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Nonlinear lateral vibration of a fluid-conveying pipe with end mass

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Abstract— The nonlinear lateral vibrations of a cantilevered pipe, which is hung vertically with an end mass and conveys fluid, are examined for the case that the upper end of the pipe is excited periodically in a horizontal direction. The fluid velocity is slightly over the critical value, above which the lateral vibration of the pipe is self-excited due to the internal flow. First, the interactions between the forced and self-excited pipe vibrations are discussed theoretically with the derived complex amplitude equations of the pipe deflections. Second, the experiment was conducted with the silicon rubber pipe conveying water. The plane vibration of the pipe was observed for the case that the upper end of the pipe is excited at the frequency which is equal to the self-excited frequency of the pipe, though the nonplanar vibration of the pipe is observed for the case of no forced excitation.

1. INTRODUCTION

Flow-induced nonplanar vibration of a fluid-conveying pipe is one of the attractive phenomena from the viewpoint of nonlinear dynamics (Paidoussis and Li, 1993). Bajaj and Sethna (1991) also studied theoretically and experimentally three-dimensional oscillatory motions of a cantilevered pipe, where small different bending stiffnesses in two mutually perpendicular directions are imposed to break the rotational symmetry. Furthermore, Copeland and Moon (1992) clarified experimentally that the addition of an end mass to a cantilevered pipe yields a rotational symmetric system with many types of nonplanar vibrations. Yoshizawa *et al.*(1998) examined theoretically and experimentally the effects of the end mass on the nonplanar pipe vibration.

In this paper, nonlinear lateral vibrations of a cantilevered pipe, which is hung vertically with an end mass and conveys fluid, are examined for the case that the upper end of the pipe is excited periodically in a horizontal plane. The fluid velocity is slightly over the critical value, above which lateral pipe vibration is self-excited due to an internal flow.

First, the four first-order ordinary differential equations governing the amplitudes and phases of the lateral deflection of the pipe are derived from the nonlinear partial differential equations of nonplanar pipe vibration. The interactions between the forced and flow-induced pipe vibrations are examined theoretically by solving numerically the obtained equations of the amplitudes and phases.

Second, the experiments were conducted with a silicon rubber pipe. As a result, the typical effect of horizontal excitation on nonplanar flow-induced pipe vibration, predicted in the theory, was confirmed qualitatively by experiment.

2. BASIC EQUATIONS

The system under consideration (Figure 1), consists of a flexible pipe with an end mass M, conveying an incompressible fluid, which is discharged into an atmosphere at the free end of the pipe. The pipe of length l, flexural rigidity EI, mass per unit length m and cross-sectional flow area S, is hung vertically under the influence of gravity $\boldsymbol{g}$ in its equilibrium state. The pipe is sufficiently long compared with its radius, and its centerline is inextensible. The internal fluid of density ρ is incompressible. The axial fluid velocity v_s relative to the pipe motion is assumed to be maintained at constant. We use two systems of co-ordinates : a fixed system XYZ, and a moving system xyz, to describe the motion of a pipe. The origin of the moving system is taken to coincide with the upper clamped end of the pipe, which is excited periodically in a horizontal direction as follows:

$$Y_0 = \delta Y \sin Nt \tag{1}$$

Let $v(s,t)$ and $w(s,t)$ be the deflections of the pipe centerline in the y and z directions respectively, which are expressed as functions of co-ordinate s along the pipe axis and time t. Then the equations governing the spatial behavior of the pipe are derived under the assumptions that v and w are small but finite, and the pipe has no torsion about its centerline (Yoshizawa, 1998 and Watanabe, 1996).

Introducing the dimensionless variables which carry the asterisk, i.e. $v = lv^*, w = lw^*, s = ls^*, t = \sqrt{(m+\rho S)l^4/(EI)}t^*$, and retaining terms up to the third order of v^* and w^*, the governing equations of $\boldsymbol{v}_i$ $(i = v, w)$ are expressed in the vector form as follows :

$$\frac{\partial \boldsymbol{v}_i}{\partial t} = L\boldsymbol{v}_i + \boldsymbol{N}_i \tag{2}$$

$$\begin{cases} s = 0 \quad : \quad B_1\boldsymbol{v}_i = \boldsymbol{0} \\ s = 1 \quad : \quad B_2\dot{\boldsymbol{v}}_i = B_3\boldsymbol{v}_i - \boldsymbol{N}_{bi} \end{cases} \tag{3}$$

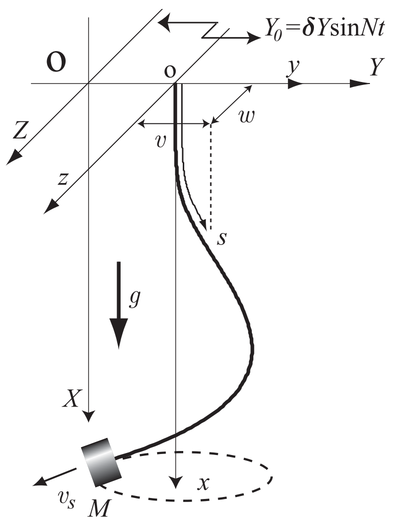

Figure 1: Analytical model

$$
\begin{aligned}
L &= \begin{bmatrix} 0 & 1 \\ L_{21} & -2\sqrt{\beta}V_s(\cdot)' \end{bmatrix} \\
\boldsymbol{N_i} &= \begin{bmatrix} 0 \\ n(i,j) + \delta_{i,j}k\nu^2 \sin \nu t \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ (\cdot)' & 0 \end{bmatrix} \\
B_2 &= \begin{bmatrix} 0 & 0 \\ 0 & \alpha \end{bmatrix}, B_3 = \begin{bmatrix} (\cdot)'' & 0 \\ (\cdot)''' - \alpha\gamma(\cdot)' & 0 \end{bmatrix} \\
\boldsymbol{N_{bj}} &= \begin{bmatrix} 0 \\ b(i,j) + \alpha k\nu^2 \sin \nu t \end{bmatrix} \\
L_{21} &= -(\cdot)'''' + \gamma\{(\alpha + 1 - s)(\cdot)'\}' - V_s^2(\cdot)''
\end{aligned}
$$

where $\dot{(\cdot)}$ and $(\cdot)'$ denote the derivatives with respect to t and s, respectively. In the equations (2) and (3), $n(i,j)$, $b(i,j)$ are expressed as the third order nonlinear polynomials with respect to v and w and $\delta_{i,j}$ is the Kronecker's delta.

There are six dimensionless parameters involved in equations (2), (3), i.e. the dimensionless velocity $V_s = v_s/\sqrt{EI/(\rho S l^2)}$, the ratio of the lumped mass to the total mass $\alpha = M/(m+\rho S)l$, the ratio of the fluid mass to the total mass $\beta = \rho S/(m+\rho S)$, the ratio of the gravity force to the elastic force of the pipe $\gamma = (m+\rho S)gl^3/EI$, the dimensionless amplitude of the excitation $k = \delta Y/l$, $(k \ll 1)$ and the dimensionless frequency of the excitation $\nu = N\sqrt{(m+\rho S)l^4/(EI)}$.

3. METHOD OF SOLUTION

3.1. LINEAR STABILITY

Neglecting the nonlinear terms with respect to v, w, and putting $k = 0$ in equations (2) and (3), $\boldsymbol{v}_v$ and $\boldsymbol{v}_w$ become independent of each other, and are described by the same equations and boundary conditions.

Letting $\boldsymbol{v}_v = \boldsymbol{q}_v e^{\lambda_v t}$, $\boldsymbol{q}_v = {}^t[\Phi_{v1}(s), \Phi_{v2}(s)]$ and substituting them into equations (2) and (3), we can cast into the eigenvalue problem. The eigenvalue λ_v, being the root of the complex characteristic equation which is symbolically described by

$$f(\lambda_v : V_s, \alpha, \beta, \gamma) = 0 \tag{4}$$

can be found numerically from the equation. The eigenvalue λ_v is equal to $i(\omega_r + i\omega_i)$, where ω_r is the linear natural frequency and ω_i corresponds to the damping coefficient.

The lowest value of V_s, at which the flow-induced pipe vibration appears for the second mode, is 6.03 and will be reffered to as the critical flow velocity V_{cr} in the case of $\alpha = 0.13, \beta = 0.26$, $\gamma = 20.6$.

An eigenvector $\boldsymbol{q}_v$ of the second mode, which is used in the following section, can be found in the form of a power series of s, and satisfies the condition $\langle \boldsymbol{q}_v, \boldsymbol{q}_v \rangle = 1$ where brackets denote the inner product $\langle \boldsymbol{x}, \boldsymbol{y} \rangle = \int_0^1 {}^t\boldsymbol{x}(s)\overline{\boldsymbol{y}(s)}ds$.

Moreover we get the equations of the adjoint vector $\boldsymbol{q}_v^* = {}^t[\Psi_{11}(s), \Psi_{12}(s)]$ of $\boldsymbol{q}_v$ from the condition:

$$\langle L\boldsymbol{q}_v, \boldsymbol{q}_v^* \rangle = \langle \boldsymbol{q}_v, L^*\boldsymbol{q}_v^* \rangle \tag{5}$$

The adjoint vector $\boldsymbol{q}_v^*$, which is expressed in the form of a power series of s, also satisfies the condition $\langle \boldsymbol{q}_v, \boldsymbol{q}_v^* \rangle = 1$.

3.2. NONLINEAR STABILITY

In this subsection, the equations governing the amplitudes and phases of $\boldsymbol{v}$ and $\boldsymbol{w}$ are derived for the case when the flow velocity V_s is near the critical velocity V_{cr}.

The Banach space, which includes $\boldsymbol{v}_v$ and $\boldsymbol{v}_w$, is expressed as $\boldsymbol{Z} = \boldsymbol{X} \oplus \boldsymbol{M}$ (Paidoussis and Li,1993). $\boldsymbol{X}$ is the eigenspace spanned by the eigenvectors $\boldsymbol{q}_v$ and $\boldsymbol{q}_w$, which correspond to the linear unstable vibration modes of $\boldsymbol{v}_v$ and $\boldsymbol{v}_w$, respectively. $\boldsymbol{M}$ is the subspace of $\boldsymbol{X}$. Therefore $\boldsymbol{v}_i$ $(i = v, w)$ are expressed as follows:

$$\boldsymbol{v}_i(s,t) = a_i(t)\boldsymbol{q}_i(s) + \boldsymbol{y}_i(s,t) + C.C. \tag{6}$$

where $\boldsymbol{y_v}$ and $\boldsymbol{y_w}$ are the elements of $\boldsymbol{M}$.

Using the projection P_i onto $\boldsymbol{X}$, the equations (2) with boundary conditions (3) are decomposed as follows:

$$P_i \frac{\partial \boldsymbol{v}_i}{\partial t} = P_i L \boldsymbol{v}_i + P_i \boldsymbol{N}_i, \qquad (i = v, w) \tag{7}$$

where $P_i\boldsymbol{x} = \langle \boldsymbol{x}, \boldsymbol{q}_i^* \rangle \boldsymbol{q}_i$.

From equations (7), the equations of a_i are derived as follows:

$$
\begin{aligned}
\dot{a}_i = \lambda_i a_i &+ (\xi_1 a_i^3 + \xi_2 a_i a_j^2 \\
&+ \xi_3 |a_i|^2 a_i + \xi_4 |a_j|^2 a_i + \xi_5 \overline{a}_i a_j^2) + f_i
\end{aligned} \tag{8}
$$

where $i = v$, $j = w$, $f_v = \xi_6 k\nu^2 \sin \nu t$, and $i = w$, $j = v$, $f_w = 0$. The constant coefficients $\xi_1, \xi_2, \cdots, \xi_6$ in equations (8), are numerically determined as functions of α, β, γ and V_s.

Letting $a_v = h_v e^{i\varphi}/2$ and $a_w = h_w e^{i\vartheta}/2$, separating the real and imaginary parts of equation (8), and averaging them by the period $2\pi/\omega_r$ under the following assumption:

$$\nu \equiv \omega_r + \sigma, \quad (|\sigma| \ll \omega_r) \tag{9}$$

where ω_r is the linear natural frequency of the unstable second mode in this paper.

Furthermore we define the phase difference Ω between a_w and a_v, and the phase difference η between Y_0 and a_v, respectively, as follows:

$$\Omega = \vartheta - \varphi, \quad \eta = \nu t - \varphi \tag{10}$$

Finally the autonomous equations governing h_v, Ω, h_w and η are expressed as follows:

$$\begin{aligned} \dot{h}_v &= -\omega_i h_v + (\xi_{4re} + \xi_{5re}) h_v^3/4 \\ &+ (\xi_{4re} + \xi_{5re} \cos 2\Omega - \xi_{5im} \sin 2\Omega) h_v h_w^2/4 \\ &+ k\omega_r^2 (\xi_{6im} \cos \eta + \xi_{6re} \sin \eta) \end{aligned} \tag{11}$$

$$\begin{aligned} h_v \dot{\eta} &= h_v \sigma - (\xi_{4im} + \xi_{5im}) h_v^3/4 \\ &- (\xi_{4im} + \xi_{5im} \cos 2\Omega + \xi_{5re} \sin 2\Omega) h_v h_w^2/4 \\ &+ k\omega_r^2 (\xi_{6re} \cos \eta - \xi_{6im} \sin \eta) \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{h}_w &= -\omega_i h_w + (\xi_{4re} + \xi_{5re}) h_w^3/4 \\ &+ (\xi_{4re} + \xi_{5re} \cos 2\Omega + \xi_{5im} \sin 2\Omega) h_w h_v^2/4 \end{aligned} \tag{13}$$

$$\begin{aligned} h_v \dot{\Omega} &= \{\xi_{5re} \sin 2\Omega + \xi_{5im} (\cos 2\Omega - 1)\} h_v^3/4 \\ &+ \{\xi_{5re} \sin 2\Omega - \xi_{5im} (\cos 2\Omega - 1)\} h_v h_w^2/4 \\ &+ k\omega_r^2 (\xi_{6re} \cos \eta - \xi_{6im} \sin \eta) \end{aligned} \tag{14}$$

where $\xi_j = \xi_{jre} + i\xi_{jim}$ $(j = 4, 5, 6)$.

4. THEORETICAL RESULT

4.1. THE CASE OF NON-FORCED EXCITATION

The transient time histories of h_v, h_w and Ω of nonplanar flow-induced pipe vibration for κ=0, V_s=6.50, α=0.13, β=0.24, γ=0.26, which are calculated numerically from equations (11),(13) and (14), are shown in Figures 2(a) and 2(b). The initial values of h_v, h_w and Ω are $1\times10^{-3}, 1\times10^{-3}$ and $2\pi/5$, respectively. After some time, the values of h_v and h_w converge to $h_{np} = 0.783$, and also the value of Ω converges to $\Omega_s = \pi/2$. As a result, the steady-state pipe motion in a horizontal plane at s=0.772 takes on a circular shape, as shown in Figure 2(c). The steady-state pipe motion don't depend on the initial conditions.

4.2. THE CASE OF FORCED EXCITATION

The transient time histories of h_v, h_w and η, as shown in Figure 3(a) and (b), are obtained numerically from equations (11) through (14) for κ=0.0077, σ=0.3, ν=16.0, V_s=6.50, α=0.13, β=0.24, γ=0.26. The initial values of h_v, h_w, Ω and η are $0.1, 0.1, 2\pi/5$ and 1, respectively. After some time, h_v converges to $h_{vs} = 1.43$, h_w converges to zero and η also converges to the constant value 5.01. Those convergent values don't depend on the initial conditions in the same manner as the case of κ=0. As a result, the stable steady-state planar pipe motion in a horizontal plane at $s = 0.772$ occurs, as shown in Figure 3(c).

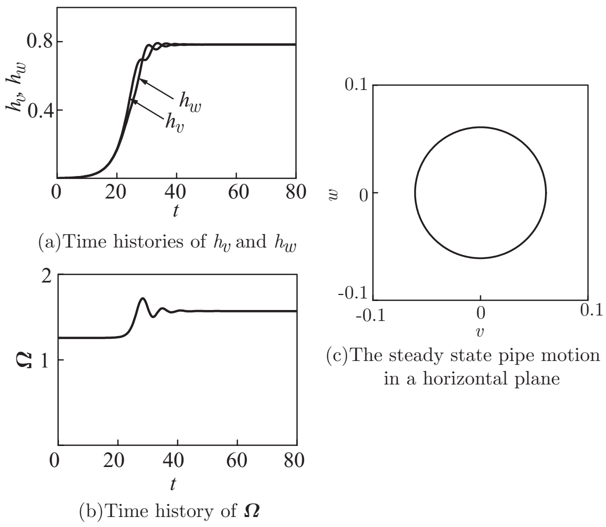

Figure 2: Transient time histories of h_v and h_w and the steady-state pipe motion in a horizontal plane without forced excitation

5. EXPERIMENTS

The experiments were conducted with the silicone rubber pipe of 12 mm external diameter, 7 mm internal diameter and 518 mm length. The equivalent bending rigidity EI is 0.01 N m^2. The flow velocity V_s is 7.2 m/s. The values of α, β and γ were determined experimentally as 0.13, 0.26 and 20.6 respectively. The spatial displacements of the flexible pipe were measured by the image processing system which can be performed measurements of the marker in three dimensional space, based on the images from two CCD cameras.

The stable nonplanar vibration of the pipe was observed at the flow velocity $v_s = 7.2$ m/sec ($V_s = 7.3$), as shown in Figure 4. Figure 4 shows the time histories of v and w, their spectrum analyses and the pipe motions in a horizontal yz plane at $s = 400$ mm ($s^* = 0.772$). The steady-state nonplanar pipe vibration with the single mode, was observed in the y-z plane as predicted in the theory.

Figure 5 is the experimental result in the case of the forced excitation $\delta Y = 4$ mm ($k = 0.00772$). The frequency of the pipe vibration is equal to the frequency N of the excitation, i.e. 2.26 Hz.

The lateral deflection v of the pipe is excited with the single vibration mode, and the amplitude of v is 68.8 mm ($v^* = 0.133$). The amplitude of w is sufficiently small compared with the amplitude of v. That is, there is almost a steady-state planar motion of the pipe, as predicted in the theory. The transition from the nonplanar motion to the planar motion is different from the quenching phenomenon which is easily predicted in the case of the planar flow-induced vibration with forced excitation (Yoshizawa *et al.*, 1988).

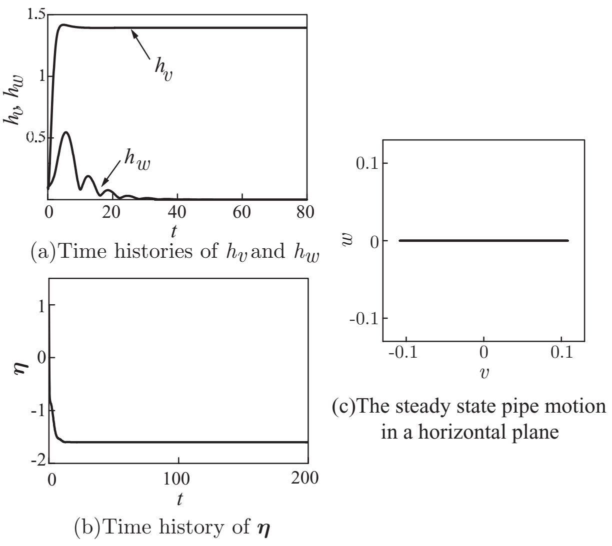

Figure 3: Transient time histories of h_v and h_w and the steady-state pipe motion in a horizontal plane with forced excitation

It is very interesting from the physical viewpoint that the lateral deflection of the pipe w in zx plane is also reduced by the forced excitation in yz plane.

6. CONCLUSION

We have studied the effect of forced excitation on the nonplanar flow-induced vibration of a cantilevered pipe, which is hung vertically with an end mass, from the viewpoint of nonlinear dynamics. That is, the upper end of the pipe is excited periodically in a horizontal plane. The fluid velocity is slightly over the critical value.

First, it has been clarified numerically that the nonplanar flow-induced pipe vibration is reduced to the planar vibration in the case of forced excitation, under the condition that the excitation frequency is nearly equal to the frequency of the flow-induced pipe vibration. The lateral deflection w of the pipe is reduced by the forced excitation perpendicular to w.

Second, the spatial behaviors of the silicon rubber pipe conveying water were observed quantitatively. As predicted in the theory, the plane pipe vibration has been observed under the condition that the excitation frequency is nearly equal to the self-excited frequency of the pipe, for the case of the nonplanar flow-induced pipe vibration without forced excitation .

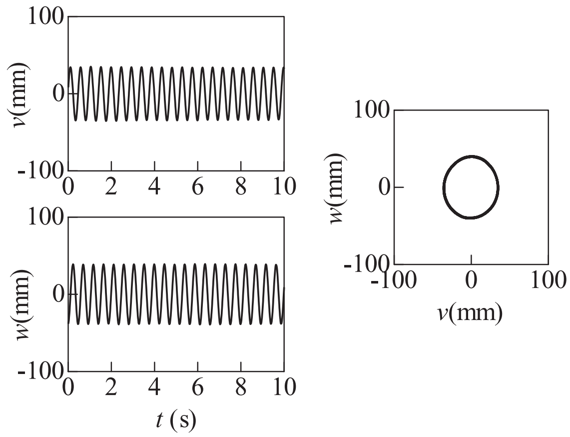

Figure 4: Time Histories and Pipe Motion in $y-z$ Plane ($Y_0 = 0mm$)

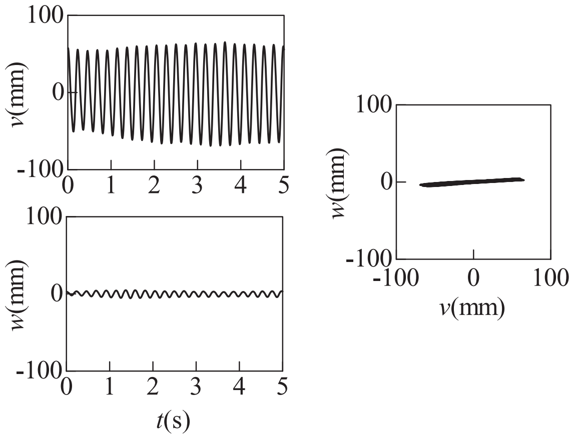

Figure 5: Time Histories and Pipe Motion in $y-z$ Plane ($N = 2.20(Hz), Y_0 = 4(mm)$)

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