2004 International Symposium on Nonlinear
Theory and its Applications (NOLTA2004)
Fukuoka, Japan, Nov. 29 - Dec. 3, 2004

Synthesis and analysis of piecewise-constant systems with sampled-data feedbacks

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Abstract—We consider a piecewise-constant systems with sampled-data feedbacks and show a synthesis procedure for a system which generates chaos and periodic solutions. The periodic solutions are stabilized "Unstable Periodic Orbits" embedded on chaos and the controlling method is based on a delayed state feedback. An analysis of the system can be performed by using the piecewise linear return map. We also provide a simple implemented circuit and typical phenomena are verified in the laboratory.

1. Introduction

Synthesis of chaotic oscillator with simple structure have received grate attention due to the increasing number of application of chaotic oscillator in engineering, as for example in communication systems [1].

Several model of simple chaotic oscillator with switching elements have been reported in some literature because of their simple structure allowing the implementation without difficulty and the use of less complex tools for its analysis. Also, some chaotic systems with sampled-data feedback loop have been studied [2]-[4].

On the other hand, periodic solutions are one of the most important phenomena in nonlinear dynamical system including many engineering systems. While stability analysis of periodic solutions is a basic problem, synthesizing a nonlinear system which exhibits a stable periodic solution is also essential. Several method for the inverse problem of synthesizing such systems have been proposed [5][6].

In this paper, we propose a novel nonlinear system which consists of a chaotic system and a dynamic controller. The proposed system exhibits some stabilized Unstable Periodic Orbits (abbr. UPO)which are embedded on chaos attractor of the original chaotic system. Generally, the procedure to stabilize UPOs is called Controlling Chaos [7][8]. Our proposal is a synthesis of a nonlinear system which generates periodic solutions based on chaos.

The basic principle is using a feedback of delayed states, but is not included on a category of Delayed Feedback Control (abbr. DFC) [8]-[11] well-known as an one of a controlling chaos methods. However a proposed system has an advantage such that no preliminary calculation of the UPOs is required, similarly to the DFC.

First, we consider a Nonautonomous Piecewise-Constant System as the basic chaotic system. The system dynamics is governed by a 1-D return map. By using the return map, we can accomplish the both of synthesis and analysis of the system. Chaos generation can be guaranteed theoretically. Second, we construct a controlled system based on the return map. We provide a condition to stabilize UPOs and a domain of attraction. Some theoretical results are verified in the experimental circuit.

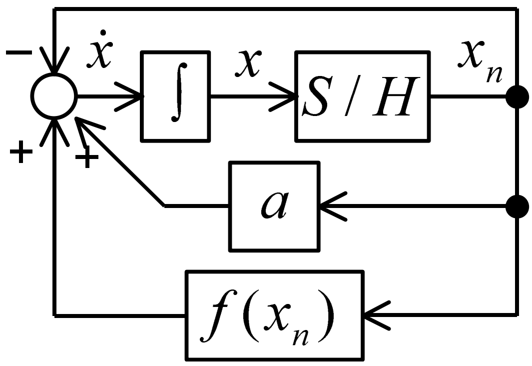

Figure 1: Nonautonomous piecewise-constant systems

2. A nonautonomous PWC with sampled-data feedbacks

A nonautonomous piecewise-constant system (abbr. NMPL) is shown in Fig. 1, where the block of S/H samples x when the normalized time τ is equal to n ($n = 1, 2, 3, \cdots$) and stores the sampled value until $\tau = n + 1$. Let the output of S/H be x_n:

$$x_n = x(n), \quad \text{for} \quad n \leq \tau < n + 1. \tag{1}$$

Then the system dynamics is described as the following:

$$\dot{x} = f(x_n) + (A - 1)x_n, \quad \text{for} \quad n \leq \tau < n + 1. \tag{2}$$

Letting the nonlinear function $f(\cdot)$ be time-invariant, the righthand side of the equation is piecewise-constant and the trajectory of the solution is piecewise-linear. Figure 2 shows an example of the time-domain wave form, where a solid and broken line shows $x(\tau)$ and x_n, respectively.

Here, focusing the state $x(\tau)$ at $\tau = n$, we can define the return map $F(x_n)$ from $x(n)$ to $x(n+1)$. The return map can be described explicitly as the following:

$$F : x_n \mapsto x_{n+1}, \quad F(x_n) = f(x_n) + Ax_n. \tag{3}$$

Changing the nonlinear function $f(x_n)$, we can construct any 1 dimensional (abbr. 1-D) return map $F(x_n)$. In this paper, we consider the case that the nonlinear characteristic $f(x_n)$ is define as the following:

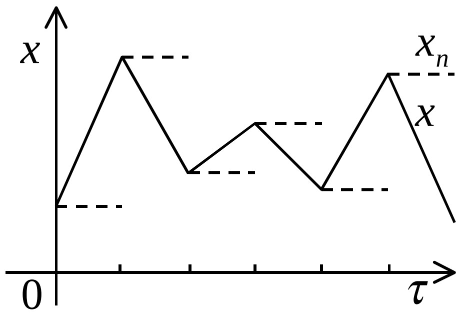

Figure 2: A time-domain waveform

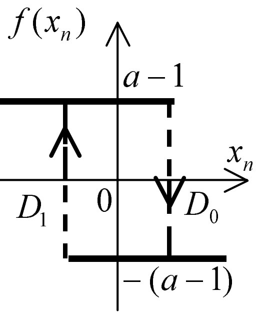

Figure 3: a hysteresis characteristic

$$f(x_n) = \begin{cases} (a-1), & \text{for} \quad x_n \leq D_0, \\ -(a-1), & \text{for} \quad x_n > D_1. \end{cases} \tag{4}$$

where the parameters a, D_0 and D_1 are constant. The hysteresis characteristic is shown in Fig. 3. $f(x_n)$ switches from $a-1$ to $-(a-1)$ if x_n reaches the right threshold D_0, $f(x_n)$ switches from $-(a-1)$ to $a-1$ if x_n reaches the left threshold D_1. In this case, the return map $F(x_n)$ is to be a Hysteresis return map [14] as shown in Fig. 4;

$$F(x_n) = \begin{cases} F_0 = ax_n + (a-1), & \text{if} \quad x_n \leq D_1, \\ \text{or} \quad D_1 < x_n \leq D_0, x_n = F_0(x_{n-1}), & \\ F_1 = ax_n - (a-1), & \text{otherwise.} \end{cases} \tag{5}$$

In the case of $1 < a < \frac{2}{1+D_0}$ and $D_0 = D_1$, the discrete time system (5) exhibits chaos. In the experimental circuit, the integrator in the Fig. 1 is realized by using R, C and OP-amp. (TL074), and S/H is composed of a capacitor and a CMOS-IC for *sample and hold* circuits (LF389). The output voltage v of the integrator and v_n of S/H corresponds to x and x_n, respectively. Figure 5 shows the chaos attractor of the return map (5) and the laboratory measurement.

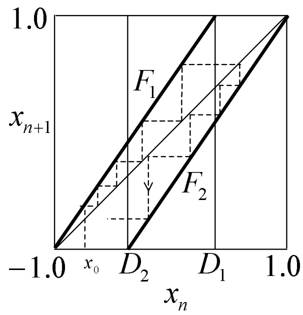

Figure 4: A hysteresis return map

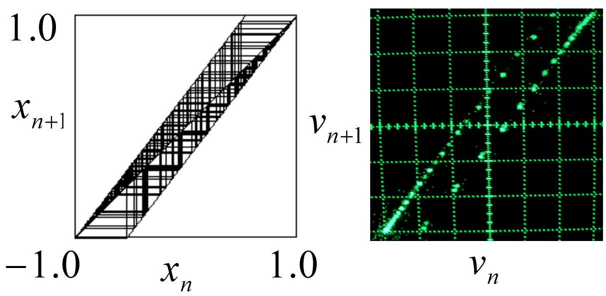

Figure 5: A hysteresis return map($a = 1.3$、$200mV$/div)

3. A NMPL with a dynamic controller

We introduce some basic definitions for the return map.

Definition: A point x_l is said to be a l-periodic point if $F^l(x_l) = x_l$ and $F^k(x_l) \neq x_l$ for $0 < k < l$, where F^l denotes the l-fold composition of F. A l-periodic point x_l is said to be stable (respectively, unstable) if $|DF^l(x_l)| < 1$ (respectively, $|DF^l(x_l)| > 1$), where DF^l denotes derivative of F^l. A sequence of a l-periodic point, $\{x_l F(x_l), \cdots, F^{l-1}(x_l)\}$, is said to be a periodic sequence with period l. We refer to a stable periodic sequence as a periodic attractor. Hereafter we abbreviate an unstable periodic point by UPP and abbreviate an unstable periodic sequence with l-period by l-p UPS.

Figure 6 shows a block diagram of a NMPL with dynamic controller and the timing charts for SW, S/H and S/H in this diagram. The switch SW is closed if the clock signal shown as the charts in Fig. 6. is high level. Also, S/H and $S/H2$ sample the input signal at the moment when the corresponding clock signal turns to high level and stores the sampled value until during the clock signal stays on low level. Letting the period of the clock signal for $S/H2$ be l, the switch SW is closed during only first 1 of l:

$$SW : \begin{cases} ON, & \text{for} \quad ln \le \tau < ln + 1, \\ OFF, & \text{for} \quad ln + 1 \le \tau < l(n+1). \end{cases} \tag{6}$$

The dynamic of the NMPL with dynamic controller is described by follows:

$$\dot{x} = \begin{cases} f((1-k)x_n + kz_{n-l}) \\ \qquad +A((1-k)x_n + kz_{n-l}) - x(n), \\ \qquad\qquad ln \le \tau < ln + 1, \\ f(x(n)) + Ax(n) - x(n), \\ \qquad\qquad ln + 1 \le \tau < l(n+1). \end{cases} \tag{7}$$

By using similar procedure in Sec. 2, we can derive the corresponding discreet-time system as the following:

$$x_{n+1} = \begin{cases} F((1-k)x_n + kz_{n-l}), n = kl, \\ F(x_n), \text{otherwise.} \end{cases} k = 1, 2, \cdots \tag{8}$$

Letting the z_{n-l} be represented by y_n, the return map from $(x(ln), y(ln))$ at $\tau = ln$ to $(x(l(n+1)), y(l(n+1)))$ at $\tau = l(n+1)$ can be given by

$$\begin{aligned} x_{n+l} &= F^l((1-K)x_n + K \cdot y_n) \\ y_{n+l} &= (1-K)x_n + K \cdot y_n. \end{aligned} \tag{9}$$

Note that if x_n is identical to y_n, the discreet-time system (9) is equivalent to (3).

Let ξ be an one UPP of l-p UPS of the system (3). letting $\hat{x}_n$ be defined by $x_n - \xi$, letting $\hat{y}_n$ be defined by $y_n - \xi$ and letting $A \equiv (\partial/\partial x_n)F^l|_{x_n=\xi}$, the linearized system of (9) in the neighbor of ξ is described by

$$\begin{bmatrix} \hat{x}(n+l) \\ \hat{y}(n+l) \end{bmatrix} = \begin{bmatrix} A(1-K) & AK \\ 1-K & K \end{bmatrix} \begin{bmatrix} \hat{x}(n) \\ \hat{y}(n) \end{bmatrix}. \tag{10}$$

Here, if we set the gain of a controller to

$$K = -\frac{A}{1-A},$$

then all of the characteristic root of the linearized system (10) are identical to zero, that is, the system is to be stable in the neighbor of ξ. The solution x_n started from the neighbor of ξ at $\tau = 0$ must converge to l-p UPP at $\tau = 2l$.

Figure 7 shows simulation results and laboratory measurements of generating 16 periodic solution by setting parameters as $A = 1.3$, $l = 16$ and $K = \frac{-1.3^{16}}{1-1.3^{16}}$. The solution is identical to the unstable periodic orbit of the original chaotic system (3). These attractors as shown in Fig. 7 are each co-existence depended on the initial conditions.

4. Conclusions

We proposed a novel nonlinear system which consists of a chaotic system and a dynamic controller. The proposed system exhibits some stabilized Unstable Periodic Orbits (abbr. UPO)which are embedded on chaos attractor of the original chaotic system. Now we try the generalization of the system and consider engineering applications.

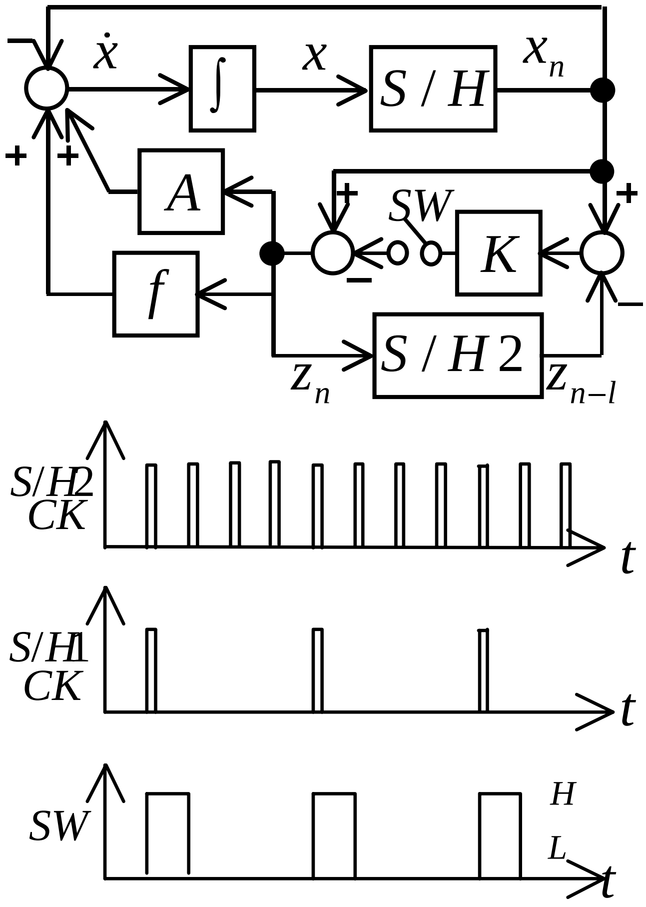

Figure 6: Nonautonomous PWC with a dynamic controller

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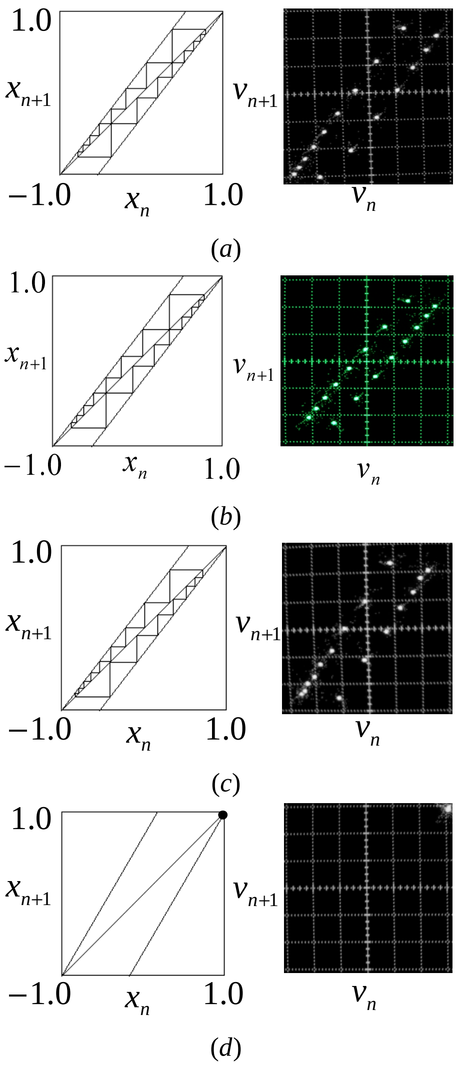

Figure 7: 16-periodic attractor($a = 1.3, l = 16, A = 1.3^{16}, K = \dfrac{-1.3^{16}}{1-1.3^{16}}, 200mV/\text{div}$)

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