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Design Procedure for Generalized Class E Amplifiers with Implicit Circuit Equations

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Abstract—This paper presents a novel design procedure for class E amplifier with implicit circuit equations. It is possible to use circuit simulators since implicit circuit equations are allowed in the proposed design procedure. Therefore, the proposed procedure achieves the simple, easy and accurate design. We can show the validity of the proposed design procedure from an experimental result.

1. Introduction

Class E switching-mode tuned power amplifiers [1]–[5] have become increasingly valuable building blocks in many applications, e.g., radio transmitters and switching-mode dc power supplies. Because of class E switching, namely, both zero voltage and zero slope of voltage switching, the efficiency of energy conversion is very high at high frequencies. Therefore, a high density of dc/dc power processing can be achieved, reducing the size and weight of the equipment.

Since the introduction of the class E amplifier, many design procedure of this circuit have appeared [1]–[5]. In [1], the design values are derived analytically via circuit analyses with some assumptions, e.g., an ideal switch, infinite output network Q (i.e., sinusoidal output current), and an RF choke in the dc supply and so on. If the design values are expressed as a function, it is easy and fast to derive the design values. However, the accuracy of design values is low because of the assumptions and much effort is needed to analysis class E amplifier. The design values are derived numerically from analytical waveform equations by using the design procedure in [2]. This design procedure needs to solve circuit equations analytically, which need much effort. In [3], the design values are expressed as a function of design parameters that is derived by fitting the data of design values. In this procedure, easy, fast, and accurate designs are possible if the design functions can be derived. The data of design values, however, should be derived other design procedures.

The design procedures in [4] and [5] are very simple and easy design procedure compared with above schemes. These schemes require only circuit equations and carry out the other process of design with aid of computer. The design scheme in [4], however, requires that circuit equations have to be piecewise linear expressions since linear differential equations are solved numerically using eigenvalues of matrix from circuit equations. Therefore, it is not allow a non-linear operation of each element. On the other hand, the scheme in [5] allows any expressions of circuit equations. By the way, the many designers would like to use circuit simulators, e.g., SPICE to design of class E amplifier. It is, however, impossible since all previous design schemes requires explicit circuit equations. If implicit circuit equations are allowed to design class E amplifier, the designers uses circuit simulators. And the design of class E amplifier is more simple than the previous design procedure.

This paper presents a novel design procedure for class E amplifier with implicit circuit equations. By using design procedure it is possible to use circuit simulators and the simple, easy and accurate design can be achieved. The accuracy of design values and amount of calculations are evaluated. It is denoted that they are achieved to be same as the design procedure in [5]. We design class E amplifier by using the proposed design procedure and carry out the circuit experiment. We can show the validity of the proposed design procedure from the experimental result.

2. Principle Operation of Class E amplifier

Figure 1 depicts the circuit topology of class E amplifier. The waveforms of class E amplifier are shown in Fig. 2, when switch on duty ratio is 50%. The switch is driven by a driving pattern of D_r in Fig. 2. If the filter inductance L_f is large, the input current i_c of the amplifier is approximately constant, which is equal to its dc component. If the loaded quality factor Q is high ($Q \geq 5$), the current i_o through the LC resonant circuit is approximately a sine wave. Since the switching loss is reduced to zero by the operating requirements of zero and zero slope of switch voltage ($v_s = 0$ and $dv_s/dt = 0$) at the turn on transition, called class E switching conditions, the theoretical efficiency of class E amplifier is 100%. On the other hand, it is difficult to derive the elemental values of amplifier in order to class

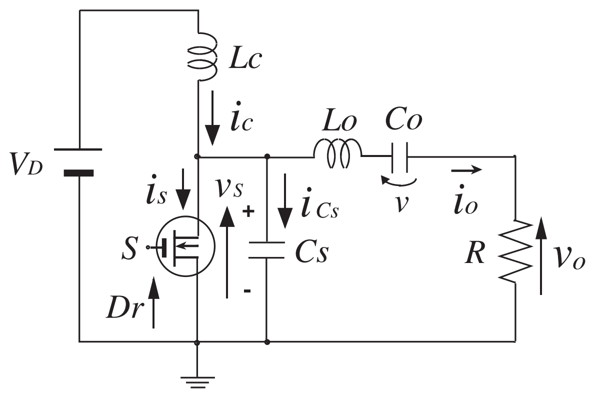

Figure 1: Circuit topology of class E amplifier.

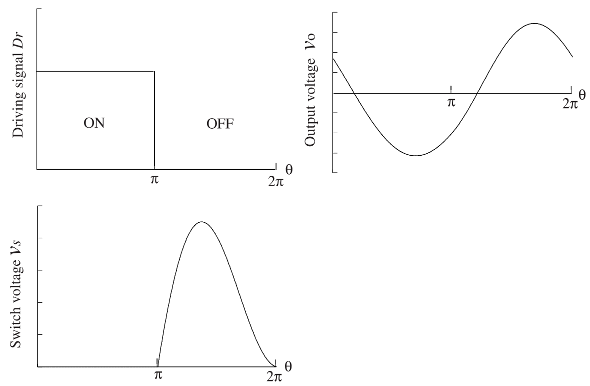

Figure 2: Optimum waveform of class E amplifier.

E switching conditions since class E switching conditions are strict.

3. Theory of Design Procedure

In this section, the theory of the proposed design procedure is presented.

3.1. Circuit Description

Let us consider a dynamic circuit described by a differential equations :

$$\frac{d\boldsymbol{x}}{dt} = f(t, \boldsymbol{x}, \boldsymbol{\lambda}), \tag{1}$$

where $t \in \boldsymbol{R}$, $\boldsymbol{x} \in \boldsymbol{R}^n$, and $\boldsymbol{\lambda} \in \boldsymbol{R}^m$ denote the time, an n-dimensional state and an m-dimensional system parameter, respectively. In this paper, For simplicity,

$$\begin{array}{rcl} f : \boldsymbol{R} \times \boldsymbol{R}^n \times \boldsymbol{R}^m & \rightarrow & \boldsymbol{R}^n \\ (t, \boldsymbol{x}, \boldsymbol{\lambda}) & \mapsto & f(t, \boldsymbol{x}, \boldsymbol{\lambda}). \end{array} \tag{2}$$

is assumed as C^∞ mapping and is periodic in t with period t_T :

$$f(t + t_T, \boldsymbol{x}, \boldsymbol{\lambda}) = f(t, \boldsymbol{x}, \boldsymbol{\lambda}). \tag{3}$$

We also assume that (1) has a solution $\boldsymbol{x}(t) = \boldsymbol{\varphi}(t, \boldsymbol{x_0}, \lambda)$ defined on $-\infty < t < \infty$ with every initial condition $\boldsymbol{x_0} \in \boldsymbol{R}^n$ and every $\boldsymbol{\lambda} \in \boldsymbol{R}^m$: $\boldsymbol{x}(0) = \boldsymbol{\varphi}(0, \boldsymbol{x_0}, \lambda) = \boldsymbol{x_0}$.

Here, it is unnecessary to find the differential equations 1 explicitly. This is a crucial difference from previous design procedures in [4] and [5]. The design procedure in this paper requires that the response of $\boldsymbol{x}$, namely $\boldsymbol{\varphi}$, can be observed and the system parameters $\boldsymbol{\lambda}$ and $\boldsymbol{x}(0)$ are given arbitrarily. It is recognized that these requirements are not particular for using circuit simulators.

3.2. Poincaré Map and Fixed Point

By the periodic hypothesis (3), we can naturally define a C^∞ diffeomorphism T from state space $\boldsymbol{R}^n$ into itself :

$$\begin{array}{rcl} T : \boldsymbol{R}^n & \rightarrow & \boldsymbol{R}^n \\ \boldsymbol{x_0} & \mapsto & T(\boldsymbol{x_0}, \lambda) = \boldsymbol{\varphi}(t_T, \boldsymbol{x_0}, \lambda). \end{array} \tag{4}$$

The mapping T is often called the Poincaré mapping.

If a solution $\boldsymbol{x}(t) = \boldsymbol{\varphi}(t, \boldsymbol{p_0}, \lambda)$ is periodic with period t_T, the point $\boldsymbol{p_0} \in \boldsymbol{R}^n$ is a fixed point of T :

$$T(\boldsymbol{p_0}, \lambda) = \boldsymbol{p_0}. \tag{5}$$

If $\boldsymbol{p_0} = \boldsymbol{x_0}$, (5) corresponds to the transient conditions.

3.3. Other Conditions

For design of the amplifier, we often consider several conditions, i.e. zero voltage switching, zero current switching, class E switching conditions, and so on. If the number of conditions is $N(\leq m)$, the conditions that consist of each condition g_k are expressed as

$$G(\boldsymbol{x_0}, \lambda) = \begin{bmatrix} g_1(\boldsymbol{x_0}, \lambda) \\ g_2(\boldsymbol{x_0}, \lambda) \\ \vdots \\ g_N(\boldsymbol{x_0}, \lambda) \end{bmatrix} = \boldsymbol{0}, \qquad \in \boldsymbol{R}^N. \tag{6}$$

In this case, we can find N design parameters. Therefore, the other $(m - N)$ parameters must be given as the design specifications. In the design procedure of **??**, the conditions g_k have to be defined on a fixed time, for example $\theta = 2\pi$. However, the presented design procedure allows all conditions if only the conditions g_k are functions of $\boldsymbol{x_0}$ and $\boldsymbol{\lambda}$. For example, statistical conditions can be given as other conditions. We recognize that the design of the amplifier boils down to the derivation of the solution of the algebraic equations (5) and (6). These equations are rewritten as follows:

$$F(\boldsymbol{x_0}, \lambda) = \begin{bmatrix} T(\boldsymbol{x_0}, \lambda) - \boldsymbol{x_0} \\ G(\boldsymbol{x_0}, \lambda) \end{bmatrix} = \boldsymbol{0}, \qquad \in \boldsymbol{R}^{n+N} \tag{7}$$

where, $T(\boldsymbol{x_0}, \lambda)$, $\boldsymbol{x_0}$ and $\boldsymbol{\lambda}$ are expressed as $T(\boldsymbol{x_0}, \lambda) = [T_1(\boldsymbol{x_0}, \lambda), T_2(\boldsymbol{x_0}, \lambda), \cdots, T_n(\boldsymbol{x_0}, \lambda)]^T$, $\boldsymbol{x_0} = \boldsymbol{x}(0) = [x_1(0), x_2(0), \cdots, x_n(0)]^T$, and $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \cdots \lambda_m]^T$. Moreover, we define $\boldsymbol{\lambda_u} \in \boldsymbol{R}^N$ as

$$\begin{array}{rl} \boldsymbol{\lambda_u} = & \{\lambda_{u1}, \lambda_{u2}, \cdots, \lambda_{uN} \mid \lambda_{uk}\ (k = 1, 2, \cdots, N) \\ & \text{are unknown design parameters in } \lambda.\} \end{array} \tag{8}$$

3.4. Computation of Design Values

We solve the equations (7) by using Newton's method that is the general algorithm to solve the algebraic equations. Since the unknown values of (7) are expressed as $\boldsymbol{u} \in \boldsymbol{R}^{n+N} : \boldsymbol{u} = [\boldsymbol{x_0}^T, \boldsymbol{\lambda_u}^T]^T$, the computations

$$\boldsymbol{u}^{k+1} = \boldsymbol{u}^k - \frac{F(\boldsymbol{u}^k)}{F'(\boldsymbol{u}^k)} \tag{9}$$

are iterated for $\|\boldsymbol{u}^{k+1} - \boldsymbol{u}^k\| < \delta$ in order to find the unknown values, where $F' \in \boldsymbol{R}^{(n+N)\times(n+N)}$ means Jacobian matrix of F, that is,

$$F'(\boldsymbol{u}^k) = \begin{bmatrix} \frac{\partial T_1(\boldsymbol{u}^k)}{\partial x_1(0)} - 1 & \frac{\partial T_1}{\partial x_2(0)} & \cdots & \frac{\partial T_1}{\partial x_n(0)} & \frac{\partial T_1}{\partial \lambda_{u1}} \cdots \frac{\partial T_1}{\partial \lambda_{uN}} \\ \frac{\partial T_2(\boldsymbol{u}^k)}{\partial x_1(0)} & \frac{\partial T_2}{\partial x_2(0)} - 1 \cdots & & \frac{\partial T_2}{\partial x_n(0)} & \frac{\partial T_2}{\partial \lambda_{u1}} \cdots \frac{\partial T_2}{\partial \lambda_{uN}} \\ \vdots & & & & \\ \frac{\partial T_n(\boldsymbol{u}^k)}{\partial x_1(0)} & \frac{\partial T_n}{\partial x_2(0)} & \cdots & \frac{\partial T_n}{\partial x_n(0)} - 1 & \frac{\partial T_n}{\partial \lambda_{u1}} \cdots \frac{\partial T_n}{\partial \lambda_{uN}} \\ \frac{\partial g_1(t_{c1}, \boldsymbol{u}^k)}{\partial x_1(0)} & \frac{\partial g_1}{\partial x_2(0)} & \cdots & \frac{\partial g_1}{\partial x_n(0)} & \frac{\partial g_1}{\partial \lambda_{u1}} \cdots \frac{\partial g_1}{\partial \lambda_{uN}} \\ \vdots & & & & \\ \frac{\partial g_N(t_{cN}, \boldsymbol{u}^k)}{\partial x_1(0)} & \frac{\partial g_N}{\partial x_2(0)} & \cdots & \frac{\partial g_N}{\partial x_n(0)} & \frac{\partial g_N}{\partial \lambda_{u1}} \cdots \frac{\partial g_N}{\partial \lambda_{uN}} \end{bmatrix}, \tag{10}$$

k is a iteration number and $\delta \ll 1$, i.e. $\delta = 10^{-9}$ in this paper. Then $\boldsymbol{u}^{k+1}$ is a solution of (7).

For computations of (9), we can derive $T(\boldsymbol{u}^k)$ from $\boldsymbol{\varphi}$ that is output of simulators. On the other hand, Jacobian matrix $F'(\boldsymbol{u}^k)$ of (10) cannot be derived from the first-order variational equations like [5] since the circuit equations are implicit. Therefore, we propose the numerical approximation is used to determine the element values of $F'(\boldsymbol{u}^k)$. When the new matrix $\boldsymbol{u_{\varepsilon i}}$ is defined as

$$\boldsymbol{u_{\varepsilon i}} = [u_1, u_2, \cdots, u_i + \varepsilon, \cdots, u_{n+N}], \tag{11}$$

the approximate values of partial differential of $F'(\boldsymbol{u}^k)$ are calculated by using the equation;

$$\frac{\partial g(\boldsymbol{u}^k)}{\partial u_i} = \frac{g(\boldsymbol{u_{\varepsilon i}}^k) - g(\boldsymbol{u}^k)}{\varepsilon}. \tag{12}$$

In (12), $\varepsilon \ll 1$ mean a minute variation. Moreover, $g(\boldsymbol{u_{\varepsilon i}})$ can be derived from response on a circuit system that is same as one for derivations of $g(\boldsymbol{u})$ by substituting $\boldsymbol{u_{\varepsilon i}}$ for $\boldsymbol{u}$.

From above computations, the unknown parameters $\boldsymbol{u}$ can be found, and the design values, that is, $\boldsymbol{\lambda_u}$ are determined. Hence, we can design class E amplifier by using the proposed design procedure.

The proposed design procedure requires only circuit configurations and conditions. Since variational equations are not needed, explicit circuit equations are also not needed. This means that the circuit simulators can be applied to derive response of circuit. For example, SPICE has many elemental models. If the designers would like to commute the kinds of MOSFET, they can reflect the affect of it in the design easily by using the elemental models from a library of SPICE. Compared with the previous design procedure, it is unnecessary to formulate new circuit equations and to make a model of new element. Therefore, we would like to emphasis that computer aided design can be achieved from the configuration of the circuit to the derivation of the design values and numerical waveforms. As a result, the design of class E amplifier is more simple than previous design procedure.

4. Evaluations for the Proposed Design Procedure

In this section, the proposed design procedure is compared with the previous one in [5] in order to evaluate the proposed procedure.

4.1. Accuracy of the design values

In the proposed design procedure, it is necessary to solve the algebraic equations (7) that is same as those in [5]. Both design procedures apply Newton's method for it. Therefore, the accuracy of the design values from the proposed design procedure is complete same as that from [5] when both procedures use the identical circuit equations, namely identical waveforms. In [5], it is denoted that high accuracy of design can be achieved. Therefore, we can conclude the design values can be derived with sufficient accuracy by using the proposed design procedure.

On the other hand, the derivations of accurate waveforms are quite important. The accuracy of the waveforms determines that of the design values and depends on the accuracy of the elemental models. For example, it need much effort to make the high accurate model of MOSFET in the design procedure of [5]. If the model in the library of circuit simulator can be applied to the design, it is easy to derive the high accurate waveforms. Therefore, the proposed procedure can derive accurate design values with fewer effort than the procedure in [5].

4.2. Amount of calculations

The difference between the proposed design procedure and the procedure in [5] is the approximations of Jacobian matrix $F'(\boldsymbol{u}^k)$. Therefore, we consider the amount of calculations for the derivation of Jacobian matrix $F'(\boldsymbol{u}^k)$ in order to evaluate amount of calculations. Approximate equations (12) are calculated in order to derive Jacobian matrix $F'(\boldsymbol{u}^k)$ in the proposed procedure while first-order variational equations are used at the procedure in [5]. The first-order variational equations are $n(n + N)$-dimensional differential equations. On the other hand, it is necessary for the derivation of $g(\boldsymbol{u}^k)$ to solve $(n + N)$ kinds of waveforms governed by n-dimensional circuit equations. As a

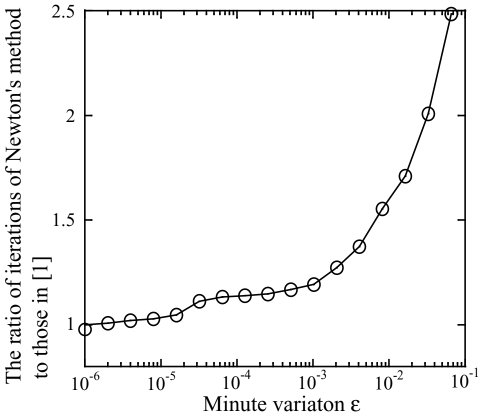

Figure 3: The ratio of iterations of Newton's method in the proposed procedure to the those in [5] as a function of a minute variation ε.

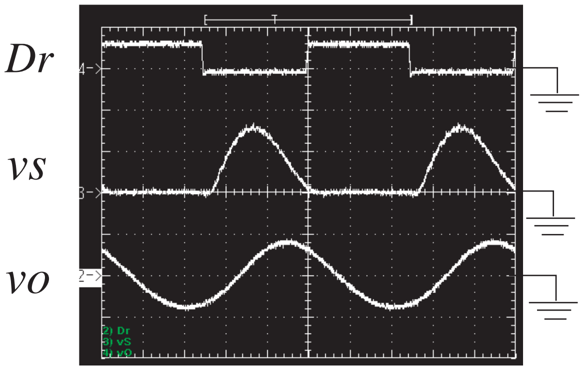

Figure 4: Experimental waveforms of class E amplifier. Horizontal : 200 ns/div. Vertical of Dr and v_o : 5 V/div, v_S : 10 V/div

result, the amount of calculations to derive Jacobian matrix in the proposed procedure is same as that in the procedure in [5].

Next, we consider the amount of iteration of Newton's method. Figure 3 shows the ratio of iterations of Newton's method in the proposed scheme to the those in [5] as a function of a minute variation ε. This ratio are derived from the designs of class E amplifiers under 100 kinds of design specifications. The higher accuracy the approximate equation (12) is, the smaller a minute variation ε is. From Fig. 3, the amount of iterations is same as the scheme in [5] for small ε. From these considerations, it is find that the amount of calculations for the design of class E amplifier by using the proposed design procedure is almost same as that by using the design procedure in [5] in case of small ε.

5. Design Example

Following the proposed design procedure, the design of class E amplifiers is carried out. In the design, Runge-kutta method is applied to circuit equations that is unknown to the designer and these calculations are regarded as a circuit simulators. Figure 4 shows the experimental waveforms of class E amplifier. From this figure, we can confirm the experimental waveform v_s is achieved class E switching conditions and the validity of the proposed design procedure.

6. Conclusion

This paper has presented a novel design procedure for class E amplifier with implicit circuit equations. It is possible to use circuit simulators since implicit circuit equations are allowed in the proposed design procedure. Therefore, the proposed procedure achieves the simple, easy and accurate design. We can show the validity of the proposed design procedure from the experimental result.

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