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Subharmonic Oscillations Related to Switching Nonlinearity in TCR-SVC

Takashi Hikihara, Ryo Toyoshima and Tsuyoshi Funaki

Department of Electrical Engineering, Kyoto University
Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8510 Japan
Email: hikihara@kuee.kyoto-u.ac.jp

Abstract—A static var compensator with thyristor controlled reactor is one of widely used apparatuses in power systems to compensate their reactive power through controlling the firing angle. This paper confirms a discontinuous change of conduction angle experimentally and numerically. The discontinuous change is sometimes accompanied with unexpected subharmonic oscillations. The subharmonics are discussed through the single phase numerical simulations, experiments and also in three phase.

1. Introduction

The increase of distributed power sources requests us to prepare apparatuses, which can control the reactive power in conventional power systems. A TCR-SVC is well known as one of the FACTS apparatuses in power systems to compensate the reactive power through controlling the firing angle of thyristor [1, 2]. The principal operation is realized by averaging state variables on fundamental components in the switching period. The switching will yield the transient responses governed by circuit elements in voltage and current. They are depending on the initial state at the switching and global structure of phase space, which is discontinuously conjugated. Then, the state shows the transient behavior between equilibrium points with higher harmonics caused by the resonance during the switching period [1].

In the 1990s, the appearance of bifurcation in conduction angle was reported [3, 4] and called the switching time bifurcation. It was also discussed in the contexts of hybrid systems [5]. The turn-off operation of thyristor is given by the state without relation to gate current. It causes the anomalous switching behavior in its circuit operation. These changes of states are classified into border collision bifurcation [6]. In despite of the propagation of TCR–SVC, the phenomenon has not been examined sufficiently. The reason seems that the nonlinear phenomena in multi-phase systems are not interested in because of the topological complexity of circuits and the limit of power application.

In this paper, the discontinuous changes of conduction angle are discussed with relation to the transient current waveform. In the states, the appearance of subharmonic components is confirmed through the single phase numerical simulations, experiments, and also in three phase. The coexistence of subharmonic components is also discussed with relation to the discontinuous change. The following discussions are based on the instantaneous value model of TCR-SVC with respect to the appearance of discontinuous phenomenon.

2. TCR-SVC and Principles of Operation

2.1. Configuration of TCR-SVC

The simplified power system with TCR-SVC is shown in Fig.1. The TCR–SVC is composed of an integrated combination of TCR and shunt capacitor. The shunt capacitor supplies the leading reactive power under constant ac voltage and the TCR regulates lagging reactive power through controlling the firing angle of thyristor valve. The base values of the circuit parameters are given in Table 1. These values are set based on the SVC, which is installed in Izushi Substation, in Hyogo, Japan [9].

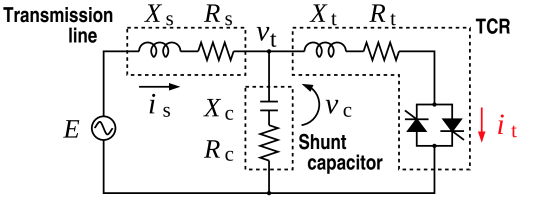

Figure 1: Simplified circuit configuration of TCR-SVC.

2.2. Operation of TCR-SVC

A TCR consists of a reactor and anti-parallel thyristor valves which are connected in series. The firing angle α of thyristor valve delays with respect to the phase of the ac voltage v_t in each half-cycle. The ideal valve current i_t to the sinusoidal ac voltage $v_t(t) = E \sin \omega_0 t$

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[1] Practical systems are designed with filters to avoid the influence of harmonics.

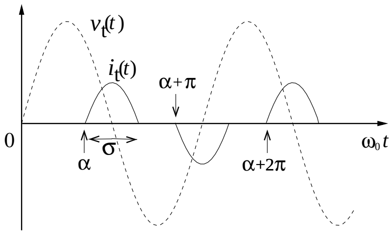


Figure 2: Ideal relation between ac voltage and TCR current; α denotes firing angle and σ conduction angle. The firing of thyristor valve is given in each half-cycle.

Table 1: Base values of circuit parameters

Base parameters for per unit.		Base value
Supply voltage	E	77 kV
Transmission line	X_{s}, R_{s}	100 MVA
TCR	X_{t}, R_{t}	30 MVA
Shunt capacitor	X_{c}, R_{c}	10 MVA

on the TCR is given by:

$$\begin{cases} i_{\mathrm{t}}(t) = 0 & \left(0 \le t \le \dfrac{\alpha}{\omega_0}\ ,\ \dfrac{2\pi-\alpha}{\omega_0} \le \dfrac{2\pi}{\omega_0}\right), \\ i_{\mathrm{t}}(t) = \dfrac{1}{L}\displaystyle\int_{\alpha/\omega_0}^{t} v(t)dt & \\ \quad = \dfrac{E}{X_{\mathrm{t}}}(\cos\alpha - \cos\omega_0 t) & \left(\dfrac{\alpha}{\omega_0} \le t \le \dfrac{2\pi-\alpha}{\omega_0}\right), \end{cases} \tag{1}$$

where E and ω_0 denote the amplitude and the angular frequency of the ac voltage, respectively. $X_{\mathrm{t}} = \omega_0 L$ and the firing angle $\alpha \in [\pi/2, \pi)$ in normal operation. R_{s} in transmission line and R_{t} in reactor are neglected.

The typical relation between voltage and current in TCR is shown in Fig. 2. The thyristor valve turns off when current reaches to zero. Then the conduction interval expressed by angle σ is defined by $\sigma = 2(\pi - \alpha)$ from Eq.(1) because $\cos\alpha - \cos(\alpha + \sigma) = 0$. This implies that the relation between firing angle α and conduction angle σ is uniquely determined, and σ changes depending on the continuous variation of α.

The valve current i_{t} can be decomposed into fundamental and harmonic components as functions of α:

$$\begin{cases} I_1(\alpha) &=& \dfrac{E}{X_{\mathrm{t}}}\left(\dfrac{2}{\pi}\alpha - \dfrac{1}{\pi}\sin 2\alpha - 2\right), \\ I_{\mathrm{n}}(\alpha) &=& \dfrac{E}{X_{\mathrm{t}}}\dfrac{4}{\pi}\dfrac{\cos\alpha\sin n\alpha - n\sin\alpha\cos n\alpha}{n(n^2-1)}, \\ && n = 2k+1\,(k = 1, 2, 3\cdots). \end{cases} \tag{2}$$

Then the TCR current causes the ac voltage distortion in the practical power circuit.

The differential equation of the circuit in conduction state is formulated as follows:

$$\begin{pmatrix} \dot{i}_{\mathrm{s(on)}} \\ \dot{i}_{\mathrm{t(on)}} \\ \dot{v}_{\mathrm{c(on)}} \end{pmatrix} = \begin{pmatrix} -\frac{R_{\mathrm{s}}+R_{\mathrm{c}}}{L_{\mathrm{s}}} & \frac{R_{\mathrm{c}}}{L_{\mathrm{s}}} & -\frac{1}{L_{\mathrm{s}}} \\ \frac{R_{\mathrm{c}}}{L_{\mathrm{t}}} & -\frac{R_{\mathrm{t}}+R_{\mathrm{c}}}{L_{\mathrm{t}}} & \frac{1}{L_{\mathrm{t}}} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{pmatrix} \times \begin{pmatrix} i_{\mathrm{s(on)}} \\ i_{\mathrm{t(on)}} \\ v_{\mathrm{c(on)}} \end{pmatrix} + \begin{pmatrix} \frac{E\sin\omega_0 t}{L_{\mathrm{s}}} \\ 0 \\ 0 \end{pmatrix}, \tag{3}$$

where, $i_{\mathrm{s(on)}}$, $i_{\mathrm{t(on)}}$ and $v_{\mathrm{c(on)}}$ denote the state variables of the conduction state. And the differential equation of the circuit in nonconduction state is formulated as follows:

$$\begin{pmatrix} \dot{i}_{\mathrm{s(off)}} \\ \dot{v}_{\mathrm{c(off)}} \end{pmatrix} = \begin{pmatrix} -\frac{R_{\mathrm{s}}+R_{\mathrm{c}}}{L_{\mathrm{s}}} & -\frac{1}{L_{\mathrm{s}}} \\ \frac{1}{C} & 0 \end{pmatrix} \begin{pmatrix} i_{\mathrm{s(off)}} \\ v_{\mathrm{c(off)}} \end{pmatrix} + \begin{pmatrix} \frac{E\sin\omega_0 t}{L_{\mathrm{s}}} \\ 0 \end{pmatrix}, \tag{4}$$

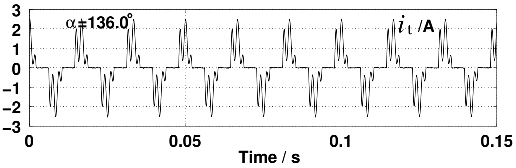
where $i_{\mathrm{s(off)}}$ and $v_{\mathrm{c(off)}}$ denote the state variables of nonconduction state. Numerical estimation is based on these equations and boundary conditions at every switching instance.

3. Subharmonic Oscillations in TCR-SVC

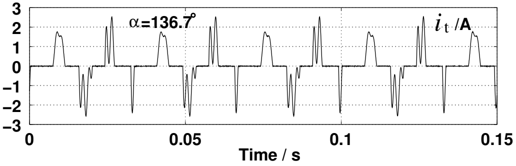
3.1. Estimation of solutions in single phase model

Three typical subharmonic oscillations of current in experiments are shown in Fig. 3. The distortion of the current waveform induces the subharmonic oscillation according to the increase of a firing angle. There also appear the asymmetric and symmetric subharmonics. Similar results can be confirmed in simulation, so that the existence of the subharmonics seems to be inevitable. The numerical and theoretical considerations on the occurrence of subharmonics are given in Fig. 4. Multiple points at the same firing angle imply the occurrence of subharmonics. The occurrence is caused by the disappearance of stable solutions without negative parts in its conduction term. The gray solid line in Fig. 4(c) implies the conduction angles which induces the anomalous turn-off behavior by thyristor operation. The S-shaped curve denotes the possibilities of anomalous jump of the conduction angle in circuit operation, even if the switch conducts in both directions.

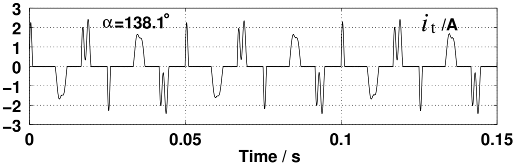
The step change of firing angle causes the convergence to coexisting different solutions depending on the initial conditions. Fig. 5 shows the numerical estimation to show the examples. In this case, the solutions without negative current exist around 146 degree.



(a) extremely distorted valve current



(b) asymmetric 1/2-subharmonic oscillation



(c) 1/3-subharmonic oscillation

Figure 3: Experimental results of subharmonic oscillation in single phase TCR-SVC.

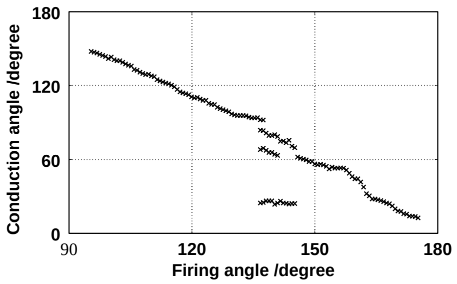
However, there appears a sub-harmonic oscillation in the numerical estimation under the monotonous decrease of firing angle. It implies that the subharmonic oscillation is stable even if the system keeps the solutions without negative current. Fig. 5(c) shows that the adequate step change of firing angle makes it possible to select the coexisting solutions. The results strongly depend on the initial conditions of firing angle at the instance of control.

3.2. Estimations of solutions in three phase model

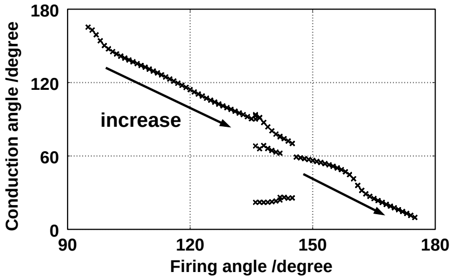
In the previous section, the experimental results in single phase circuits are sufficiently explained through numerical and theoretical estimation. Here, let us discuss the more practical system configurated in three phase. The three phase circuit configuration is shown in Fig. 6. The circuit topology and operation are symmetric in $2\pi/3$. The experiment and numerical estimation show the appearance of subharmonic oscillation as in a single phase system. The typical case is shown in Fig. 7. The similar waveforms with phase shift are observed in the other phases. The monotonous increase of firing angle shows the occurrence of discontinuous jump and subharmonics in conduction angle (Fig. 8).

4. Conclusion

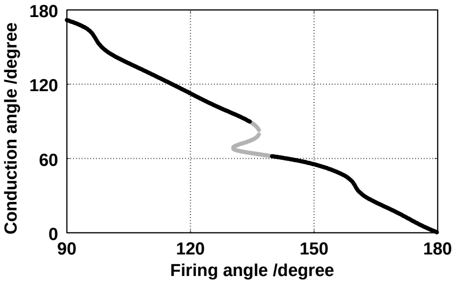
The anomalous jump of conduction angle in TCR-SVC is confirmed experimentally, numerically, and theoretically, both in single phase and three phase models The experimental results showed the subharmonic oscillations possibly occur at the appearance of jump. Moreover, the subharmonic oscillations are caused by the coexisting solution and the initial condition of switching operation. That is, the dynamics under the discontinuous switching by thyristor operation are globally governed by the coexisting solutions of TCR-SVC.



(a) experimental results



(b) numerically simulated results (firing angle is monotonically increased.)



(c) theoretical results

Figure 4: Conduction angle σ traced to firing angle α (Case-1). In this case the conduction angle, which keeps valve current without zero contact, does not exist.

Even in the case of self turn off switches, which allows the negative current in the switching device, the similar phenomenon possibly occurs. The global structure of phase space is the important clues to grasp the hybrid dynamics in TCR-SVC.

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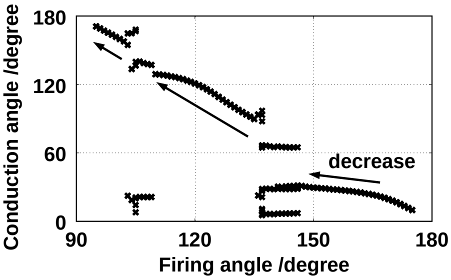
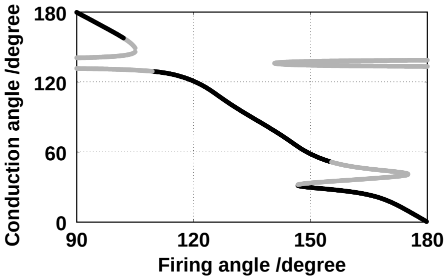
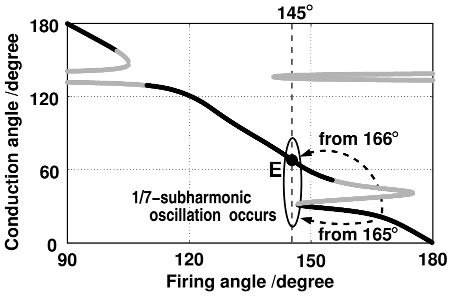


(a) numerically simulated results for monotonously decreased firing angle



(b) theoretical results



(c) generation of 1/7-subharmonic oscillation

Figure 5: Conduction angle σ traced to firing angle α (Case-2). In this case the conduction angle, which keeps valve current without zero contact, exists but discontinuously changes.

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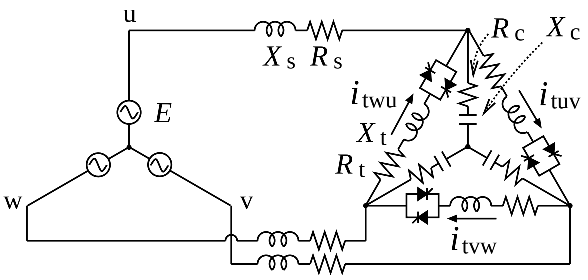

Figure 6: Simplified three phase circuit configuration of TCR-SVC.

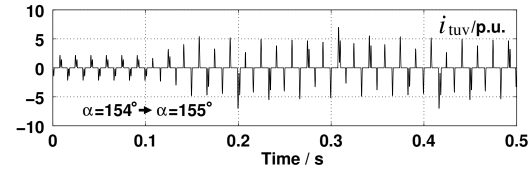

Figure 7: Three phase 1/5-subharmonic oscillation after the disappearance of asymmetric oscillations in i_{uv}.

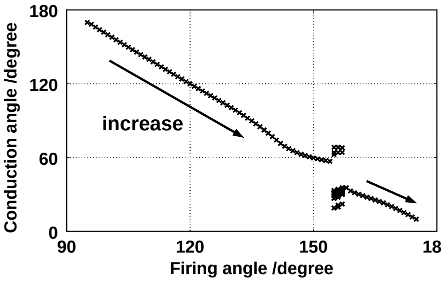

Figure 8: Conduction angle σ at monotonously increasing of α in three phase model.

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