2004 International Symposium on Nonlinear
Theory and its Applications (NOLTA2004)
Fukuoka, Japan, Nov. 29 - Dec. 3, 2004

Superiority of Markovian Spreading Sequences in Code Acquisition Performance

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Abstract—Superiority of Markovian spreading sequences in code acquisition performance is presented. We discuss acquisition method based on a posteriori probability for an asynchronous direct sequence/code division multiple access (DS/CDMA) systems, where single-user matched filter receiver is used and sampling rate is twice per chip. Since bit error rate of Markovian codes has already been shown to be better than linear feedback shift register (LFSR) codes in many papers, Markovian codes are a strong candidate for spreading sequences.

1. Introduction

Markovian spread spectrum (SS) codes were proven to minimize the average interference parameter (AIP) in asynchronous direct sequence/code division multiple access (DS/CDMA) systems [1]. It is reported in [1] that minimum bit error rate (BER) of Markov codes is lower than that of independent and identically distributed (i.i.d.) codes as well as linear feedback shift register (LFSR) sequences such as Kasami and Gold codes. This was also supported by [2, 3, 4]. In [3], a variance of multiple access interference (MAI) with respect to code symbols was discussed and SS codes generated by some Markov chains were shown to be superior to the ones generated by sequences of i.i.d. random variables in an asynchronous state.

Complete synchronization of the receiver is assumed in [1]-[4]. However, the question arises whether code acquisition performance of Markovian SS codes is superior or inferior to the i.i.d. codes. Serial search acquisition was employed for acquisition of Markovian SS codes [5, 6]. We have proposed an acquisition method based on *a posteriori* probability [10], where a chip-synchronous DS/CDMA system was assumed and matched filter output was sampled once each chip time. This paper shows that method can be also applied to the asynchronous systems by increasing the sampling rate up to twice per chip.

In [11], a post-filter, which is a digital filter placed after the matched filter, was shown to reduce the variance of MAI for i.i.d. codes up to the level of Markovian codes. Thus, the two codes have the same performance as far as post-filter is applied. This paper considers, however, matched filter receivers without post-filtering. Simulation result shows that code acquisition of Markov codes is faster than that of i.i.d. codes. This result leads us to conclude that Markovian codes are a strong candidate for SS codes.

This work was supported in part by Grants-in-Aid for Scientific Research of Japan Society for the Promotion of Science, no.16016269 and no.15360206.

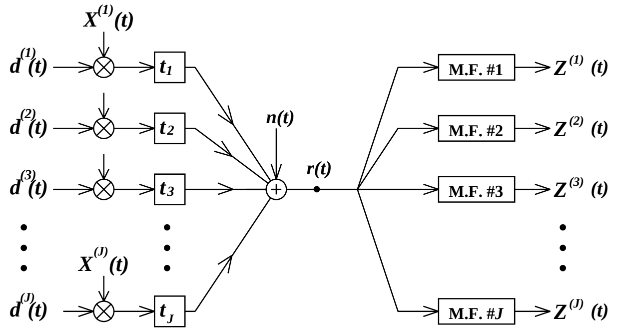

Figure 1: asynchronous DS/CDMA system: M.F. denotes matched filter.

2. Matched Filter Output Model

Fig. 1 shows an asynchronous DS/CDMA system with J users. Let $d^{(j)}(t)$ and $X^{(j)}(t)$ be data and code signals of j-th user defined by $d^{(j)}(t) = \sum_{p=-\infty}^{\infty} d_p^{(j)} u(t/T - p)$ and $X^{(j)}(t) = \sum_{q=-\infty}^{\infty} X_q^{(j)} u(t/T_C - q)$, where T is data period, T_C is chip duration and $u(t) = 1$ for $0 \le t < 1$ and $u(t) = 0$ otherwise. Assume that SS codes $\boldsymbol{X}^{(j)} = \{X_q^{(j)}\}_{q=-\infty}^{\infty}$ has period N and that $T = NT_C$. Since signals from different users arrive asynchronously at the receiver, received signal is given by

$$r(t) = \sum_{j=1}^{J} d^{(j)}(t - t_j) X^{(j)}(t - t_j) + n(t), \qquad (1)$$

where t_j denotes time delay of j-th user and $n(t)$ is common channel noise assumed to be white Gaussian with two-sided power spectral density $N_0/2$. Output of the i-th user's correlator at time instant τ is

$$z^{(i)}(\tau) = \int_0^T X^{(i)}(t) r(\tau + t) \mathrm{d}t$$

$$= \sum_{j=1}^{J} \zeta^{(i,j)}(\tau; t_j) + \eta^{(i)}(\tau), \qquad (2)$$

where $\eta^{(i)}(\tau) = \int_0^T n(t+\tau)X^{(i)}(t)\mathrm{d}t$ and

$$\zeta^{(i,j)}(\tau; t_j) = \int_0^T d^{(j)}(t - t_j + \tau)X^{(j)}(t - t_j + \tau)X^{(i)}(t)\mathrm{d}t. \qquad (3)$$

For $j \neq i$, $\zeta^{(i,j)}$ is called multiple access interference (MAI) and $\zeta^{(i,i)}$ is referred to as auto-correlation or self-interference. We denote $\zeta^{(i,i)}$ as $\xi^{(i)}$ for convenience.

It follows from (3) that $\zeta^{(i,j)}(\tau; t_j) = \zeta^{(i,j)}(\tau + \Delta; t_j + \Delta)$ holds for any Δ. If $0 \leq t_j < T$ and $\tau = pT$, then (3) becomes

$$\zeta^{(i,j)}(pT; t_j) = d^{(j)}_{p-1}\rho_{ij}(t_j) + d_p\rho_{ij}(T - t_j), \qquad (4)$$

where $\rho_{ij}(\tau) = \int_0^{T-\tau} X^{(i)}(t)X^{(j)}(t+\tau)\mathrm{d}t$ is a cross-correlation of SS code signal with *continuous* time delay τ. For an integer $0 \leq \ell \leq N-1$ and $0 \leq s < T_C$, we have

$$\begin{aligned}\rho_{ij}(\ell T_C + s) &= (T_C - s)R^A_N(\ell; \boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)}) \\ &\quad + sR^A_N(\ell + 1; \boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)}), \qquad (5)\end{aligned}$$

where $R^A_N(\ell; \boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)})$ is Pursley's aperiodic cross correlation function with *discrete* delay defined by $R^A_N(\ell; \boldsymbol{X}^{(i)}, \boldsymbol{X}^{(j)}) = \sum_{n=0}^{N-\ell-1} X^{(i)}_n X^{(j)}_{n+\ell}$. For simplicity, $\boldsymbol{X}^{(i)}$ and $\boldsymbol{X}^{(j)}$ are denoted, in brief, as $\boldsymbol{X}$ and $\boldsymbol{Y}$ hereinafter.

SS codes with Markovity have attracted considerable attention because they show smaller BER than i.i.d. codes as well as LFSR sequences. The following is a statistical property of cross- and auto-correlation functions, if $\{X^{(j)}_q\}_{q>0}$ is a Markov chain.

2.1. Markovian SS codes

Let $\boldsymbol{X} = \{X_n\}_{n=0}^{\infty}$ and $\boldsymbol{Y} = \{Y_n\}_{n=0}^{\infty}$ be sequences of $\{-1, +1\}$-valued binary random variables. Suppose $\boldsymbol{X}$ and $\boldsymbol{Y}$ are mutually independent, stationary 2-state Markov chains with 2-dimensional transition matrix P. Let $\mathrm{Prob}\{X_n = -1\} = \mathrm{Prob}\{Y_n = -1\} = \mathrm{Prob}\{X_n = 1\} = \mathrm{Prob}\{Y_n = 1\} = \frac{1}{2}$. For simplicity consider irreducible, aperiodic Markov chains, then for $\ell, m, k \geq 0$ we have [3]

$$\mathbf{E}_X[X_n] = \mathbf{E}_Y[Y_n] = 0, \mathbf{E}_{XY}[X_nY_{n+\ell}] = 0, \qquad (6)$$

$$\mathbf{E}_X[X_nX_{n+\ell}] = \lambda^{\ell}, \mathbf{E}_X[X_nX_{n+\ell}X_{n+\ell+k}] = 0, \qquad (7)$$

$$\mathbf{E}_X[X_nX_{n+\ell}X_{n+\ell+k}X_{n+\ell+k+m}] = \lambda^{l+m} \qquad (8)$$

where $\mathbf{E}_Z[\cdot]$ denotes the expected value with respect to the distribution of a random variable Z, and λ is the eigenvalue of P other than 1.

2.2. Previous Results: Variance of Matched Filter Outputs

Applying (6)-(8) to Pursley's aperiodic cross- and auto-correlation function, we get for $0 \leq \ell \leq N-1$, $0 \leq \ell + k \leq N-1$ and $k \geq 0$ [3, 7]

$$\mathrm{E}_{XY}[R^A_N(\ell; \boldsymbol{X}, \boldsymbol{Y})] = 0, \qquad (9)$$

$$\mathrm{E}_X[R^A_N(\ell; \boldsymbol{X}, \boldsymbol{X})] = (N - \ell)\lambda^{\ell}, \qquad (10)$$

$$\begin{aligned}&\mathrm{Cov}_{XY}[R^A_N(\ell; \boldsymbol{X}, \boldsymbol{Y}), R^A_N(\ell + k; \boldsymbol{X}, \boldsymbol{Y})] \\ &\quad = (N - \ell - k)\left(k + \frac{1+\lambda^2}{1-\lambda^2}\right)\lambda^k + \varepsilon_1 \qquad (11)\end{aligned}$$

$$\begin{aligned}&\mathrm{Cov}_X[R^A_N(\ell; \boldsymbol{X}, \boldsymbol{X}), R^A_N(\ell + k; \boldsymbol{X}, \boldsymbol{X})] \\ &\quad = (N - \ell - k)\left\{\left(k + \frac{1+\lambda^2}{1-\lambda^2}\right)(1 - \lambda^{2\ell}) \right. \\ &\qquad \left. -2\ell\lambda^{2\ell}\right\}\lambda^k + \varepsilon_2 \qquad (12)\end{aligned}$$

where ε_1 and ε_2 are negligible. It follows from (6) and (9) that expectation of matched filter output with respect to SS codes is equal to that of autocorrelation function, i.e.,

$$\begin{aligned}\mathrm{E}_{XY}[z^{(i)}(\tau)] &= \sum_{j=1}^{J} \mathrm{E}_{XY}[\zeta^{(i,j)}(\tau; t_j)] + \mathrm{E}_X[\eta^{(i)}(\tau)] \\ &= \mathrm{E}_X[\xi^{(i)}(\tau; t_i)]. \qquad (13)\end{aligned}$$

Assuming data sequence of one user is independent of that of other user, we get $\mathrm{Var}_X[z^{(i)}(\tau)] = \mathrm{Var}_X[\xi^{(i)}(\tau; t_j)] + \sum_{j\neq i} \mathrm{Var}_X[\zeta^{(i,j)}(\tau; t_j)] + \mathrm{Var}_X[\eta^{(i)}(\tau)]$. The variance of MAI and self-interference were respectively given in [3, 7].

Theorem 1 *The variance of normalized multiple-access interference is [3]*

$$\mathrm{Var}_{XYT_j}\left[\frac{\sqrt{N}}{T}\zeta^{(i,j)}(\tau; t_j)\right] = \frac{2}{3}\frac{1+\lambda+\lambda^2}{1-\lambda^2}, \qquad (14)$$

where T_j is a random variable for t_j uniformly distributed in $[0, T]$.

Theorem 2 *The variance of normalized self-interference is given by[7]*

$$\begin{aligned}&\mathrm{Var}_X[\tfrac{\sqrt{N}}{T}\xi^{(i)}(pT; (\ell_S + \varepsilon_S)T_C)] = (1 - \varepsilon_S)^2\mathcal{G}_+(\ell_S) \\ &\quad + \varepsilon_S^2\mathcal{G}_+(\ell_S + 1) + 2\varepsilon_S(1 - \varepsilon_S)\mathcal{H}_+(\ell_S) \\ &\quad + d^{(i)}_p d^{(i)}_{p-1}\left\{(1 - \varepsilon_S)^2\mathcal{G}_-(\ell_S)\right. \\ &\qquad \left. + \varepsilon_S^2\mathcal{G}_-(\ell_S + 1) + 2\varepsilon_S(1 - \varepsilon_S)\mathcal{H}_-(\ell_S)\right\}, \qquad (15)\end{aligned}$$

where $\mathcal{G}_+(\ell_S) = \frac{1+\lambda^2}{1-\lambda^2} - (2\ell_S + \frac{1+\lambda^2}{1-\lambda^2})\lambda^{2\ell_S}$, $\mathcal{H}_+(\ell_S) = \frac{2\lambda}{1-\lambda^2} - (2\ell_S + 1 + \frac{1+\lambda^2}{1-\lambda^2})\lambda^{2\ell_S+1}$, $\mathcal{G}_-(\ell_S) = \frac{2\ell}{N}(N - 2\ell + \frac{1+\lambda^2}{1-\lambda^2})\lambda^{N-2\ell}$ *and* $\mathcal{H}_-(\ell_S) = \frac{2\ell+1}{N}(N - 2\ell - 1 + \frac{1+\lambda^2}{1-\lambda^2})\lambda^{N-2\ell-1}$.

3. Acquisition Method Based on *a posteriori* Probability Threshold Rule

We proposed an acquisition method based on *a posteriori* probability for a chip-synchronous DS/CDMA system [10]. This section shows that the method can also be applied to asynchronous system by increasing the sampling rate up to $2/T_C$.

Consider sampling the matched filter output to acquire the i-th user's time delay. For accuracy of estimated time delay and avoiding too much complexity, sampling rate $2/T_C$ is employed in this paper. We can generalize the following discussion to the case of sampling rate M/T_C, where M is an integer. Let $z_m^{(i)}$ be m-th sample of i-th matched filter, i.e.,

$$z_m^{(i)} \stackrel{\text{def}}{=} z^{(i)}(mT_c/2) = \sum_{j=1}^{J} \zeta_m^{(i,j)}(t_j) + \eta_m^{(i)}, \quad (16)$$

where $\zeta_m^{(i,j)}(t_j) \stackrel{\text{def}}{=} \zeta^{(i,j)}(mT_C/2; t_j)$ and $\eta_m^{(i)} \stackrel{\text{def}}{=} \eta^{(i)}(mT_C/2)$.

3.1. Expectation of Matched Filter Outputs

Since matched filter output is sampled once each $T_C/2$, difference between the actual delay of i-th user and the closest sample time is less than or equal to $T_C/4$. There is one acquisition timing in every data period. Thus, p-th correct acquisition timing, denoted by $m = m_p^*$, satisfies

$$|t_i + pT - m_p^* T_C/2| \le T_C/4. \quad (17)$$

Let $s_i = t_i + pT - m_p^* T_C/2$, then we have $\xi_{m_p^*}^{(i)}(t_i) = \xi^{(i)}(pT; s_i)$. Hence,

$$\mathrm{E}_{X,T_i}[\xi_{m_p^*}^{(i)}(t_i)] = \frac{2}{T_C}\int_{-T_C/4}^{T_C/4} \mathrm{E}_X\left[\xi^{(i)}(pT; s_i)\right] \mathrm{d}s_i, \quad (18)$$

where T_i is a random variable for t_i which is uniformly distributed in (17). From (4) and equation $\xi^{(i)}(pT; -s_i) = \xi^{(i)}((p+1)T; T - s_i)$, we have

$$\xi^{(i)}(pT; s_i) = d_p^{(i)} \rho_{ii}(T - |s_i|) + d_{p\pm1}^{(i)} \rho_{ii}(|s_i|), \quad (19)$$

where $\pm$ takes $+$ for $s_i \ge 0$ and $-$ for $s_i < 0$. Averaging (19) over $s_i \in [-T_C/4 : T_C/4]$ and substituting (10) gives

$$\begin{aligned} \mathrm{E}_{X,T_i}[\xi_{m_p^*}^{(i)}(t_i)] &= d_p^{(i)} \frac{(7+\lambda)N - \lambda}{8} T_C \\ &\quad + (d_{p-1}^{(i)} + d_{p+1}^{(i)}) \frac{\lambda^{N-1}}{16} T_C, \end{aligned} \quad (20)$$

which approaches to $d_p^{(i)} \frac{7+\lambda}{8} T$ when $N \to \infty$.

Variance of self-interference with respect to codes corresponding to the correct acquisition timing is relatively small, that is, from (15) and letting $\ell_S = 0$,

$$\begin{aligned} \mathrm{Var}_{X,T_i}\left[\frac{\sqrt{N}}{T}\xi_{m_p^*}^{(i)}(t_i)\right] &= \frac{2}{T_C}\int_{-T_C/4}^{T_C/4} s_i^2(1-\lambda^2)\mathrm{d}s_i \\ &= \frac{1}{192}(1-\lambda^2). \end{aligned} \quad (21)$$

Thus, this paper regards variance of self-interference at $m = m^*$ to be zero. On the other hand, for $m \ne m^*$ it is regarded as $\frac{2}{3}\frac{1+\lambda+\lambda^2}{1-\lambda^2}$, i.e., the same value as variance of MAI. We can verify this by averaging (15) over the time delay.

We regard expectation of auto-correlation function is zero for $m \ne m^*$, though it is not exactly zero because of the Markovity of SS codes. This approximation makes calculations of conditional probability less accurate. However, the complexity of the acquisition system decreases since the receiver's task is reduced to classify distribution of matched filter output into only two types; f_{on} and f_{off}, respectively corresponding to presence and absence of synchronization.

Concerning i-th user's signal, two cases are considered: i) data is modulated and ii) data is not modulated. In the former case, $D^{(i)}$ is introduced to represent a random variable for $d_p^{(i)}$, where $\Pr(d_p^{(i)} = +1) = \Pr(d_p^{(i)} = -1) = 1/2$. In the latter case, $d_p^{(i)} = +1$ with probability 1. Such a data sequence is called *training sequence*. Since code acquisition is performed in advance of data transmission, utilization of training sequence is possible and, as a consequence, acquisition process is accelerated. This paper investigates the training sequence case.

Let $\mathcal{N}(z|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)$ denote normal distribution function with mean μ and variance σ^2. Then $f_{\text{on}}(x) = \mathcal{N}(x|\mu_{\text{on}}, \sigma_{\text{on}}^2)$ and $f_{\text{off}}(x) = \mathcal{N}(x|0, \sigma_{\text{off}}^2)$ when $N \to \infty$, where $\mu_{\text{on}} = \frac{7+\lambda}{6}\sqrt{N}$, $\sigma_{\text{off}} = J \cdot \sigma_{\text{MAI}}^2 + N_0/2$, $\sigma_{\text{on}}^2 = (J-1)\sigma_{\text{MAI}}^2 + N_0/2$ and $\sigma_{\text{MAI}}^2 = \frac{2}{3}\frac{1+\lambda+\lambda^2}{1-\lambda^2}$.

3.2. A Posteriori Probability Based Threshold Rule

Suppose first M matched filter outputs are observed, where superscript (i) is omitted for $z_m^{(i)}$ for simplicity. Then the conditional (or, *a posteriori*) probability that n-th sample time ($n = 0, 1, \ldots, 2N-1$) is correct acquisition timing is given by

$$\mathrm{Prob}(n|\{z_m\}_{m=0}^{M-1}) = \frac{\prod_{m \bmod N = n} g(z_m)}{\sum_{\ell=0}^{2N-1}\prod_{m \bmod N = \ell} g(z_m)}, \quad (22)$$

where $g(x) = f_{\text{on}}(x)/f_{\text{off}}(x)$. If $\mathrm{Prob}(n|\{z_m\}_{k=0}^{M-1})$ exceeds threshold p_{th} (e.g. 0.95 or 0.99), we declare that $t = nT_C/2$ is the correct acquisition timing. Otherwise *a posteriori* probability is updated using the next output and process continues until the threshold is crossed.

Two outputs z_m and z_{m+1} actually are correlated but we ignored this correlation and treated it as independent in (22). In spite of this inaccuracy, the proposed acquisition method works well, as shown in simulation results.

4. Simulation Results and Conclusions

Numerical simulation is performed using i.i.d. and Markovian SS codes with $N = 63$ and threshold $p_{th} = 0.95$. For Markovian codes, $\lambda = -2 + \sqrt{3}$ is employed which is known as the eigenvalue minimizing variance of MAI. Note that i.i.d. codes can be regarded as Markovian codes with $\lambda = 0$.

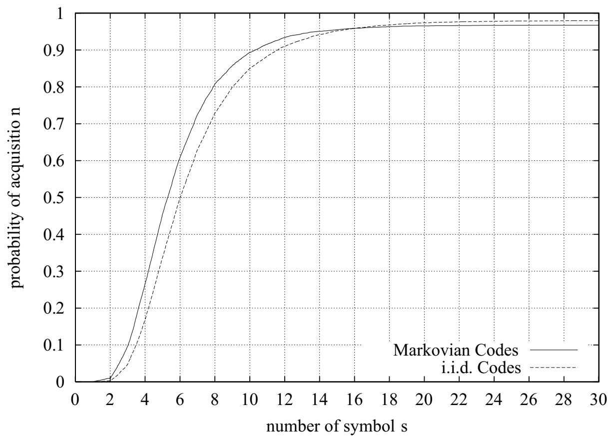

Figure 2: Probability of Correct Acquisition v.s. number of symbols: Spreading factor and number of user are $N = 63$ and $J = 30$.

Simulation results shows that code acquisition of Markovian SS codes is faster than i.i.d. codes (Fig. 2). Probability of correct acquisition using 30 symbols is 0.967 for Markovian codes, which is inferior to i.i.d.codes (0.980). This is because we approximate the expectation and variance of self-interference and assume z_m's are independent. However, for both codes, correct acquisition probability is more than the threshold. Using 6 data periods, about 60% of correct acquisition is declared for Markov codes, which is better than i.i.d. codes by 10%. Hence Markovian codes show superiority in code acquisition performance.

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