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# Modeling and Dynamics of a High Voltage DC-DC Converter: A nonlinear approach

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**Abstract**—One of the most important research areas in the industry of devices used in power electronics consists in founding semiconductor devices able to conduct high current in the ON phase and simultaneously to support high voltages when they are in the OFF phase. New schemes of converters have been developed to overcome shortcomings in solid-state switching device ratings so that they can be applied to high-voltage electrical systems. On the other hand, recently, power electronic converters are shown to undergo many nonlinear phenomena. In this paper a nonlinear approach is used to model a multilevel high voltage converter controlled by constant frequency Pulse Width Modulation (PWM). This model is then used to predict some instability phenomena that can undergo the system.

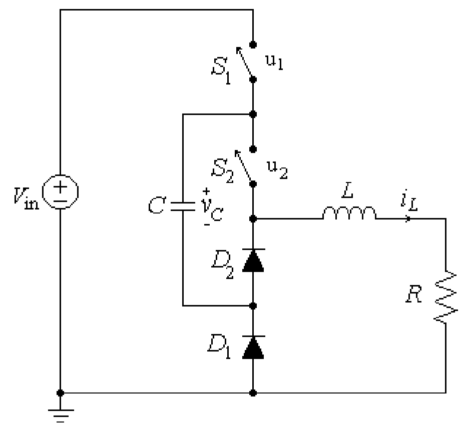



Figure 1: Schematic circuit diagram of a multilevel high voltage DC-DC buck converter

## 1. Introduction

One of the most important research areas in the industry of devices used in power electronics consists in founding semiconductor devices able to conduct high current in the ON phase and simultaneously to support high voltages when they are in the OFF phase. New schemes of converters have been developed to overcome shortcomings in solid-state switching device ratings so that they can be applied to high-voltage electrical systems. Applications include such uses as medium voltage adjustable speed motor drives, dynamic voltage restoration, harmonic filtering. Because distributed power sources are expected to become increasingly prevalent in the near future, the use of such a converter to control the current and voltage output directly from renewable energy sources will provide significant advantages because of its fast response and autonomous control. Additionally, they can also control the real and reactive power flow from a utility connected renewable energy source. On the other hand, recently, power electronic converters are shown to undergo many nonlinear phenomena. To study and to control these phenomena, an appropriate model is needed. Different Modeling approach can be used. The most traditionally used *averaged model* was found to fail in predicting some bifurcation phenomena like period doubling and chaotic oscillations. Even for elementary DC-DC converters with two configurations during one switching cycle, this model is shown to be inaccurate to predict some kinds of dynamical behavior s [1, 2]. Although low frequency instability in the form of a Hopf bifurcation can be predicted by the averaged model, some accuracy is lost as this model destroys the main non-linearity of the real system by averaging the state variables during a switching cycle. An alternative is the *discrete-time* modeling approach. The objective of this work is to construct a discrete time model able to predict accurately the dynamical behavior of a high voltage converter under different control strategies.

## 2. System Description

The studied converter is shown is Fig. 1. It is based on the well known buck converter. In order to improve the ability to high voltages, the usual converter is modified by using a controlled DC voltage source. This source is done by a simple capacitor whose charge and discharge currents are controlled by independent switches ($S_1$ and $S_2$). The analysis can be simplified by assuming that the voltage across the capacitor is constant. However, we will use a more accurate analysis for this circuit. Each pair formed by a switch $S_i$ and a diode $D_i$ is activated in a complementary manner in such away that when $S_i$ is ON, $D_i$ is OFF and *vice versa*. In order to obtain optimum waveforms for the inductor current, a phase shift of $180^o$ is introduced between the two sawtooth signals. In order to perform the switch mode operation of the converter, the durations of the time intervals $T_i$, $(i = 1...4)$ must be controlled. Pulse Width Modulation technique plays the basic control method. The duty cycles of the command signals $u_1$ and $u_2$ are varied proportionally to the error signals to control the output signals in such away that when the error $e_i$ is greater (resp. smaller) than the ramp voltage $v_{\text{ramp,i}}$, the switch is OFF (resp ON). We will suppose also that there exist a phase shift of $180^o$ between the two ramp signals signal in order to obtain optimum waveforms for the inductor current [3]. More precisely, the signal $e_1 = k_i(i_L - I_{\text{ref}}) + k_v(v_C - V_{\text{ref}})$ is compared with a ramp signal $v_{\text{ramp}}(t)$ while the signal $e_2 = k_i(i_L - I_{\text{ref}}) - k_v(v_C - V_{\text{ref}})$ is compared with a ramp signal $v_{\text{ramp}}(t - T/2)$, where $T$ is the period of the ramp signal. Figure 2 shows the block diagram of the controller described.

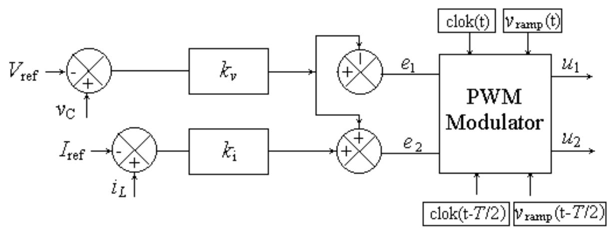



Figure 2: Block diagram of the controller used for the multilevel DC-DC buck converter

There are basically three modes of operation for the system depending on the duty cycle. In this paper we will focus on the behavior of the system for duty cycles between 0.5 and 1. In this case it can be shown that the nominal periodic behavior of the system is characterized by switching between three configurations during four intervals. Figure 3 shows the stationary waveforms of the system under stationary normal periodic operation for a value of duty cycle between 0.5 and 1.

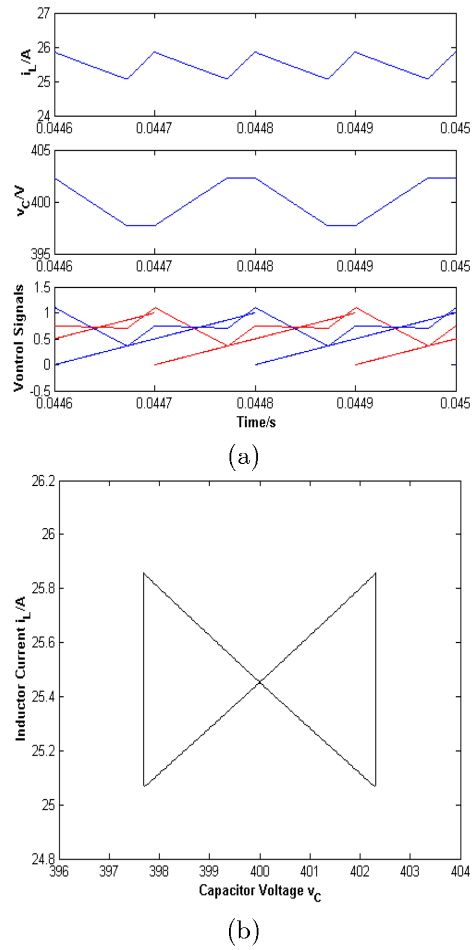



Figure 3: Operation of the system in stationary normal periodic behavior for a value of the duty cycle between 0.5 and 1. (a) Time domain waveforms. (b) the orbit in the state plane.

## 3. Some Bifurcation Phenomena Observed in the System

In this section we will present some bifurcation behavior of the system under the variation of the inductor current loop gain $k_i$ and the reference current $I_{\text{ref}}$. The illustration of the results are supported by time domain waveforms, trajectory in the state plan $(v_C, i_L)$, Poincaré sections and bifurcation diagrams. We begin by plotting a bifurcation diagram for the system by varying $k_i$ in the range (0.5, 1) $\Omega$. The parameter values used are: Input voltage $V_{\text{in}} = 800$ V, capacitance of the capacitor $C = 400$ $\mu$F inductance of the inductor $L = 10$ mH, load resistance $R = 20$ $\Omega$, gain of the voltage loop $k_v = 0.075$. Reference voltage $V_{\text{ref}} = 400$ V and reference current $I_{\text{ref}} = 24$ A. The bifurcation diagram corresponding to this set of parameter values is shown in Fig. 4. We can observe clearly that for some critical value of the current gain,

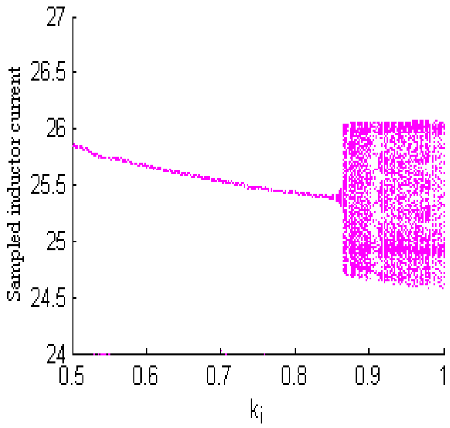
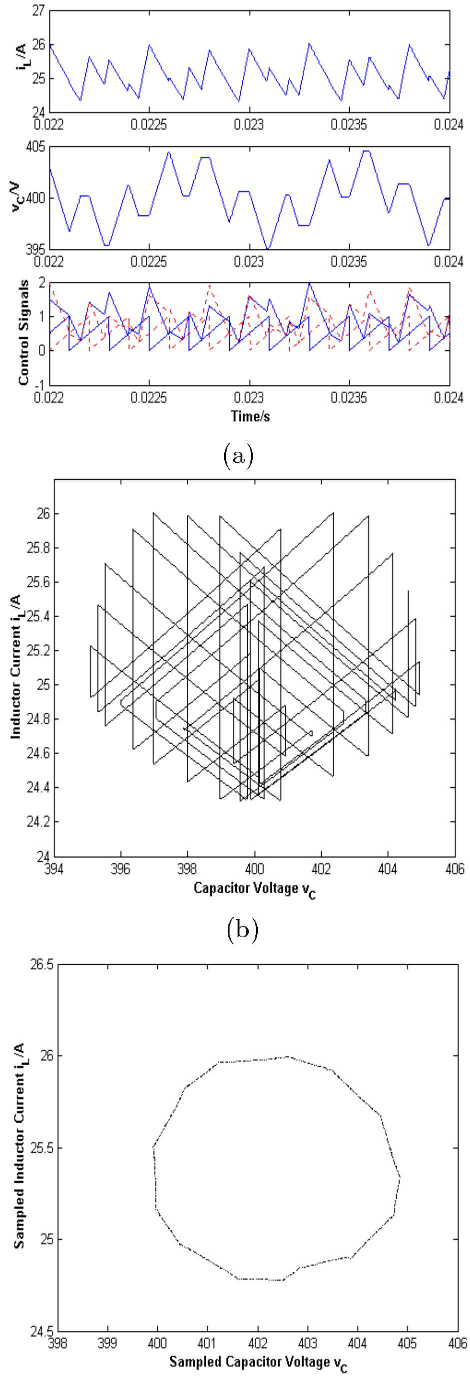



Figure 4: Bifurcation diagram of the multilevel high voltage DC-DC buck converter taking the current loop gain $k_i$ as a bifurcation parameter

a periodic a orbit undergo a Hopf bifurcation and the resulting attractor is quasiperiodic (Fig. 5-a). The attractor in the state plane becomes a Torus (Fig. 5-b) and its Poincaré section is an invariant closed curve (Fig. 5-c). A similar behavior is obtained when the reference current $I_{\text{ref}}$ is decreased.

## 4. Discrete Time Modeling Approach

The accurate study of the complex behavior of a switched dynamical system can be carried out by constructing a nonlinear model which conserves the main nonlinearity of the system, i.e, the switching action. Discrete time model is the more accurate in predicting any kind of instability. In this section we will present a systematic way to obtain the nonlinear discrete time model for the multilevel high voltage DC-DC buck converter studied in this paper. Without lose of generality we will suppose that duty cycle is between 0.5 and 1. In this case, the nominal periodic behavior is characterized by toggling among three different configurations during four subintervals within a switching cycle. For each configuration, the system equations are linear time invariant in the form of $\dot{x} = Ax + B$, where $x = (i_L, v_C)$ is the vector of the state variables. The three different configurations corresponds to the different states of the switches $S_1$ and $S_2$ and diodes $D_1$ and $D_2$. When the duty cycle is between 0.5 and 1, the following sequence is obtained for $(S_1, S_2)$: (OFF,ON) → (ON,ON) → (ON,OFF) → (ON,ON). As the diode are complementarily activated to the switches, the sequence for $(D_1, D_2)$ during a switching cycle is: (ON,OFF) → (OFF,OFF) → (OFF,ON) → (OFF,OFF). Hereafter, the different configurations are identified by the state of the switches $S_1$ and $S_2$. The different configurations and their equations are:




(a)

(b)

(c)

Figure 5: Quasiperiodic behavior obtained after the system undergoes a Hopf bifurcation. (a) Time domain waveforms. (b) the orbit in the state plane, (c) Poincaré section

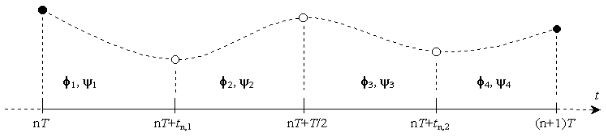



Figure 6: The switching instants during one switching cycle and their associated matrices $\Phi_i$ and vectors $\Psi_i$

- Configuration (OFF,ON). During this configuration, the capcitor is charged while the inductor is discharged. The system matrix $A$ and vector $B$ for this configuration are

$$A_1 = \begin{pmatrix} -\frac{R}{L} & \frac{1}{L} \\ \frac{1}{C} & 0 \end{pmatrix}, \ B_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

- Configuration (ON,ON). During this configuration, the capcitor charge is maintained and the inductor is charged. The system matrix $A$ and vector $B$ for this configuration are:

$$A_2 = \begin{pmatrix} -\frac{R}{L} & 0 \\ 0 & 0 \end{pmatrix}, \ B_2 = \begin{pmatrix} \frac{V_{\text{in}}}{L} \\ 0 \end{pmatrix} \tag{2}$$

- Configuration (ON,OFF). During this configuration, both capcitor and inductor are discharged. The system matrix $A$ and vector $B$ for this configuration are

$$A_3 = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L} \\ -\frac{1}{C} & 0 \end{pmatrix}, \ B_3 = \begin{pmatrix} \frac{V_{\text{in}}}{L} \\ 0 \end{pmatrix} \tag{3}$$

During the fourth sub-interval, the system uses the same matrix and vector $A_2$ and $B_2$. Then we have $A_4 = A_2$ and $B_4 = B_2$.

The discrete time approach is based on stroboscopic sampling of the state variables at the beginning of each switching cycle. The consecutive states $x_{n+1} := x((n+1)T)$ and $x_n := x(nT)$ must be related by a two dimensional map $P$, i.e $x_{n+1} = P(x_n)$. The aim of this section is to obtain the analytical expression of such a map. When the multilevel DC-DC buck converter is working for duty cycle greater between 0.5 and 1, it switches between three different configurations during four subintervals (Fig. 6). These are:

$$\begin{array}{lll} \dot{x} = A_1 x + B_1 & \text{for} & t \in [nT, nT + \tau_{n,1}] \\ \dot{x} = A_2 x + B_2 & \text{for} & t \in [nT + \tau_{n,1}, nT + \frac{T}{2}] \\ \dot{x} = A_3 x + B_3 & \text{for} & t \in [nT + \frac{T}{2}, nT + \tau_{n,2}] \\ \dot{x} = A_3 x + B_4 & \text{for} & t \in [nT + \tau_{n,2}, (n+1)T \end{array} \tag{4}$$

where $\tau_{n,i}$ $(i = 1..2)$ are the switching instants within the switching period. $x$ is the vector of the state variables and $A_k$ and $B_k$ are the system matrices during phase $k$ $(k = 1...4)$. As it was already mentioned, during each sub-interval the system equations are linear and time invariant. In this case, the solution during each phase interval is available and takes the following form:

$$x(t) = e^{A(t-t_k)} x(t_k) + \int_{t_k}^{t} e^{A(t-\alpha)} B d\alpha \tag{5}$$

where $t_k$ is the instant at which the system switches from one configuration to another, $x(t_k)$ is the state vector at the switching instant $t_k$. The switching functions can be expressed as the difference between the amplified error signals $e_i$ and the ramp signal $v_{\text{ramp,i}}$, i.e:

$$\sigma(x, t) := e_i(t) - v_{ramp,i}(t) \tag{6}$$

The system switches from one configuration to another whenever this switching function crosses zero. There are mainly two switching instants to be determined from the switching fucntion while the third one is always fixed to $T/2$. In order to construct a generalized map, let us write Eq. (5) in the following form for convenience.

$$x(t) = \Phi(t - t_k) x(t_k) + \Psi(t - t_k) := \phi(t, t_k) \tag{7}$$

where $\Phi(t) = e^{At}$ and $\Psi(t) = \int_{t_k}^{t} e^{A(t-\alpha)} B d\alpha$. The mapping that relates the state variables $x_n$ at the beginning of an entire cycle to $x_{n+1}$, those at the end of the same cycle, can be build in the following way:

$$\begin{array}{l} P : \mathbf{R}^2 \mapsto \mathbf{R}^2 \\ \quad x_n \mapsto x_{n+1} := P(x_n) \\ \quad = \phi_1(\tau_{n,1}, x_n) \circ \phi_2(\frac{T}{2} - \tau_n, x(\tau_{n,1}))\circ \\ \quad \phi_3(\tau_{n,2} - \frac{T}{2}, x(\frac{T}{2})) \circ \phi_4(T - \tau_{n,2}, x(\tau_{n,2})) \end{array} \tag{8}$$

The general switching equation may be written in the following form:

$$\begin{array}{l} \sigma(x_n, \tau_n) := \begin{bmatrix} \sigma_1(x_n, \tau_{n,1}) \\ \sigma_2(x_n, \tau_{n,1}\tau_{n,2}, ) \end{bmatrix} \\ := \begin{bmatrix} K_1(\phi_1(\tau_{n,1}, x_n)) - v_{\text{ramp}}(\tau_{n,1}) \\ K_2(\phi_3(\tau_{n,2} - \frac{T}{2}, \phi_2(\frac{T}{2} - \tau_{n,1}), \phi_1(\tau_{n,1}, x_n)) \\ -v_{\text{ramp}}(\frac{T}{2} - \tau_{n,2}) \end{bmatrix} \\ = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array} \tag{9}$$

where $K_1 = (k_i, -k_v)$ and $K_2 = (k_i, k_v)$. The map (Eq. (8)) and the constraint (Eq. (9)) define the discrete-time model for the multilevel DC-DC buck converter under the PWM control. The stability of the nominal periodic orbit of the system can be studied by analyzing the stability of the fixed points $x^*$ of the map $P$. Thses can be obtained by enforcing the periodicity: $x_n = x_{n+1} = x^*$. In doing so, $x^*$ can be expressed in terms of switching times and system matrices $\Phi_k$. The obtaining of the fixed point $x^*$ requires solving a transcendental equation $\sigma(\tau^*) = 0$. Therefore a root finding algorithm should be applied. Once the fixed points are located, their stability analysis may be carried out by studying the local behavior of the map $P$ near these fixed points. Denoting $x_n - x^*$ by $\tilde{x}_n$ and noting that by definition $P(x^*) = x^*$, the linearized map can be written as:

$$\tilde{x}_{n+1} \approx DP(x^*)\tilde{x}_n \tag{10}$$

where $DP(x^*)$ is the Jacobian matrix of $P$. The eigenvalues $\lambda_i$ of $DP$ give the amount of expansion or contraction near the fixed point $x^*$ when the map is once iterated. Therefore they determine the stability asymptotic of the fixed point and hence of its underlying periodic orbit. A sufficient condition for stability is that all characteristic multipliers lie inside the unite circle. The expression of the Jacobian matrix $DP$ evaluated at the fixed point is

$$DP(x^*) = \frac{\partial P}{\partial x_n} + \frac{\partial P}{\partial \tau_n}\frac{\partial \tau_n}{\partial x_n}\bigg|_{x^*} \tag{11}$$

By implicit derivation using the implicit function theorem we get:

$$DP\bigg|_{x^*} = \frac{\partial P}{\partial x_n} - \frac{\partial P}{\partial \tau_n}\left(\frac{\partial \sigma}{\partial \tau_n}\right)^{-1}\frac{\partial \sigma}{\partial x_n}\bigg|_{x^*} \tag{12}$$

By calculating each term in Eq. (12) using Eq. (8) and Eq. (9), $DP$ can be obtained in a straightforward manner. Details are omitted here for limited space.

## 5. Results of Stability Analysis Using the Jacobian Matrix

Power electronic systems are usually characterized by having complex eigenvalues. As a parameter is varied a pair of complex conjugate eigenvalues can leave the unit disc. In this case the system is said to undergo a Hopf bifurcation. As a result, a periodic orbit bifurcate to a torus. In the corresponding discrete time system a Neimark-Sacker bifurcation which causes a fixed point to bifurcate to an invariant closed curve. The multilevel DC-DC buck converter is a two dimensional system being able therefore to have a pair of complex eigenvalues. Figure 7 shows the evolution of the eigenvalues of the Jacobian matrix evaluated at the fixed point of the discrete time model of the system when $k_i$ and $I_{\text{ref}}$ are varied. The current loop gain is increased until the stability of the system is lost. This

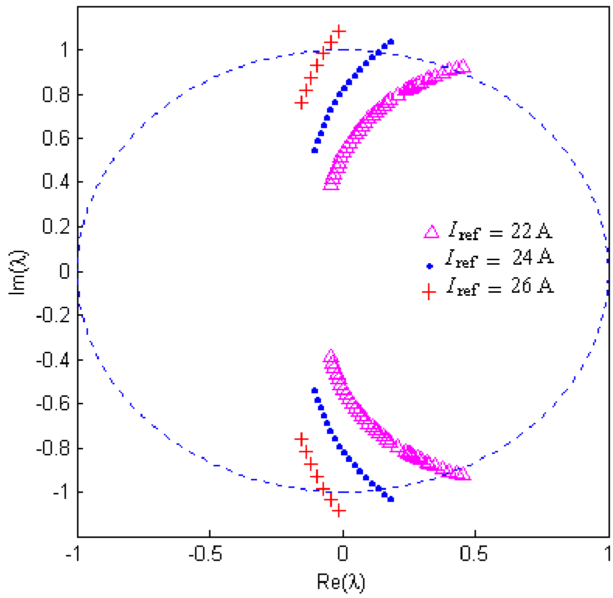



Figure 7: Evolution of the eigenvalues of the Jacobian matrix when $k_i$ and $I_{\text{ref}}$ are varied

was repeated for three values of the reference current which are indicated in Fig. 7. It can be observed that for a fixed value of the reference current $I_{\text{ref}}$, as the current loop gain is increased, two complex conjugate eigenvalues leaves the unit circle indicating that a Hopf bifurcation occurs at a critical value $k_{i,cri}$ which decrease as $I_{\text{ref}}$ is increased. For $I_{ref} = 24$ A, the critical value of the feedback coefficient $k_{i,cri} = 0.86$ A. This value agree well with bifurcation diagram of Fig. 4. Theses results allow to obtain accurately the stability boundary in the design parameter $(k_i, I_{\text{ref}})$. This boundary is plotted in Fig. 8. The result of this figure match well with the simulation of the circuit by using the switched model.

Another type of bifurcations that can undergo this system is the well known period doubling bifurcation. This behavior corresponds to the case when an eigenvalue crosses the unite circle from the point (-1,0) as a parameter is varied. Border collision bifurcations are also observed by numerical simulations and they are due essentially to the change of operating mode. The detailed phenomena observed in this circuit will be present in a further study.

## Conclusions

This paper presented a systematic way to obtain the discrete time model of a high voltage multilevel DC-DC buck converter. This model can be used to study accurately the stability of the system. Based on this

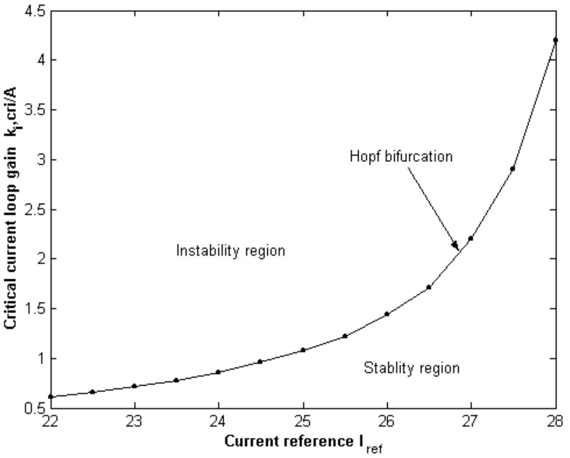



Figure 8: Boundary between stable and unstable region in the design parameter $(I_{\rm ref}, k_i)$

study, the boundary between the stable and the unstable region can be obtained. The stability region is determined by analyzing the eigenvalues of the Jacobian matrix of the stroboscopic Poincaré map. Two design parameters are varied and it was obtained that as the current loop gain is increased, the system loses stability by a typical Hopf bifurcation. The same behavior is obtained when the reference current is decreased. The results on the stability analysis by using the Jacobian matrix are confirmed by time domain simulations, phase plane trajectories, Poincaré sections and bifurcations diagrams obtained from the switched model. Future work will deal with the study of other nonlinear phenomena in this circuit which were already detected like period doubling and border collision bifurcations. Experimental confirmation is also the subject of further study.

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