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EMI-Performance of DC-DC Converters – Criteria and Spectral Optimisation

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Abstract—DC-DC Converters are known to be potential sources of electromagnetic radiation due to their discontinuous working principle. A periodic switching operation will result in a discrete power density spectrum and hence in a poor EMI performance. In order to improve this performance various methods are being applied. In this paper we focus on clock period and clock frequency modulation by use of pattern generators. We introduce performance indices in order to evaluate the EMI produced by the converter. Using these indices we propose methods to find optimised modulation patterns of finite length.

1. Introduction

DC-DC converters produce AC current components at the supply line due to their discontinuous working principle, i.e. switching voltages and currents. This causes electromagnetic interference (EMI) with other systems. Conventional DC-DC converters are clocked periodically which results in a discrete power density spectrum (PDS) with poor EMI performance. To improve EMI several methods exist e.g. supply line filtering, pulse shaping, periodic[1], chaotic[2],[3] and randomised modulation schemes[4]. As most systems today contain a digital processor periodic modulation using a pattern generator with finite memory is very common. This paper investigates the synthesis of optimised modulation sequences of finite length especially appropriate for use in pattern generators. Therefore in section 4 we introduce performance indices to rate the PDS of a modulation sequence. We propose an option of considering EMC conventions by the performance index. In section 5 we propose two methods of synthesising optimised modulation sequences and compare them in the application of a the DC-DC converter.

2. DC-DC Converter

Fig. 1 depicts the DC-DC converter schematic. It operates using hysteresis control. The switch S is switched on when the inductor current i_L reaches the reference current I_r and off when i_L exceeds the upper limit $I_r + I_n$. In the steady state I_r is constant. In the sequel the PDS of the inductor current will be optimised. This is done by changing the distance I_n in every switching cycle. Fig. 2 draws the corresponding time waveform of i_l. Every time the switch closes the pattern generator provides a new value I_n. The pattern generator has a finite memory and hence the modulation waveform is periodic.

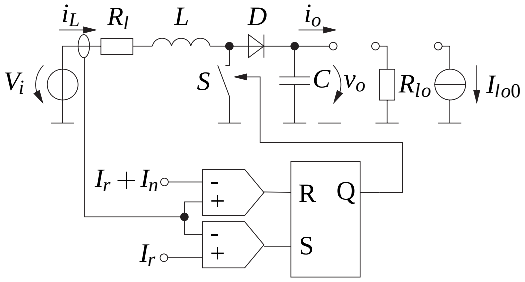


Figure 1: Boost converter with hysteresis control.

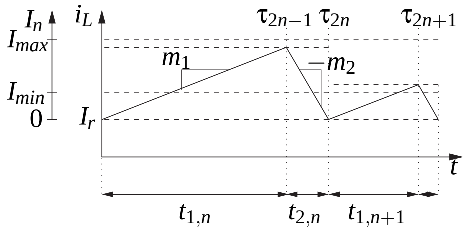


Figure 2: Hysteresis control inductor current.

3. Spektrum calculation

The fourier coefficients of the periodic time function $i_L(t)$ are given by:

$$c_k = \frac{m_1 + m_2}{k^2 \omega_0^2 \tau_{2N}} \sum_{n=1}^{N} \left(e^{jk\omega_0 \tau_{2n-1}} - e^{jk\omega_0 \tau_{2n}} \right),\ k \in \mathbf{Z} \tag{1}$$

where N is the length of the sequence, $\tau_{2n-1} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) \sum_{\mu=1}^{n-1} I_\mu + \frac{1}{m_1} I_\mu$, $\tau_{2n} = \left(\frac{1}{m_1} + \frac{1}{m_2}\right) \sum_{\mu=1}^{n} I_\mu$ and $\omega_0 = \frac{2\pi}{\tau_{2N}}$. The corresponding power spectral line of the inductor current is

$$p_k = 2|c_k|^2,\ k \in \mathbf{N}_+ \tag{2}$$

In [5] the statistical analysis of this kind of signals is explained in detail. The PDS consists of delta pulses. As a result of the resolution bandwidth of the analyser which is larger than zero the measuerd PDS will be finite. When the resolution bandwidth is small enough compared to ω_0 the measured PDS is the calculated power of the spectral line. This we will assume in the sequel and use the Power spectrum instead of the PDS.

4. Performance indices

4.1. Indices

The performance index *id* provides a goal function for optimisation procedures. Several approaches are possible depending on the objectives of the optimisation:

1. Power within a bandlimited interval

$$id = \sum_{k_{min}}^{k_{max}} p_k \tag{3}$$

2. Maximum power coefficient in an interval

$$id = \max(p_k),\ k \in [k_{min}, k_{max}] \tag{4}$$

3. Difference between maximum and minimum power coefficient in an interval

$$id = \max(p_k) - \min(p_k),\ k \in [k_{min}, k_{max}] \tag{5}$$

4. Power difference between neighboring power coefficients in an interval

$$id = \sum_{k_{min}}^{k_{max}-1} |p_{k+1} - p_k| \tag{6}$$

To optimise the EMI performance we use index 2.

4.2. EMC conventions

EMC conventions define an upper limit curve for the measured PDS. A typical shape [6] is depicted in Fig. 3. To consider the EMC conventions the Power spectrum will be weighted with the EMC shape of Fig. 3, i.e. we divide the power coefficients by $H_{EMC}(\omega)$ and determine performance index 2 from the result. This index has to be minimised.

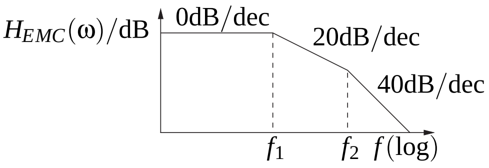


Figure 3: Typical EMC shape.

5. Pattern optimisation methods

A natural approach is to take all N subsequent values of the sequence I_n as optimisation variables. This results in an N-dimensional optimisation problem. However the goal function turns out to have multiple minima so that iteration procedures tend to find local minima rather than the global one.

In the sequel we discuss two methods to overcome this problem.

5.1. Polynomial waveform generation

5.1.1. Method

We use a polynomial sequence waveform given by:

$$I_n = \begin{cases} I_{min} + a\left(\frac{n}{N}\right)^{\alpha} & 0 < n \leq \frac{1}{4}N \\ I_{max} - b\left|\frac{1}{2} - \frac{n}{N}\right|^{\beta} & \frac{1}{4}N < n \leq \frac{3}{4}N \\ I_{min} + a\left(1 - \frac{n}{N}\right)^{\alpha} & \frac{3}{4}N < n \leq N \end{cases} \tag{7}$$

where:

$$a = \frac{I_{max} - I_{min}}{2} 4^{\alpha}, \quad b = \frac{I_{max} - I_{min}}{2} 4^{\beta} \tag{8}$$

This approach can also be found in [1] in principle and includes the goal functions therein. Fig. 4 depicts three sequence waveform examples.

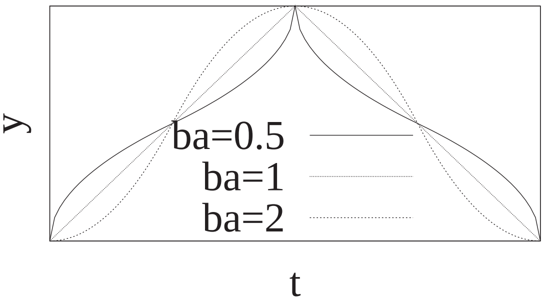


Figure 4: Polynomial waveform.

5.1.2. Results

The converter parameters used for power spectrum optimisation are shown in Tab. 1. We investigate the following cases:

Table 1: DC-DC converter parameters

m_1	170 A/ms
m_2	340 A/ms
I_{min}	1 A
I_{max}	1.5 A

1. $I_n = 1.25$ A $= const.$, classical periodic case as reference.
   Result: The power of the first and largest harmonic is 61 mA^2.
2. Polynomial waveform generation (Eq. (7)) without EMC shape consideration, $N = 50$.
   Result: The optimum exponents are $\alpha = \beta = 0.58$. Fig. 5 depicts the corresponding Power spectrum. The maximum power in the region of the mean clock frequency is reduced by approximately 9 dB.
3. Polynomial waveform generation (Eq. (7)) with EMC shape consideration, $N = 50$, $f_1 = 80$ kHz and $f_2 = 300$ kHz in Fig. 3.
   Result: The optimum exponents are $\alpha = \beta = 0.84$. The corresponding Power spectrum and the EMC shape are depicted in Fig. 6.

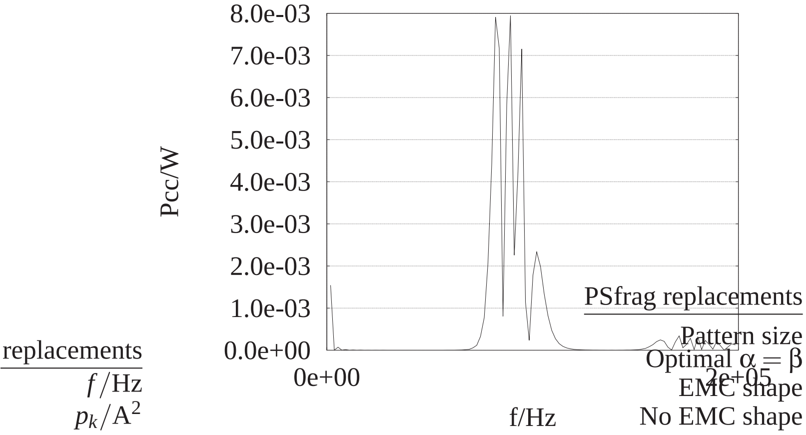


Figure 5: Converter Power spectrum, polynomial waveform, no EMC shape.

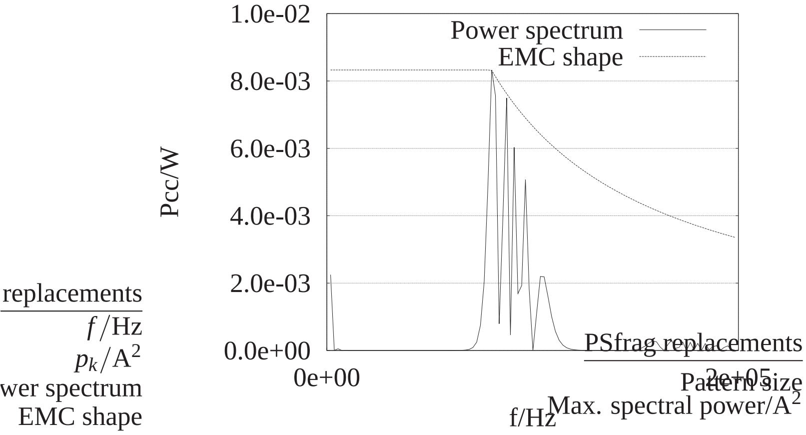


Figure 6: Converter Power spectrum, polynomial waveform, EMC shape.

Fig. 7 depicts the dependence between the pattern length N and the optimum exponents for the cases 2 and 3. For small pattern length it is a large value as a result of fast modulation effects [2]. Note that the result of the optimisation is $\alpha = \beta$. This is no longer true if the pattern generator and switch are not synchronised. The maximum value of the optimised Power spectrum versus pattern size N is depicted in Fig. 8.

5.2. Iterative optimisation procedure

5.2.1. Method

We start with an initial waveform. It can be arbitrarily chosen or determined by the procedure in 5.1. Then we vary every single value of the modulation sequence until a minimum of the goal function is found. This procedure is iterated until no further improvement is achieved. Nevertheless the minimum remains to be a local one. It depends on the initial sequence where the procedure starts.

5.2.2. Results

We investigate the following cases:

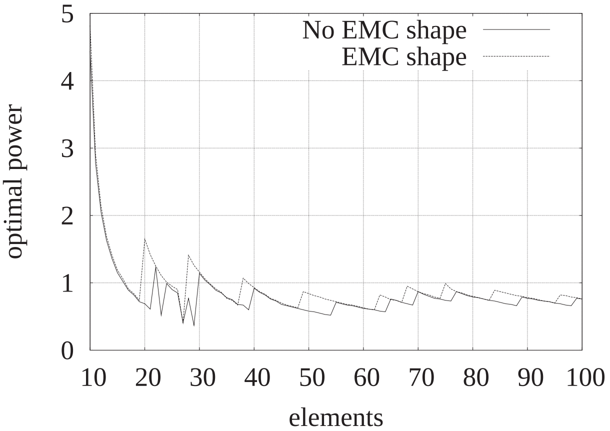


Figure 7: Optimal exponent versus pattern size N, polynomial waveform.

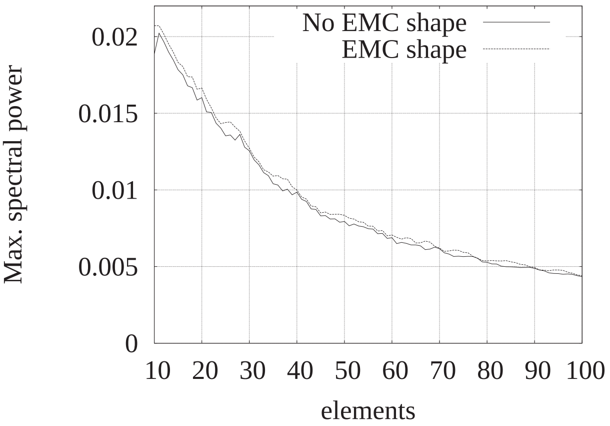
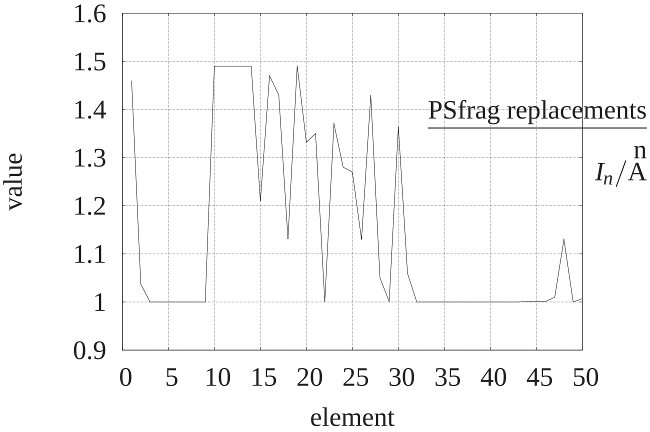


Figure 8: Maximum spectral power versus pattern size N, polynomial waveform.

1. Initialisation with classical periodic sequence, $I_n = 1$ A, $N = 50$, no EMC shape consideration.
   Result: Fig. 9 depicts the optimised control sequence. It is close to binary modulation schemes, see [2]. The Power spectrum is depicted in Fig. 10 by the thin solid line. The maximum power is reduced by 12 dB compared to periodic clock which is about 3 dB better than the polynomial fitted waveform.
2. Initialisation with classical periodic sequence, $I_n = 1$ A, $N = 50$, EMC shape consideration, $f_1 = 80$ kHz and $f_2 = 300$ kHz.
   Result: The power spectrum is depicted in Fig. 10 by the thick dashed line. As a result of the larger number of parameters this method is much better appropriate to fit the control sequence to requested EMC conventions than the polynomial waveform generation.
3. Initialisation with triangular sweep, $N = 50$, no EMC shape consideration.
   The resulting control sequence is drawn in Fig. 11 by the thin solid line. The maximum power is reduced by 11 dB comparing to the periodic clock.
4. Initialisation with optimised polynomial waveform of section 5.1, $N = 50$, no EMC shape consideration.

Result: Fig. 11 depicts the optimised control sequence by the thick dotted line. The maximum power here is 10 dB below the classical periodic case.

When triangular sweep and polynomial waveform are used as initialisation sequence the optimisation procedure results in similar waveforms (Fig. 11) i.e. that the performance indices of these waveforms are closed to local minima.

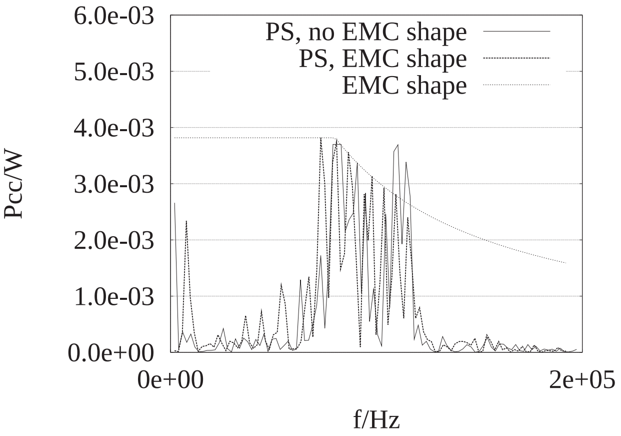


replacements

n

I_n/A

Figure 9: Iterative optimised classical periodic sequence.



replacements

f/Hz

p_k/A^2

Figure 10: Power spectra of iterative optimised classical periodic sequence.

As the iterative procedure results only in a local minimum depending on the initialisation sequence several runs with different initialisation sequences should be performed. In our examples the periodic case produced the best result which is about 3 dB better than the fitted exponential waveform and 4 dB better than the triangular sweep.

As the triangular sweep is standard in modulated clocking schemes it can be used as performance reference. The iteration procedure results in substantial improvement for small pattern lengths (30 values or less). From varius experiments we found that the iterative procedure is always worthwhile to be applied if an EMC shape is considered as a result of its higher fitting performance.

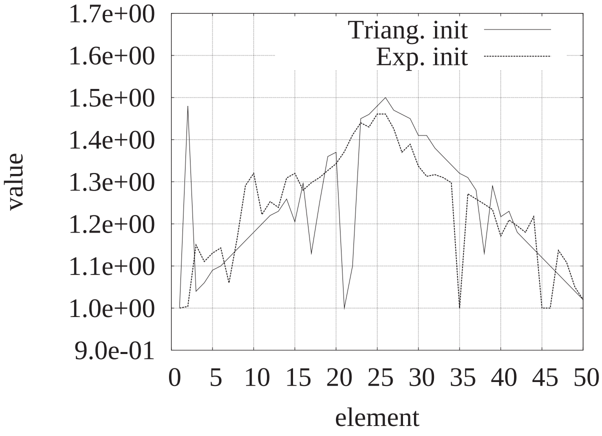


Figure 11: Iterative optimised waveforms.

6. Conclusions

Modulation is an efficient method to adjust the EMI performance of DC-DC converters. In the paper we introduced performance indices as a base of optimisation procedures. Two methods of optimising finite control sequences have been proposed. Examples have been provided. For optimisation a software tool has been implemented. Extended waveform generations using further parameters moving the boundaries between the segments of Eq. 7 and algorithmic optimisation of binary periodic and nonperiodic modulation schemes are due to future work. Further research on evolutionary methods to find a global optimum can improve the method.

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