_K . \tag{16}$$

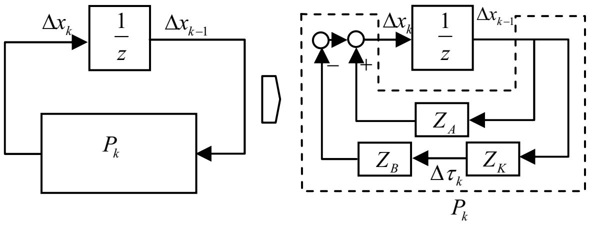

Fig.4 Stability analysis of passive dynamic walking

From the above discussion, we can say that, as shown in Fig.4, a feedback structure is included in the Poincare map. This feedback structure plays an important rule in stability of passive walking. In other words, this is the reason why the passive dynamic walking is stable.

3. Quasi-Passive Dynamic Walking

In the previous chapter, the stability of passive dynamic walking was discussed. This is a very interesting feature. But to develop a walking "robot" instead of "machine", we have to imbed a kind of controller in the robot. In this chapter, we introduce a control law developed by us based on the passive dynamic walking.

3.1 Quartet III

Fig.5 shows the walking robot “Quartet-III” which was developed to study of Quasi-Passive Dynamic Walking.

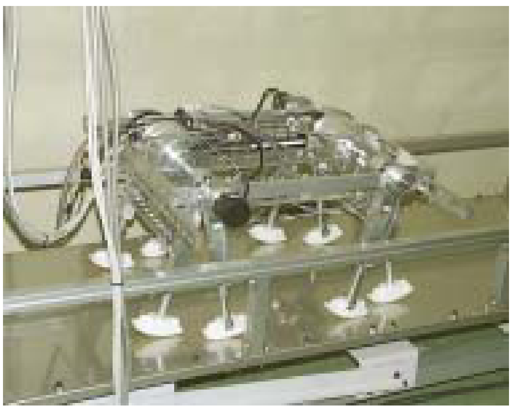

Fig.5 QUARTET III[6]

The robot has the following features.

- The robot has a pair of legs. Each leg consists of the four legs which are connected each other by links. Therefore, QUARTET III can be regarded as a simple compass type walker shown in Fig.3.
- The robot has two Direct Drive Motors.
- Each leg has a DC motor for expansion and contraction of the length of the leg.
- The shape of the each foot is circle type. We knew through simulations that this shape is suited for passive walking.

The experimental setup of this system is shown in Fig.6.

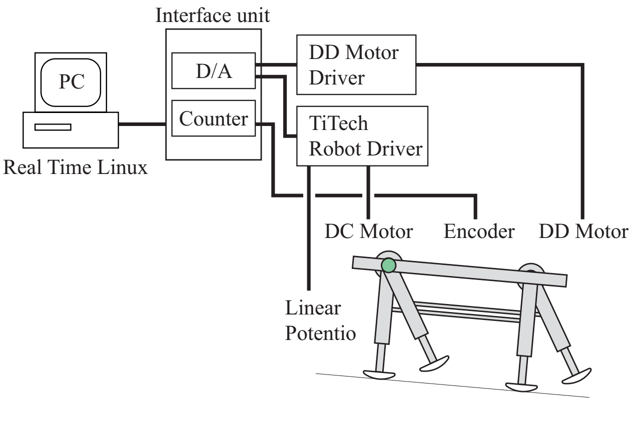

Fig.6 Experimental setup

3.2 Control Laws

In this section, we show the newest control method of Quasi-Passive Dynamic Walking. The control method is derived from the two previous control methods of Quasi-Passive Dynamic Walking. The one is named as “Weekly guidance control method”[6] and the other is named as “Discrete-Delayed Feedback Control based control method”[7]. Because of the limitation of the space, we omit the explanation of the two methods.

The control law which we proposed is the following[8].

--------Continuous-DFC based control method-------

$$\tau_k = K_f(\delta_k)\left[K_v(\dot{\theta}_{k-1} - \dot{\theta}_k) + K_p(\theta_{k-1} - \theta_k)\right] \quad (17)$$

$$\delta_k = \left\| p(k) - p(k-1) \right\|_\phi \quad (18)$$

$$K_f(x) = \begin{cases} 1 & \|x\| \geq \gamma \\ \left(1 - \cos\left(\dfrac{x\pi}{\gamma}\right) + 1\right) \Big/ 2 & \|x\| \leq \gamma \end{cases} \quad (19)$$

where θ_k is k th step's $\theta = [\theta_p, \theta_w]^T$, $\phi \in \boldsymbol{R}^3$ is a constant matrix and $\|\bullet\|_M$ is a norm defined by $\|x\|_M = \sqrt{x^T M x}$ and a constant matrix $M \in \boldsymbol{R}^3$. See Fig.7 for $K_f(x)$.

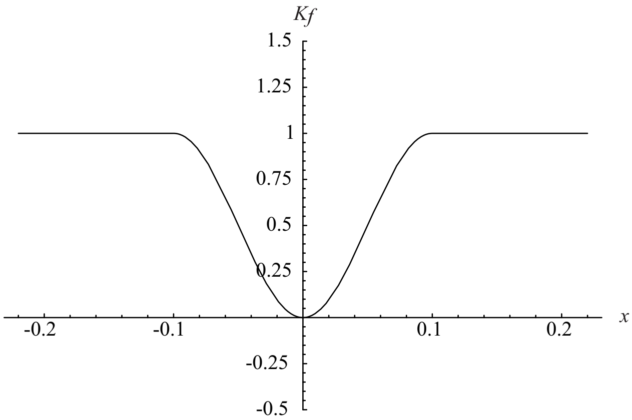

Fig.7 $K_f(x)$ (case of γ =0.1)

This control method is developed by making use of the characters of the two previous our proposed control methods skillfully, and it have the following two big features.

- It regards $\|p(k)-p(k-1)\|_{\phi}$ as a function of the stability of robot's walking and regulates the tracking control gain depending on it.
- It realizes tracking control not with a reference trajectory which is made in advance but with the $(k-1)-th$ trajectory of the robot. As a result, the reference trajectory is updated in each step.

From the first feature, we can expect that a robot with the proposed control method will realize continuous walking easily than that with Discrete-DFC based control method. And, because of the term K_f, the robot's walking will be able to converge to PDW easily. From the second feature, we can expect that, during a robot walks continuously, its walking will converge to the ideal trajectory $r_{id}=[\theta_{id},\dot{\theta}_{id}]^T$ without making correctly design of the PDW's trajectory. The proposed control method needs to make an initial reference trajectory $r_0=[\theta_0,\dot{\theta}_0]^T$. However, since it can expected that the robot itself makes the ideal trajectory r_{id} during the walking, it is enough to make roughly design of r_0 with which robot's walking can start without falling down. This situation means the proposed control method overcomes Weekly guidance control method's problem which is how to make the reference trajectory.

3.3 Experimental Results

To show the effectiveness of the proposed control law, we carried out some experiments. The main part of the control law which we used is the following.

$$\tau_k=\begin{cases} K_f(\delta_k)\left[K_v(\dot{\theta}_{k-1}-\dot{\theta}_k)+K_p(\theta_{k-1}-\theta_k)\right] \\ \quad if\ T_p(k-1)\le t\le 2T_p(k-1)-T_p(k-2) \\ 0 \quad otherwise \end{cases} \quad . \tag{20}$$

This control law means that the input torque is set as 0 when the time from the (k-1)-th impact time $T_p(k-1)$ is longer than (k-1)-th step's walking period, that is, when there are no corresponding reference trajectories, and otherwise, input torques of Eq.(20) are equivalent to that of Eq.(17).

Before carrying out the walking experiments, we obtained the trajectories of PDW r_{id-sim} by computer simulations in which the slope angle α was set as 3.0[deg] and we used them as the first reference trajectory r_0. The parameters in Eq.(20) were set as the following.

$$K_p=\begin{bmatrix}25 & 0\\ 0 & 25\end{bmatrix}, K_v=\begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix} \tag{21}$$

$$\phi=\begin{bmatrix}5.0 & 0.0 & 0.0\\ 0.0 & 0.1 & 0.0\\ 0.0 & 0.0 & 0.1\end{bmatrix}, \quad \gamma=0.5 \tag{22}$$

In Fig.8, we show the experimental result. The horizontal axis is time. And the vertical axis is torque of the two legs.

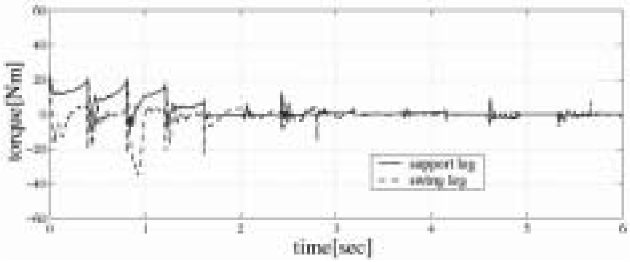

Fig.8 Experimental result

As this figure shows, we can see that the torque of the legs convergence to zero. That is, we could achiev Quasi-Passive Dynamic Walking.

4. Conclusion

In this paper, we introduced that the phenomenon of passive dynamic walking is stable. And then, showed a kind of simple control law based on delayed feedback control. We called the control law as Continuous-DFC based control method, and showed the effectiveness of the method through experiments.

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