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Performance of FM-DCSK Communication System Over a Multipath Fading Channel with Delay Spread

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Abstract—Multipath performance is an important consideration for chaos-based communication systems. In this paper the performance of the FM-DCSK communication system over a two-ray independent Rayleigh fading channel is evaluated by computer simulations. The low-pass equivalent model of the FM-DCSK system is considered. Based on this model, we analyze the bit error performance and the effects of system parameters on the bit-error performance.

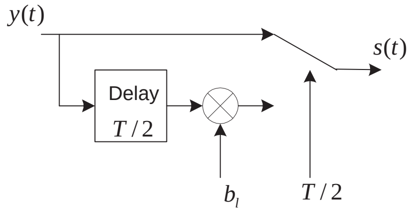

Figure 1: The DCSK modulator.

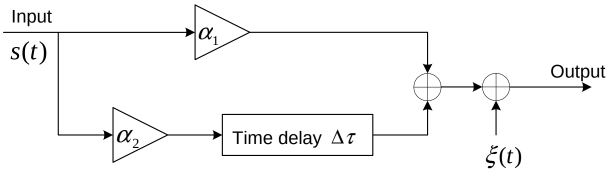

Figure 2: Two-ray independent Rayleigh fading channel model.

1. Introduction

The frequency-modulated differential chaos-shift-keying (FM-DCSK) communication system is a simple and practical candidate for spread spectrum (SS) communication applications [1]. Such a system under an additive white Gaussian noise (AWGN) environment has been thoroughly studied (see [2] for a survey). In wireless communications, however, the transmission environment is much more complex than what is covered by the simple AWGN model [3]. The reflecting objects and scatterers in a wireless channel dissipate the signal energy, leading to multiple versions of the transmitted signal arriving at the receiver with different amplitudes, phases, and time delays. These multipath waves combine at the receiver, causing the received signal to vary greatly in amplitude and phase. Such multipath fading therefore limits the performance in wireless applications. It is generally known that spread-spectrum systems perform significantly better than narrowband systems in a multipath environment. Since chaos-based systems are spread-spectrum systems, their performance in multipath environments should be taken into important practical consideration.

The multipath performance analysis and data for the chaos-based communication system are generally unavailable. The earliest study of multipath performance was taken by Kennedy *et al.* [4] for the FM-DCSK system. Their study was simulation-based and each path in the two-ray channel model was assumed an ideal constant gain value. In practice, however, each path suffers from random fading, which should be duly incorporated in the channel model [3]. Recently, Mandal and Banerjee [5] analyzed the performance of the DCSK system over a channel with Rayleigh fading or Ricean fading. However, the multipath time delay has not been considered. In a spread spectrum communication system such as DCSK and FM-DCSK, it is necessary to model the effects of multipath delay spread as well as fading. Furthermore, Xia *et al.* studied the multipath performance of coherent CSK and noncoherent DCSK systems [6, 7]. The two-ray independent Rayleigh fading channel model is considered, which includes the effects of channel fading and multipath delay spread. For the FM-DCSK system, however, similar data are still unavailable.

In this paper, we study the performance of the FM-DCSK system over the multipath fading channel, taking into account the effects of both channel fading and multipath delay spread.

2. System Model

2.1. FM-DCSK System

To our knowledge, there is no analytical expression available for the performance of the FM-DCSK system over a multipath channel. The computer simulation method has to be used to obtain the BER results. The FM-DCSK system is a RF band-pass system, which is based on the DCSK system as shown in Fig. 1. The multipath chan-

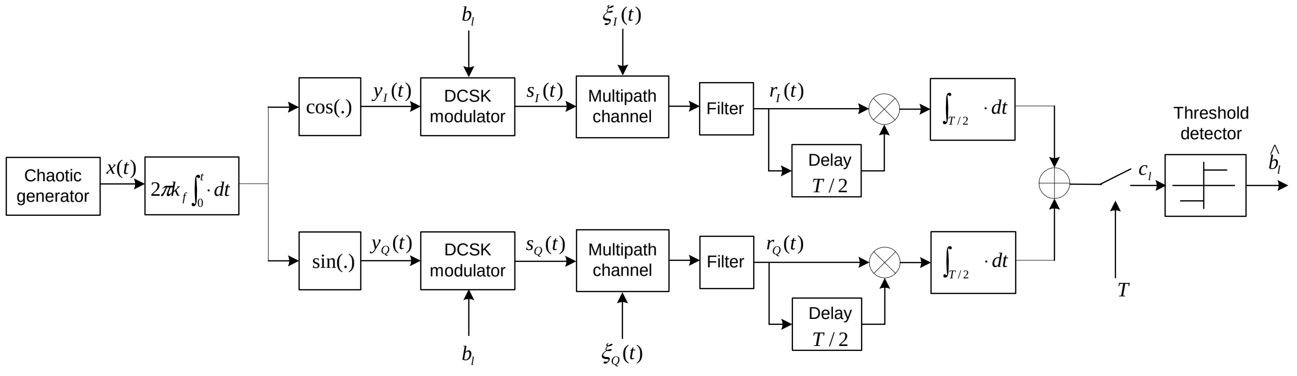

Figure 3: The FM-DCSK system.

nel model used in this study is a two-ray Rayleigh fading channel as shown in Fig. 2. To simulate such a system directly, we need a high sampling rate and hence a rather long simulation time. For realistic and fast simulations, we consider the low-pass equivalent model of the FM-DCSK system [8]. The system model is shown in Fig. 3. In the low-pass equivalent model, the chaotic signal is generated by the chaotic generator with the chip duration T_c, and is fed into the FM modulator. The FM modulator is divided into two branches, namely, the I branch and Q branch. The outputs of the FM modulator are given by

$$y_I(t) = \cos\left(2\pi k_f \int_0^t x(t)dt\right) \tag{1}$$

$$y_Q(t) = \sin\left(2\pi k_f \int_0^t x(t)dt\right). \tag{2}$$

These signals are fed into the DCSK modulator. The binary information bit, b_l, is modulated in this modulator, as shown in Fig. 1. After being modulated, every transmitted bit is represented by two chaotic signal segments. The first one serves as the reference whereas the second one carries the information. If “+1” is to be transmitted, the information-bearing segment will be identical to the reference segment, while if “–1” is to be transmitted, the information segment will be the inverted version of the reference, i.e.,

$$s(t) = \begin{cases} y(t) & (l-1)T \le t < (l-1)T + T/2 \\ b_l y(t - T/2) & (l-1)T + T/2 \le t < lT \end{cases} \tag{3}$$

where T denotes the bit duration.

The transmitted signal $s(t)$ passes through the channel and is distorted, as a result of the fading and multipath delay spread. Before reaching the receiver, the transmitted signal is also corrupted by AWGN $\xi(t)$.

At the receiver, the received signal $r(t)$ is demodulated by a differential coherent demodulator, after being filtered by the low-pass filter. The decision variable is then obtained by

$$c_l = \int_{T/2} r(t)r(t - T/2)dt. \tag{4}$$

Finally the decoded information bit is determined according to the following rule:

$$\hat{b}_l = \begin{cases} +1 & \text{if } c_l \ge 0 \\ -1 & \text{if } c_l < 0. \end{cases} \tag{5}$$

2.2. Multipath Fading Channel Model

In the analysis of chaos-based communication systems, an AWGN channel model is often assumed. This assumption is valid for some practical communication systems and makes the analysis computationally tractable. However, in spread-spectrum wireless communication systems, the channel is more complex and the transmitted signal will suffer from fading and multipath delay spread in addition to the effect of noise. Under this condition a commonly used channel model is the *two-ray independent Rayleigh fading channel* model [3]. Fig. 2 shows the block diagram of such a channel. The output of the channel is given by

$$\text{output} = \alpha_1 s(t) + \alpha_2 s(t - \Delta\tau) + \xi(t) \tag{6}$$

where α_1 and α_2 are independent and Rayleigh distributed random variables, and $\Delta\tau > 0$ is the time delay between the two paths. Also, $\xi(t)$ is the AWGN with mean equal to zero and power spectral density $N_0/2$.

In a narrowband communication system, if the signals from two paths are out of phase, they cancel each other, resulting in a large attenuation. Moreover, they may reinforce each other if the signals are in phase. This effect is called *multipath-related nullings and reinforcements* [2]. In the FM-DCSK system, however, one information bit is divided into several chips. Also, the frequencies of different chips are different as a result of applying FM. Suppose the variance of the carrier frequency is σ_f. It is readily shown that if $\Delta\tau \ll T_c$ and $\sqrt{\sigma_f}\Delta\tau \ll 1$, “approximate” nullings (reinforcements) occur when the phase shifts due to the delay in the two paths are close to $\pi, 3\pi, 5\pi, ...$ (0, 2π, 4π, ...). When these conditions are not met (which is the case for most practical situations), no multipath-related nullings or reinforcements would occur. This is one of the reasons why the FM-DCSK system performs better in a multipath environment than narrowband systems do.

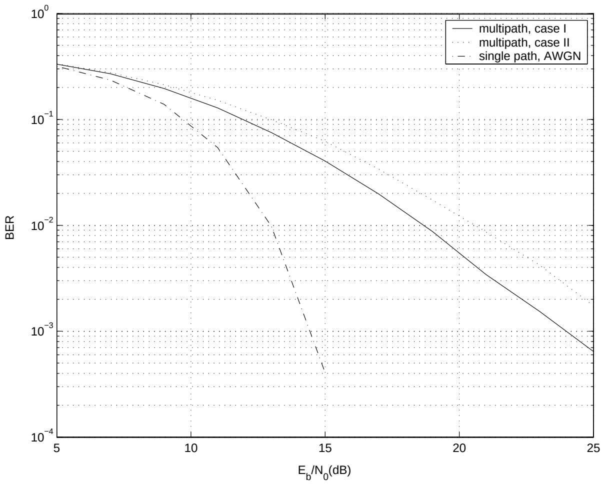

Figure 4: BER performance of the FM-DCSK system over a two-ray Rayleigh fading channel.

3. Simulation Results

3.1. Discrete-time Equivalent Model

The system described above is a continuous-time system. In order to expedite computer simulations, we employ a simple equivalent model which is a discrete-time version of the original continuous-time system sampled at a rate of 40 MHz. The discrete-time chaotic signal is generated by the logistic map at a clock rate of 20 MHz. Thus, there are two samples per chip.

3.2. BER Performance

The multipath performance of the FM-DCSK system is shown in Fig. 4. Here we set $\Delta\tau = 100$ ns and $T = 2$ μs. In the figure two cases with different path gain ratios are considered, and the single-path case (i.e., AWGN channel) is also shown for comparison.

Case I: The two paths have identical average power gain. In this case the average power gain in each path is 0.5, i.e.,

$$\mathrm{E}\left\{(\alpha_1)^2\right\} = \mathrm{E}\left\{(\alpha_2)^2\right\} = \frac{1}{2}. \qquad (7)$$

Case II: The average power gain of the second path is 10 dB below that of the first path. In this case, the average powers of the two paths are

$$\mathrm{E}\left\{(\alpha_1)^2\right\} = \frac{10}{11} \quad \text{and} \quad \mathrm{E}\left\{(\alpha_2)^2\right\} = \frac{1}{11}. \qquad (8)$$

Clearly, Fig. 4 shows that the performance of the FM-DCSK system over a multipath fading channel is much worse than that of the case over a single-path AWGN channel. When BER=10^{-3}, the performance degradation is almost 10 dB (case I) or even more (case II).

3.3. Effect of Parameters

In the FM-DCSK system, the different selection of values of system parameters will influence the system performance greatly. Here we consider two parameters, namely, the bit duration T and the time delay between the two paths $\Delta\tau$.

First, we show the effect of bit duration on the system performance in Fig. 5. We assume that the two paths have identical average power gain. The time delay $\Delta\tau$ is fixed to 100 ns. Fig. 5 (a) plots the curves of BER versus E_b/N_0, with $T = 1, 2, 4$ μs. In Fig. 5 (b), the effect of T on the BER performance is shown with E_b/N_0 fixed to 25 dB. By reducing the bit duration, the BER performance can be improved. The similar result has been got in the single-path environment [4]. Another merit of reducing the bit duration is that the data rate can be increased. However, when the bit duration decreases, the system will be sensitive to timing recovery errors. Thus, in practice, a tradeoff of these two factors should be considered.

Another parameter which can affect the system performance is $\Delta\tau$. It should be noted that in a specific application environment, $\Delta\tau$ has a typical value. For example, for large warehouses the typical value of $\Delta\tau$ is 91 ns, and for office buildings the typical value is 75 ns [4]. Here we do not specify the application. By changing the value of $\Delta\tau$, we can observe the general effect of this parameter on the system performance. The results are shown in Fig. 6. We assume that the two paths have identical average power gain. The bit duration T is fixed to 2 μs. Fig. 6 (a) shows the BERs versus E_b/N_0, with $\Delta\tau = 50, 100, 200$ ns. In Fig. 6 (b) the effect of $\Delta\tau$ on the BER performance is shown directly with E_b/N_0 fixed to 25 dB. From these two figures we can see that when $\Delta\tau$ increases, the system performance degrades. This result is consistent with the fact that the inter-symbol interference (ISI) increases with $\Delta\tau$. Note that the multipath-related nullings do not occur here, as explained previously in Section 2.2.

4. Conclusions

The FM-DCSK system has been considered as a practical candidate for chaos-based communication systems. In this paper the performance of the FM-DCSK system over a two-ray Rayleigh fading channel is evaluated by computer simulations. The results show the degradation of the system performance due to the channel fading and multipath delay spread. This paper also reveals the effects of system parameters on the bit-error performance.

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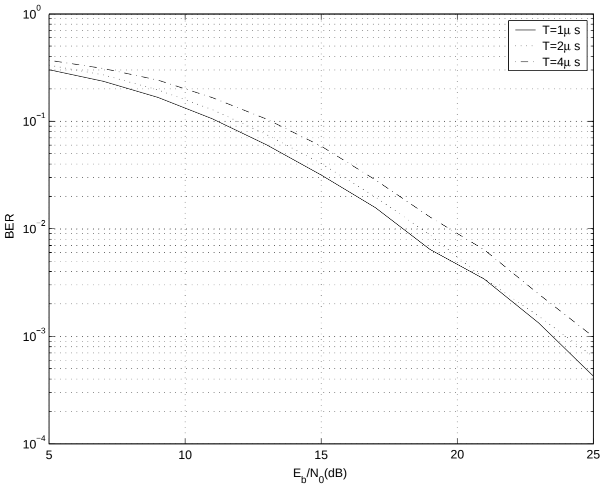
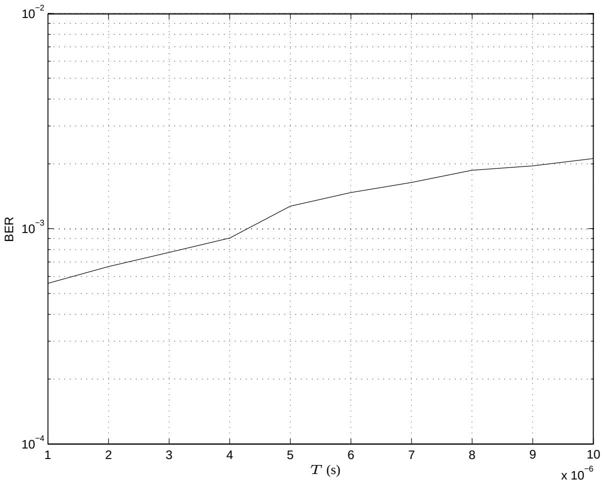


(a) BER versus E_b/N_0, with $T = 1, 2, 4$ μs



(b) BER versus T, with $E_b/N_0 = 25$ dB

Figure 5: Effect of T on the BER performance of the FM-DCSK system over a two-ray Rayleigh fading channel, with $\Delta\tau = 100$ ns.

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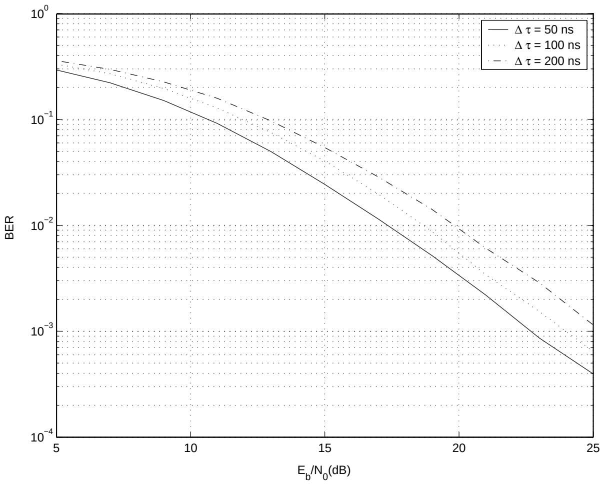
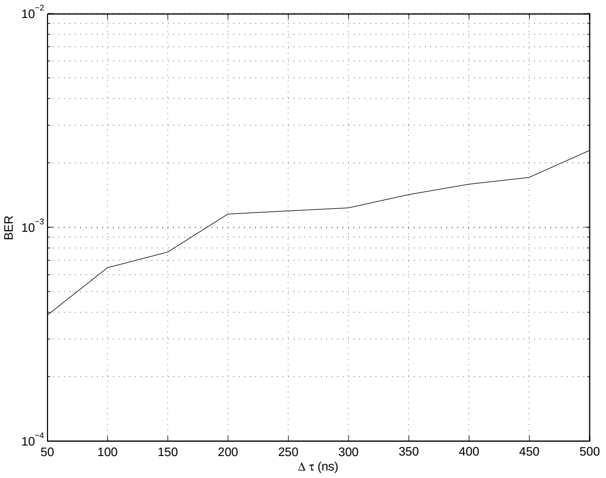


(a) BER versus E_b/N_0, with $\Delta\tau$ = 50, 100, 200 ns



(b) BER versus $\Delta\tau$, with $E_b/N_0 = 25$ dB

Figure 6: Effect of $\Delta\tau$ on the BER performance of the FM-DCSK system over a two-ray Rayleigh fading channel, with $T = 2$ μs.

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