Developing Parametrically-Resonant MEMS for Sensing

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Abstract– MEMS and NEMS provide a novel and exciting medium where one can observe and utilize mathematical phenomena not often present in macro-scale systems. By understanding thoroughly the dynamics of resonant MEMS, we can not only predict device behavior to eliminate unwanted effects, but also use nonlinear effects to design better sensors and systems. We present a basis for the development of sensors based on the parametric resonance phenomena. As an example, a mass sensor, comprised of a single-crystal silicon micro-oscillator with sensitivity at the pico-gram (10^{-12} g) level when operating in air. Is demonstrated This mass sensor detects mass change by measuring frequency shift at the boundary of the first order parametric resonance 'tongue'. High sensitivity is achievable due to the sharp jump in amplitude caused by parametric resonance at predictable drive frequencies. The sensor results show that the sensitivity can be > 1 order higher than the same oscillator working at simple harmonic resonance mode in air. Higher sensitivity is expected when the oscillator design is optimized and dimensions are scaled.

1. Introduction

Resonant mode operation is common for many applications including on-chip mechanical filters/switches/mixers in the RF range [1], scanning probe microscopy [2], small force detection, among other sensors. When operated in the linear regime as simple harmonic oscillators (SHO) there exists a tradeoff between the selectivity and the bandwidth of operation. In addition, at the microscale, the quality factor (governing selectivity) is primarily controlled by viscous damping which depends on the ambient conditions and cannot be controlled in open loop implementations. Parametrically resonant devices can be used to avoid a direct relationship between quality factor and selectivity/sensitivity for some of these applications.

Parametric resonance has been well established in many areas of science, including the stability of ships, the forced motion of a swing and Faraday surface wave patterns on water. We have previously investigated a linear parametrically driven torsional oscillator [3] and along with other groups have mentioned applications including mass sensing, parametric amplification, and others. As an example, we demonstrate a mass sensor which utilize parametric resonance.

2. Example: Mass Sensor

2.1. Background

As the technology of miniaturization develops rapidly, building micro/nano scale oscillators becomes possible with much smaller mass and much higher frequencies than traditional mechanical oscillation systems. The concept of tracking resonant frequency (or phase) shifts of micro/nano-oscillators in the simple harmonic resonance mode to measure mass change has become a well–established technology in applications of chemical and biological sensing. Since the fundamental resonant frequency of simple harmonic resonance depends on the mass and stiffness of the oscillator, mass change can cause resonance frequency to shift and so can be tracked by monitoring this frequency shift. Eq.(1) and (2) describe the relationship between mass change and frequency shift. High sensitivity can be achieved in mass sensing using an oscillator with extremely small mass and high resonant frequency, thus attracting attention for sensor applications. In a microcantilever array, information on beam frequency shifts can be used for recognition of a variety of chemical substances.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

(1)

$$\frac{dm}{m} = -2\frac{df_0}{f_0}$$

(2)

Theoretically, any mass change in the sensing oscillator can cause a certain amount of frequency shift. However, the detection limit of frequency shift is governed by noise and the quality factor. Quality factor (Q), which denotes the sharpness of frequency response curve of simple harmonic resonance, is one of the important factors that

limit the sensitivity of the simple harmonic resonance (SHR) based mass sensors [5].

In our previous work, we have reported the conceptual basis of mass sensing using the parametric resonance phenomenon [4]. In this approach, mass change is monitored by tracking the frequency shift at the stability boundary of the first order parametric resonance 'tongue'. The frequency transition at this boundary is very sharp [3], thereby making small frequency changes easily detectable and the frequency shift resolution high. The sharpness of the boundary does not depend on the quality factor. Therefore, very small mass change can be detected in high-pressure environments, such as in air or even in water, where the sensitivity can be as high as in high vacuum.
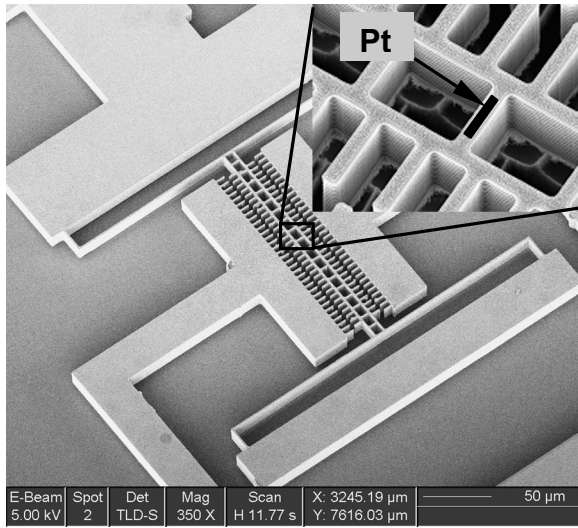

Figure 1. *A SEM picture of the mass sensor. It has a backbone, 4 springs with folded beams to provide recovery force and one set of non-interdigitated comb-fingers to drive the oscillator. Pt deposition to change mass is schematically shown in this picture.*

2.2. Theoretical Basis of Operation

The micro-oscillator, shown in Fig.1, can be simplified as a mass-spring system with electrostatic force as the driving force. When voltage is applied on the non-interdigitated comb-fingers, the electrostatic force generated is dependent on the position of the oscillator[14]. In the experiments presented here, we use a square rooted AC voltage signal ($V_A \left(1 + \cos 2\omega t\right)^{1/2}$) to isolate the parametric resonance from direct harmonic response [3]. The movement of the device is governed by the nonlinear Mathieu-Hill equation with cubic nonlinearity[4].

$$\frac{d^2 x}{d\tau^2} + \alpha \frac{dx}{d\tau} + (\beta + 2\delta \cos 2\tau)x + (\delta_3 + \delta_3' \cos 2\tau)x^3 = 0$$

$$(3)$$

Here $\qquad \alpha = \frac{2c}{m\omega} \qquad \beta = \frac{4(k_1 + r_1 V_A^2)}{m\omega^2} \qquad \delta = \frac{2r_1 V_A^2}{m\omega^2}$

$$\delta_3 = \frac{4k_3 + 4r_3 V_A^2}{m\omega^2} \qquad \delta_3' = \frac{4r_3 V_A^2}{m\omega^2}$$

where m, k_1 and k_3 are the mass, linear and cubic mechanical stiffness of the oscillator respectively, c is the damping coefficient, r_1 and r_3 are linear and cubic "electrostatic stiffness" and $\tau = \omega t$ is a normalized time [4].

Figure 2 schematically shows the dynamics of this nonlinear Mathieu equation in β-δ plane defined above. According to dynamic characteristics, β-δ plane can be divided into three areas. Area II (inside the "tongue") is the resonance area of the first order parametric resonance with a non-trivial solution, while area I and III are non-resonance area with one trivial solution in area I and one trivial solution plus one non-trivial solution in area III. The characteristics of solutions and phase plane in each area are schematically shown in Fig.2 as well, where "S" means stable and "U" means unstable. Here we only consider stable solution, since unstable solution cannot be observed experimentally. The dynamics can be more easily understand in an experimental frequency response as shown in Fig.3.

Figure 3 shows the frequency response of parametric resonance inside Areas I, II, and III for the device shown in Fig.1. As the driving frequency goes up, the amplitude of the oscillator increases from the left boundary of area II and keeps increasing past the boundary of area III. The frequency at the right boundary of area II, where the "jump" happens, is given by $f = \frac{1}{2\pi} \sqrt{4k + 2rV_A^2 \Big/ m}$, where k, m, V_A and r are stiffness, mass, driving voltage amplitude applied and the coefficient of electrostatic force respectively. A small mass change (Δm) in the oscillator causes this "jump" frequency to shift (Δf). Therefore, by measuring the frequency shift, this mass change can be determined by $\left|\Delta m\right| - 2m \Big|\Delta f\Big| \Big/ f_0$.

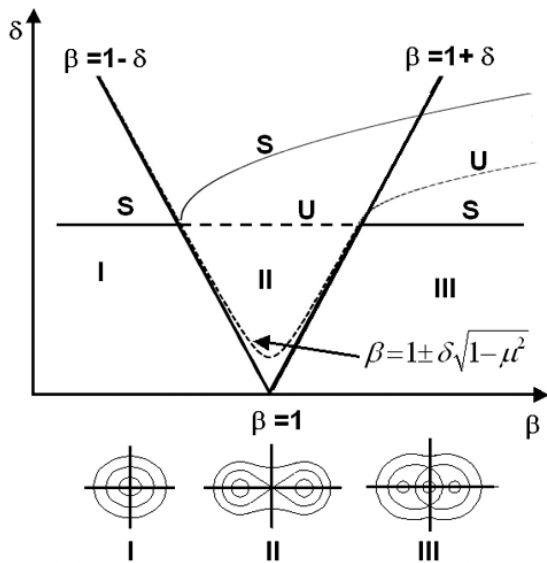*Figure 2. Dynamic characteristics of Nonlinear Mathieu Equation in the β-δ plane. β=1±δ are the transition curves, which divide β-δ plane into Area I, II, III*
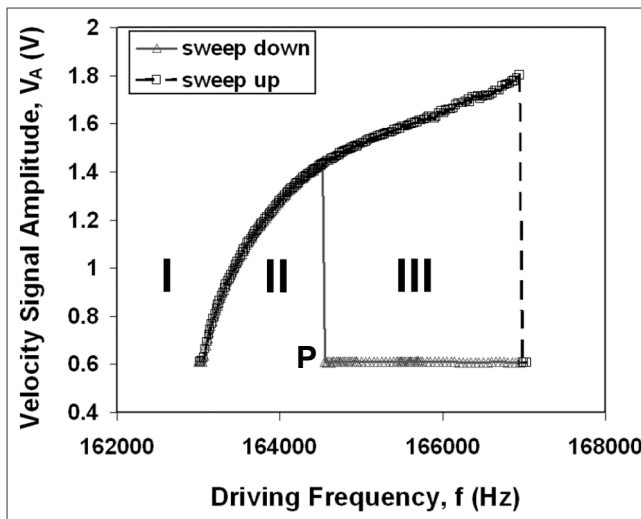

Figure 3. A frequency response curves of the first order parametric resonance inside and outside of stability region. Area II is inside of "tongue", as shown in Fig.2, while I and III are outside. When sweep frequency down, resonance only happens in area II and there is completely no movement in area I and III. A sharp jump happens at the right boundary of area II near point P.

The sensitivity of parametric resonance based mass sensor depends on the smallest frequency shift we can measure. Since the "jump" is a characteristic of nonlinear dynamics, where the characteristics of the dynamical equations change, the transition is very sharp and the change of frequency response is extremely 'fast' in the frequency domain. Therefore, the frequency at the boundary can be accurately defined. Very small frequency shifts caused by mass change can be easily detected. In previous work, a $0.001Hz$ frequency shift (in 58,000Hz) has been observed [16]. Damping affects the resonance boundary of the tongue, as shown in Fig.2. At high damping, for example in air, actuation of parametric resonance requires more energy than at low pressure. However, the characteristics of dynamics remain the same as in low damping environments and the transition is still very sharp. Therefore, the frequency at the boundary can be still accurately determined and damping has little effect on the sensitivity of parametric resonance based mass sensors.

2.3. Experimental Mass Sensing Results

To perform a test to find the ultimate sensitivity of this prototype device, the mass sensor was tested using adsorption of water vapor. The prototype oscillator is made of single crystal silicon. After it is exposed to air, a thin layer of native silicon oxide grows on the surface. This native silicon oxide can absorb water molecules when it is exposed to environment with humidity[6]. Therefore, the mass of the device changes as humidity level varies. A gas handling setup is built to control the water vapor content in testing chamber, as shown in Fig.4. A dry nitrogen gas flows through mass flow controller #1 directly to the testing chamber and another dry N_2 through mass flow controller #2 with water vapor from the water bubbler. By setting the flow rates of the two mass flow controllers, relative water content can be adjusted in the testing chamber. The frequency information of device #4 at the right boundary of the first order parametric resonance area is recorded, as water content in the testing chamber changes. By adjusting the water content, the resolution of frequency shift can be found and corresponding mass change can be determined. Since the test is performed in a relatively short time and the humidity level is varied in a controllable way, errors caused by temperature fluctuation are negligible. Figure 5 shows the results of frequency shift as the relative water content is adjusted. When water content is switched between 5% and 10% as shown in Fig.13 from A-B, the rate of frequency shift changes accordingly. As water vapor is turn on and off from B-C, the results shows good consistency between frequency shift and water vapor content change. The smallest resolvable frequency shift we can measure using this device is less than 2 Hz, as shown in Fig.13, which is equivalent to a 0.7 pg mass change.
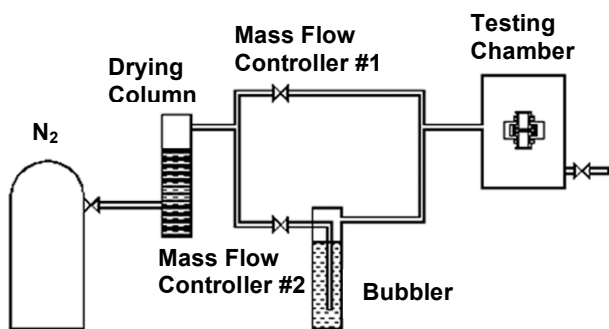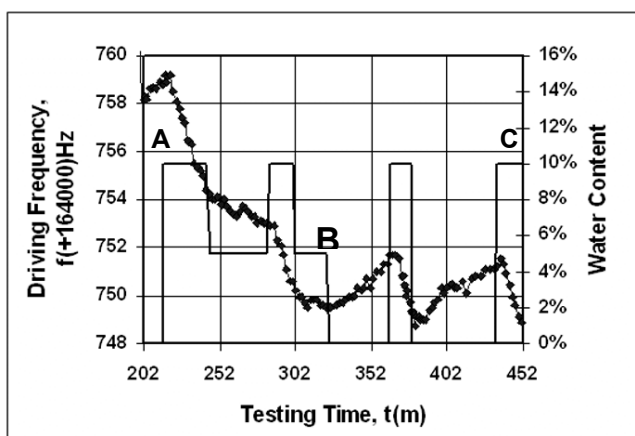
Figure 4. *Gas handling setup*

Figure 5. Frequency shifts at the right side of the first parametric resonance area as water content is adjusted in the testing chamber

2.6. Noise Effects

As with other mass sensors, noise is an important consideration when developing mass sensing using parametric resonance technology. The ability to detect ultra-fine frequency shift because of the nature of parametric resonance is compromised by noise effects. As we mentioned earlier, a 0.001 *Hz* or even smaller frequency shift has been observed **[6]**. However, in our current experiment, the frequency fluctuation is much larger than this value and the standard deviation of this frequency fluctuation we measured in these two oscillators is about 0.8 *Hz* at room temperature. To analyze noise problems in mass sensing, we consider thermal noise, Brownian motion and Driving voltage fluctuation. Because of the complexity of parametric resonance, numerical simulation method is used to calculate the effects of these noise sources. Brownian motion of oscillators is attribute to be the main noise

source in parametric resonance mass sensing. In room temperature, the standard deviation of Brownian motion is estimated about 0.5 *Å*, which brings about 0.7 *Hz* of frequency fluctuation. The result agrees well with measured result.

Certainly, there are other issues, which can be considered noise effects in mass sensing, such as frequency drift caused by humidity fluctuation and temperature fluctuation in the environment. We notice that noise caused by this frequency drift can be larger than that caused by Brownian motion even with reference sensor when the test lasts days. These issues are currently being addressed.

To find the ultimate sensitivity of this mass sensorg based on parametric resonance, we use the micro-oscillator to detect water content change in the testing chamber. By adjusting the water content, we observe the frequency shift simultaneously. Figure 8 shows the results of frequency shift of Device#4 as the relative water content is changed in air. When water content is switched between 5% and 10% as shown in Fig.8 from A-B, the rate of frequency shift changes. As water vapor is turn on and off from B-C, the results shows good consistency between frequency shift and water vapor content. The smallest controllable frequency shift as shown in Fig.8 is about 2 *Hz*, which is equivalent to mass change of 0.7 *pg*.

3. Conclusion

Parametric resonance can be very useful in micro and nanoscale sensors. We have demonstrated a mass sensor with pg sensitivity in air, which is nearly 2 orders of magnitude higher than the same sensor operated in a harmonically resonant mode. Ongoing work involves improving the resolution through miniaturizing and optimizing the design and detection processes.

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