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Code Acquisition in Asynchronous DS/CDMA Systems, Markov and i.i.d. Codes in Multiuser Scenario

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Abstract–Already synchronized users' signals are subtracted from the total received signal to aid code acquisition of new users at the base station of a direct sequence code division multiple access (DS/CDMA) system. Explicit expressions for the expectation and variance of correlator output corresponding to a synchronization chip and otherwise are derived for the asynchronous environment and a detector based on *a posteriori* probability threshold rule is proposed for code acquisition. This method can be used to acquire Spread Spectrum (SS) signals modulated by Markovian codes. Performance of Markov codes is found superior to i.i.d codes in this multiuser code acquisition system too.

1. Introduction

Markovian SS codes have recently attracted a lot of attention in the field of SS communications since it was reported in [1] that their bit error rate (BER) in asynchronous DS/CDMA systems is lower than i.i.d. random codes as well as linear feed back shift register (LFSR) sequences such as Kasami and Gold codes. This result was supported by several following papers [2-5]. In [3], variance of multiple access interference (MAI) with respect to code symbols was discussed. It was observed that in case of SS codes generated by some Markov chains, MAI's variance was less than i.i.d. codes in asynchronous state, which showed superiority of Markovian codes over i.i.d. codes in terms of BER. However, the performance of a DS/CDMA system depends both on BER and synchronization performance. Results reported in [6] and [17] about the comparison of synchronization performance of i.i.d., Gold and Markov codes showed superiority of Markov codes in singleuser case. In [18] using a parallel interference canceller, BER performance of Markov codes was shown to be better than i.i.d. codes. However, their superiority in multiuser systems, in terms of synchronization performance, is not yet established.

The process of synchronization is typically performed in two steps; i) code acquisition, by which the time delay between received signal and locally generated signature waveform is brought within a fraction of a chip, and ii) tracking, which performs fine-tuning and the delay is further reduced. It has been shown in [7] that acquisition based capacity (defined as the maximum number of simultaneous users supported by CDMA system while maintaining acceptable acquisition performance) is less than that obtained by BER or signal to noise ratio (SNR) even in singleuser case. Capacity gains (in BER) promised by multiuser detection (MUD) techniques [8] cannot be realized by using conventional code acquisition techniques which treat MAI as random. Using codes, demodulated data and timing estimates of already synchronized users, if their signals are subtracted from the received signal prior to code acquisition (see Fig. 1), MAI is canceled thus improving timing accuracy and system capacity [9-12]. In [9-11] system is assumed to be synchronous while in [12] we assumed a chip-synchronous system for simplicity. In a practical DS/CDMA system, communication from user to base station may start at any time and thus the uplink is inherently asynchronous.

A multiuser code acquisition technique based on *a posteriori* probability calculation is proposed for the uplink in asynchronous DS/CDMA systems. Proposed method uses cumulative information for decision making unlike conventional methods which only use current information; thus, resulting in capacity/mean acquisition time improvements of the order of several times over similar multiuser systems [12]. This method can be applied for code acquisition of SS signals modulated by Markovian codes. Computer simulations show that performance of Markov codes is superior to i.i.d. codes which means Markov codes are a strong candidate for multiuser systems as well.

2. System Model

Consider a DS/CDMA system with spreading factor N and J users. Let T_c be the chip time, $n(t)$ be additive white Gaussian noise, data and SS code signals of j-th user be,

$$d^{(j)}(t) = \sum_{p=-\infty}^{\infty} d_p^{(j)} u_T(t - pT),$$

$$X^{(j)}(t) = \sum_{q=-\infty}^{\infty} X_q^{(j)} u_{T_c}(t - qT_c)$$

where $u_D(t) = 1$ for $0 \le t < D$, $u(t) = 0$ otherwise and $d_p^{(j)},\ X_q^{(j)} \in \{+1, -1\}$. The received signal is given by

$$r(t) = \sqrt{2P_j} \sum_{j=1}^{J} d^{(j)}(t - t_j) X^{(j)}(t - t_j) \cos(\omega t + \varphi_j) + n(t) \quad (1)$$

where P_j denotes received power, φ_j the received phase and t_j the time delay of j-th user. Let T denotes the time period of one data symbol and $T / T_c = N$. The purpose of code acquisition process is to find the code chip corresponding to time delay t_j where $0 \le t_j < T$. We assume all users have equal powers and $T_c = 1$. Components of data sequence are assumed to be equi-probable, independent and identically distributed (i.i.d.) binary random variables, i.e., Prob $(d_p^{(j)} = +1)$ = Prob $(d_p^{(j)} = -1)$ = 1/2. The code sequences are assumed to be generated by mutually independent Markov chains whose components are equi-probable. Let λ be the eigenvalue, other than 1, of the transition probability matrix. Then, for $1 \le i, j \le J$ and $n, m \ge 0$

$$E_{X^{(i)}}[X_n^{(i)}] = 0,$$
$$E_{X^{(i)}X^{(j)}}[X_n^{(i)} X_m^{(j)}] = \begin{cases} \lambda^{|m-n|} & \text{if } i = j, \\ 0 & \text{otherwise}, \end{cases} \quad (2)$$

where $E_Z[.]$ denotes expectation with respect to the distribution of a random variable Z. It may be noted that i.i.d. codes can be regarded as a special case of Markov codes where λ=0.

3. Correlator output

Suppose $Z_n^{(j)}$ models the j-th correlator's output at time instant n, then ignoring channel noise

$$Z_n^{(j)} = \int_{nT_c}^{nT_c+T} r(t) X^{(j)}(t - nT_c) \cos(\omega t + \varphi_j) dt$$
$$= S_n^{(j)} + \sum_{j=1, j \ne i}^{J} I_n^{(i,j)}. \quad (3)$$

The correlator output is composed of desired signal plus self-interference $S_n^{(j)}$ of target user and MAI $I_n^{(i,j)}$ due to the i-th interfering user.

3.1. Expression for variance

It has been shown in [3] that variance of normalized MAI per user in asynchronous case is

$$E_D[Var_X[I_n^{(i,j)} / \sqrt{N}]] = \sigma_I^2 = \frac{2}{3} \cdot \frac{1 + \lambda + \lambda^2}{1 - \lambda^2}, \quad (4)$$

where E_D denotes expectation with respect to data.

In [13], variance of normalized self-interference is given; however, if we average that variance over time, then for non-synchronization chip, it can be regarded as one more user contributing to MAI. Thus,

$$E_D[Var_X[S_n^{(j)} / \sqrt{N}]] = \sigma_S^2 = \frac{2}{3} \cdot \frac{1 + \lambda + \lambda^2}{1 - \lambda^2}. \quad (5)$$

Corresponding to synchronization chip, self interference will be zero and variance of correlator output would only be due to MAI.

3.1.1 Effect of Data Estimation Errors

Let J be the total number of users in the system out of which, K are already synchronized and L are new users still waiting to be synchronized. Using SS codes X, data estimates $\hat{d}$ and timing estimates $\hat{t}$ of the K synchronized users, their signals are subtracted[1] from total received signal, the residual signal is

$$s(t) = r(t) - \sum_{j=1}^{K} \hat{d}^{(j)}(t - \hat{t}_j) X^{(j)}(t - \hat{t}_j).$$

Let us assume $\hat{t}_j = t_j$ i.e., perfect timing estimates. A wrongly estimated data bit of any user will increase the corresponding MAI by two and variance of MAI by four times, if the system is synchronous. In asynchronous systems, each data bit of a user overlaps with two data bits of interfering users i.e. $\hat{d}_p^{(i)}$ and $\hat{d}_{p+1}^{(i)}$ where $p = \lfloor (n - t_i) / N \rfloor$. Let

$$M_1 \underline{\underline{\Delta}} \{ j \mid \hat{d}_p^{(i)} \ne d_p^{(i)}, 1 \le j \le K \}$$
$$M_2 \underline{\underline{\Delta}} \{ j \mid \hat{d}_{p+1}^{(i)} \ne d_{p+1}^{(i)}, 1 \le j \le K \}$$

The increase in normalized correlator output variance due to $\hat{d}_p^{(i)} \ne d_p^{(i)}$ will be $(N - t_i) / N \times 4|M_1|$, where $|A|$ means cardinality of set A. Let t_i be uniformly distributed in the interval $0 \le t_i < T$ then $E_{T_i}[(N - t_i) / N] = 1/2$. Hence, total increase in normalized correlator output variance due to data estimation errors will be $2|M_1|+2|M_2|$. From (4), variance of normalized correlator output corresponding to synchronization chip after subtraction, denoted by σ^2 is

$$\sigma^2 = \sigma_I^2 (L - 1 + 2m), \quad (6)$$

where, $m = |M_1|+|M_2|$, $0 \le m \le 2K$. From (5), for non-synchronization chip, variance of correlator output is $\sigma^2 + \sigma_S^2$.

3.2. Expression for expectation

Expectation of MAI with respect to codes, from (2), is

$$E_X[I_n^{(i,j)} / \sqrt{N}] = 0.$$

If correct delay t_j is uniformly distributed between $-1/2 \le t_j \le 1/2$ then its expectation corresponding to synchronization chip is given by

$$E_{X,T_j}[S_n^{(j)} / \sqrt{N}] = \int_{-T_c/2}^{T_c/2} f(t) dt,$$

where $f(t) = \sqrt{N}\{1 - (1 - \lambda)|t|\}$ and T_j is a random variable for t_j. Hence

[1] Parallel interference canceller originally proposed by R. Kohno, H. Imai and M. Hatori [14] is applied for subtraction

$$E_{X,T_j}[S_n^{(j)}/\sqrt{N}] = \frac{3+\lambda}{4}\sqrt{N}, \quad (7)$$

and corresponding to the non-synchronization chip is

$$E_{X,T_j}[S_n^{(j)}/\sqrt{N}] = 0 \quad \text{as } N \to \infty \quad (8)$$

3.3. Density functions

Applying central limit theorem, it can be shown that the normalized correlator output $Y_n = Z_n/\sqrt{N}$ (superscript j has been ignored for simplicity) has density functions $f_c(y\,|\,\sigma^2)$ when n is the synchronization chip and $f_{inc}(y\,|\,\sigma^2+\sigma_S^2)$ otherwise, given as

$$f_c(y\,|\,\sigma^2) = \frac{1}{2}\text{nor}\,(y\,|\,\mu,\sigma^2) + \frac{1}{2}\text{nor}\,(y\,|\,-\mu,\sigma^2)$$
$$f_{inc}(y\,|\,\sigma^2) = \text{nor}\,(y\,|\,0,\sigma^2+\sigma_S^2),$$

where, $\text{nor}\,(y\,|\,\mu,\sigma^2) = 1/\sqrt{2\pi\sigma^2}\ \exp\left(-(y-\mu)^2/2\sigma^2\right)$, σ_S^2, σ^2 and μ are given by (5), (6) and (7) respectively.

4. Acquisition method

Since the proposed method is quite different from various conventional methods [15], a discussion about working of the two seems appropriate for a clear understanding. The main operative constituents of code acquisition process are a search strategy i.e. serial search or parallel search and a function to identify the presence or absence of synchronization, known as the detector. Different phases are checked by correlating the received signal with signature waveform for some time known as the dwell-time. In conventional methods, the correlator output is directly compared with a threshold value and if found greater, acquisition is declared or in some methods the code chip having largest correlator output value is declared as the synchronization chip [15]. Tests may be repeated a number of times over independent data periods for reliability thus only current information is used for decision making during each test.

The philosophy of our proposed detector is based on calculation of *a posteriori* probability. After observing an output, *a posteriori* probability of corresponding chip being the synchronization chip is updated using the entire information available at that time. If this *a posteriori* probability is found greater than the threshold value, acquisition is declared; otherwise next correlator output is added to the decision process and this continues until threshold is crossed. It is clear that probability of correct acquisition will be higher than the threshold value and time taken for code acquisition will be flexible.

Let $b(n,x,P) = \binom{n}{x}(1-P)^{n-x}P^x$. Then, probability of m already synchronized users contributing to BER at a particular chip interval is $b(2K, m, p)$, where p is the BER of synchronized users and $0 \le m \le 2K$. It can be shown that for sufficiently large integer NJ (N is codelength and J is the number of users) and optimum Markov codes i.e. $\lambda=\sqrt{3}$-2 [1], the correlator outputs are asymptotically independent. When acquisition starts in the absence of any *a priori* timing information, synchronization chip is uniformly distributed over entire code length with probability $1/N$. After observing correlator output at time instant n, when i-th chip is the synchronization chip, conditional probability of corresponding chip being synchronization chip is given by the following joint density function:

$$\varphi(\boldsymbol{y}_n\,|\,i) = \prod_{\substack{t=0\\ t\neq i \bmod N}}^{n} A_{inc}(y_t\,|\,m) \times \prod_{\substack{t=0\\ t=i \bmod N}}^{n} A_c(y_t\,|\,m) \quad (9)$$

where

$$\boldsymbol{y}_n = y_0, y_1, \ldots, y_n$$

$$A_c(y) = \sum_{m=0}^{K} b(K,m,p) f_c(y\,|\,\sigma_{(L,m)}^2), \quad (10)$$

$$A_{inc}(y) = \sum_{m=0}^{K} b(K,m,p) f_{inc}(y\,|\,\sigma_{(L,m)}^2 + \sigma_S^2), \quad (11)$$

$$\sigma_{(L,m)}^2 = \frac{2}{3}\cdot\frac{1+\lambda+\lambda^2}{1-\lambda^2}(L-1+4m).$$

(Remark 1: Above variance is a function of L (new users) and m (users having data demodulation error) therefore notation $\sigma_{(L,m)}^2$ is used rather than σ^2 as in (6). Instead of exact value of $\sigma_{(L,m)}^2$ its expectation $E_m[\sigma_{(L,m)}^2]$ can be used thus reducing complexity significantly reduced [12]). There will be a total of N distributions and the process of code acquisition is equivalent to decide that which distribution the signal belongs. From statistical decision theory [16], the best way is to chose distribution $\varphi(i)$ such that $\varphi(\boldsymbol{y}_n|i) = \max\{\varphi(\boldsymbol{y}_n|0), \varphi(\boldsymbol{y}_n|1), \ldots, \varphi(\boldsymbol{y}_n|N\text{-}1)\}$. Let P_c be the threshold value and Ψ_i be *a posteriori* probability of i-th chip being the synchronization chip, then

$$\Psi_i = \varphi(\boldsymbol{y}_n\,|\,i) \Big/ \sum_{k=0}^{N-1}\varphi(\boldsymbol{y}_n\,|\,k). \quad (12)$$

If $\Psi_i \ge P_c$, the distribution is acquired and corresponding chip is declared as the synchronization chip. If there are no distributions satisfying this condition, next correlator output i.e., at time instant n+1, is added to the decision process and joint density function and *a posteriori* probability are updated accordingly. The process continues until threshold is crossed. Hence, upper bound on acquisition error probability is equal to 1-P_c. It is clear that this technique inherently has a flexible acquisition time. (Remark 2: It was observed that in asynchronous case with sampling rate once per chip, sometimes it was not possible to detect the synchronization chip. Sampling more than once per chip would introduce correlation and evaluation by (9) would be inaccurate. Using two parallel correlators operating independently with a time difference of $T_c/2$, this problem is solved. Eqs. (9) and (12) are evaluated for correlator 1 and correlator 2 alternatively.)

5. Simulation Results

Computer simulations were performed for both i.i.d. and Markov codes with spreading factor N = 63, minimum probability of correct acquisition P_c = 99%, new users L =1, 2 and 5 and already synchronized users K =10, 20,…, 50. It can be seen from Figs. 2 and 3 that Markov codes have a higher correct acquisition probability than i.i.d. codes and their mean acquisition time is less than i.i.d. codes which shows their superiority in this multiuser system as well.

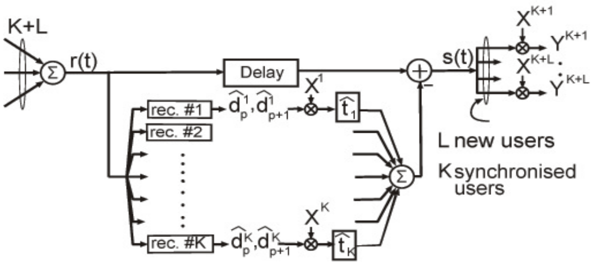

Fig. 1: Base station model using MAI cancellation,

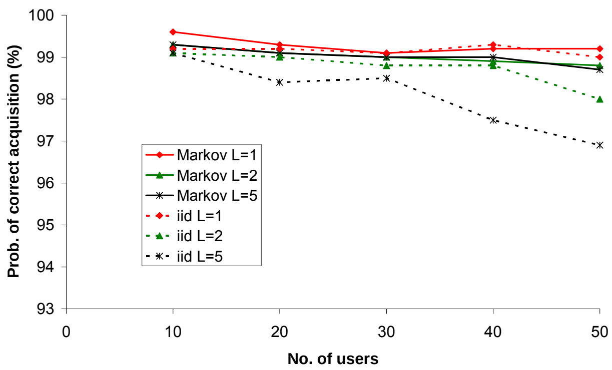

Fig. 2: Comparison of probability of correct acquisition

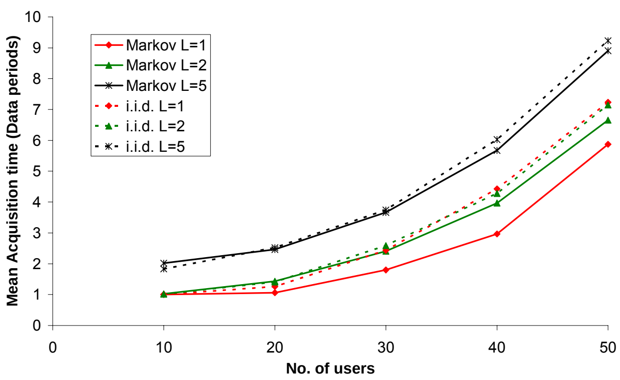

Fig. 3: Comparison of mean acquisition time

References

[1] G. Mazzini, R. Rovatti and G. Setti, "Interference minimization by auto-correlation shaping in asynchronous DS/CDMA systems: chaos-based spreading is nearly optimal," *IEE*, *Electronics Letters,* 35, pp. 1054-1055, 1999.

[2] Ling Chong and Li Shaoqian, "Chaotic spreading sequences with multiple access performance better than random sequences," *IEEE Trans. circuits and systems-1*, 47-3, pp.394-397, 2000.

[3] T. Kohda and H. Fujisaki, "Variances of multiple access interference: code average against data average," *IEE*, *Electronics Letters,* 36-20, pp. 1717-1719, Sept. 2000.

[4] C.C. Chen, E. Biglieri, K. Yao, "Design of spread spectrum sequences using Ergodic theory," 2000 *IEEE ISIT Sorrento,* Italy, June 2000.

[5] G. Mazzini, R. Rovatti and G. Setti, "A tensor approach to higher order expectations of quantized chaotic trajectories-part II: applications to chaos-based DS/CDMA in multi-path environments," *IEEE Trans. CAS*-47, no. 11, pp. 1584-1596, 2000.

[6] T. Kohda, Y. Matsumara and Y. Jitsumatsu, "Bit error rate in asynchronous DS/CDMA system using Markovian SS codes," *IEEE ISSSTA Prague,* Czech Republic, Sept. 2002.

[7] U. Madhow and M. B. Pursley, "Acquisition in direct-sequence spread-spectrum communication networks: an asymptotic Analysis," *IEEE Trans. on Commun.*, vol. 39, no. 3, pp. 903-912, May 1993.

[8] S. Verdu, *Multiuser Detection*, Cambridge University Press, 1998.

[9] J. B. Lee and S. K. Oh, "An interference-cancellar-aided code acquisition scheme for DS/CDMA systems with interference cancellation," *IEICE Trans. Commun.*, vol.E86-B, no. 9, pp. 2785-2787, Sept. 2003.

[10] V. Bharadwaj and R. M. Buehrer, "Acquisition in CDMA systems using parallel interference cancellation," *IEEE 59th VTC Milan*, May 2004.

[11] M. C. Reed, "Return link 2-D code acquisition for DS/CDMA in a high capacity multiuser system," *IEEE 59th VTC Milan*, May 2004.

[12] T. A. Khan, N. Eshima, Y. Jitsumatsu and T. Kohda, "An *a posteriori* probability threshold rule for multiuser code acquisition in DS/CDMA systems: theoretical and approximate methods," *IEEE 59th VTC Milan*, May 2004.

[13] Y. Jitsumatsu and T. Kohda, "BER of incompletely synchronized correlator in asynchronous DS/CDMA system using SS Markovian codes," *IEE*, *Electronics Letters,* 38-9, pp. 415-416, April 2002.

[14] R. Kohno, H. Imai and M. Hatori, "Cancellation techniques of co-channel interference in asynchronous spread spectrum multiple access systems," *Trans. IEICE* (Japanese), Vol. 66, pp. 416-423, May 1983.

[15] S. Glisic and B. Vucetic, *Spread Spectrum CDMA Systems for Wireless Communications*, Chapter 2, Artech House Publishers,1997.

[16] T.W. Anderson, *An introduction to multivariate statistical analysis*, John Wiley and Sons, New york, 1984.

[17] G. Setti, R. Rovatti and G. Mazzini, "Synchronization mechanism and optimization of spreading sequences in chaos-based DS/CDMA systems," *IEICE Trans. on Fundamentals,* Vol. E82-A, No.9, Sept. 1999.

[18] G. Mazzini, R. Rovatti and G. Setti, "The impact of chaos-based spreading on parallel DS/CDMA cancellers," NOLTA 2000, Dresden, Germany, Sept. 2000.