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Transient stability and discontinuous solution in electric power system with dc transmission: A study with DAE system

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Abstract—This paper focuses on transient stability of an electric power system with dc transmission. When the transient stability is numerically evaluated based on a differential-algebraic equation (DAE) system, associated trajectories become discontinuous since constraint sets are different among corresponding pre-fault, fault-on, and post-fault DAE systems. In this paper, several discontinuous solutions are numerically and analytically discussed for the DAE system.

1. Introduction

This paper addresses transient stability of an electric power system with dc transmission. DC transmission has been widely recognized as a novel technology for future power supply networks [1, 2, 3, 4]. Transient stability of ac/dc power systems is of important concern with their ability to reach an acceptable operating condition following an event disturbance. The transient stability is mainly analyzed based on the two different approaches: time-domain (numerical) simulation [1, 2] and dynamical system theory [5, 6] involving energy function method [7, 1, 6]. By combing the two approaches, we can obtain sufficient and practical information about the transient stability.

The present paper investigates discontinuous solutions in a differential-algebraic equation (DAE) system using the numerical simulation. In [5, 6] we examined transient stability boundaries of the ac/dc power system based on the DAE system. Our previous studies focused on geometric and topological structures of the stability boundaries. Unfortunately, the relation has not been clarified between the stability boundaries and possible system trajectories relative to accidental faults. The understanding of the relation is inevitable in order to apply the obtained results in [5, 6] to practical situations and reveal the transient stability. Here, to clarify the relation, we consider particular discontinuous solutions caused by accidental faults in the ac/dc system; the solutions have a potential to provide with us a clue about the relation. This paper shows several discontinuous solutions of the DAE system and evaluates them via the singular perturbation technique.

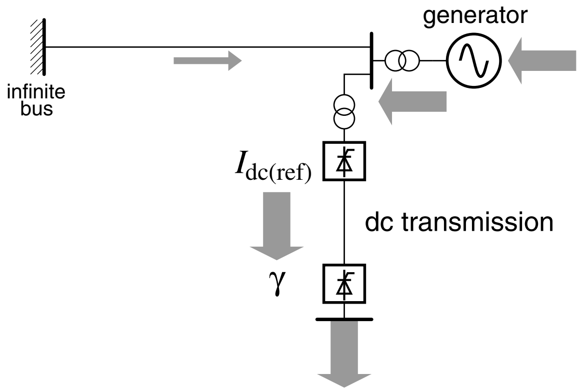

Figure 1 System model of electric power system with dc transmission

2. Differential-algebraic equation system

Figure 1 shows the system model of an electric power system with dc transmission [8]. In [6] we derive the following DAE system as a mathematical model for the transient stability analysis:

$$
\left\{
\begin{array}{rcl}
\dfrac{T'_{d0}}{L_d - L'_d}\dfrac{\mathrm{d}v'_q}{\mathrm{d}t} & = & -\dfrac{\partial \mathcal{U}_{\rm ac}}{\partial v'_q}(v'_q, \delta, \theta_{\rm r}, V_{\rm r}), \\
\dfrac{\mathrm{d}\delta}{\mathrm{d}t} & = & \Delta\omega, \\
2H\dfrac{\mathrm{d}\Delta\omega}{\mathrm{d}t} & = & -D\Delta\omega - \dfrac{\partial \mathcal{U}_{\rm ac}}{\partial \delta}(v'_q, \delta, \theta_{\rm r}, V_{\rm r}), \\
L_{\rm dc}\dfrac{\mathrm{d}I_{\rm dc}}{\mathrm{d}t} & = & -R_{\rm dc}I_{\rm dc} \\
 & & +K_V(\mathrm{e}^{V_{\rm r}}\cos\alpha(I_{\rm dc}) - V_{\rm i}\cos\gamma), \\
0 & = & -\dfrac{\partial \mathcal{U}_{\rm ac}}{\partial \theta_{\rm r}}(v'_q, \delta, \theta_{\rm r}, V_{\rm r}) \\
 & & -K_I\mathrm{e}^{V_{\rm r}}I_{\rm dc}\cos\varphi_{\rm r}, \\
0 & = & -\dfrac{\partial \mathcal{U}_{\rm ac}}{\partial V_{\rm r}}(v'_q, \delta, \theta_{\rm r}, V_{\rm r}) \\
 & & -K_I\mathrm{e}^{V_{\rm r}}I_{\rm dc}\sin\varphi_{\rm r}, \\
0 & = & K_I\mathrm{e}^{V_{\rm r}}I_{\rm dc}\cos\varphi_{\rm r} \\
 & & -\left(K_V\mathrm{e}^{V_{\rm r}}\cos\alpha(I_{\rm dc}) - \dfrac{3}{\pi}X_{\rm c}I_{\rm dc}\right)I_{\rm dc},
\end{array}
\right.
$$

where

$$\begin{cases} \alpha(I_{\rm dc}) & = & G_\alpha(I_{\rm dc(ref)} - I_{\rm dc}), \\ \mathcal{U}_{\rm ac}(v'_q, \delta, \theta_{\rm r}, V_{\rm r}) & = & -p_{\rm m}\delta \\ -\dfrac{(L'_d - L_q)\cos 2(\delta-\theta_{\rm r}) - (L'_d + L_q)}{2L'_d L_q}\dfrac{{\rm e}^{2V_{\rm r}}}{2} \\ -\dfrac{v'_q {\rm e}^{V_{\rm r}}}{L'_d}\cos(\delta-\theta_{\rm r}) - \dfrac{{\rm e}^{V_{\rm r}}V_\infty}{L_\infty}\cos\theta_{\rm r} + \dfrac{{\rm e}^{2V_{\rm r}}}{2L_\infty} \\ -\dfrac{V_0}{L_d - L'_d}v'_q + \dfrac{L_d}{L'_d(L_d-L'_d)}\dfrac{v'^2_q}{2}, \end{cases}$$

or

$$\mathsf{M}\frac{{\rm d}\boldsymbol{x}}{{\rm d}t} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}), \qquad \boldsymbol{0} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}). \qquad (1)$$

$\boldsymbol{x} \triangleq (v'_q, \delta, \Delta\omega, I_{\rm dc})^{\rm T} \in X$ and $\boldsymbol{y} \triangleq (\theta_{\rm r}, V_{\rm r}, \varphi_{\rm r})^{\rm T} \in Y$, and M denotes the positive-definite matrix: $\mathsf{M} \triangleq \mathsf{diag}(T'_{d0}/(L_d - L'_d), 1, 2H, L_{\rm dc})$. In (1) variables and parameters are normalized with the well known per unit system. Their physical meaning is given in [6].

3. Singularly perturbed and boundary-layer systems

Let us consider the associated singularly perturbed (SP) system as follows:

$$\mathsf{M}\frac{{\rm d}\boldsymbol{x}}{{\rm d}t} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{y}), \qquad \varepsilon\frac{{\rm d}\boldsymbol{y}}{{\rm d}t} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}), \qquad (2)$$

where ε is a small positive parameter. Let $L \triangleq \{(\boldsymbol{x},\boldsymbol{y}) \in X \times Y;\ \boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{0}\}$ be the constraint set of the DAE system. The dynamics of the SP system (2) have the similarity to those of the DAE system (1) in a stable component $\mathit{\Gamma}_{\rm s} \subset L$: $\mathit{\Gamma}_{\rm s}$ is a connected set such that for any point in $\mathit{\Gamma}_{\rm s}$ the Jacobian ${\rm D}_{\boldsymbol{y}}\boldsymbol{g}$ has no eigenvalue with positive and zero real parts. In addition, applying the variable transformation $s \triangleq \varepsilon/t$ to the SP system (2) and setting $\varepsilon = 0$ freeze the variables at $\boldsymbol{x} = \boldsymbol{x}^*$, and derive the following boundary-layer (BL) system:

$$\frac{{\rm d}\boldsymbol{y}}{{\rm d}s} = \boldsymbol{g}(\boldsymbol{x}^*,\boldsymbol{y}). \qquad (3)$$

The set of equilibrium points (EPs) in the BL system corresponds to L, and the set of asymptotically stable EPs also the union of stable components. This fact is directly used to justify the discontinuous solutions theoretically as discussed below.

4. Transient stability and discontinuous solutions

4.1. Faults setting

Two fault cases that we now adopt are as follows: one is a three-phase fault in the ac transmission line, and the generator operates as a result onto the dc link only during the sustained fault. The other is a three-phase fault at the infinite bus, and thereby the infinite bus voltage is fixed at zero in the fault duration. Suppose that the system is at a known asymptotically stable EP at $t = 0^-$, the fault duration is confined to the time $[0^+, t_{\rm cl}^-]$, and the fault is cleared at a time $t = t_{\rm cl}$. It is assumed for simplicity that the post-fault DAE system is identical to the pre-fault DAE one. The above two cases are now mathematically formulated in the DAE system (1) for $t \in [0^+, t_{\rm cl}^-]$: $1/L_\infty = 0$ (case-1) and $V_\infty = 0$ (case-2), respectively.

Two constraint sets are apparently different between the pre-fault (or post-fault) and fault-on DAE systems. The difference generates discontinuous solutions of the DAE system (1) at $t = 0$ and $t_{\rm cl}$; they are called *external jumps* [9, 10]. Jump behaviors or discontinuous solutions have been discussed for constrained dynamical systems in [11, 12, 9]. The previous works [11, 12] define general discontinuous solutions based on associated BL systems. In particular, when the BL systems are gradient [13], the discontinuous solutions are simply characterized based on the orbit structures of the BL systems. In the following, we use some previous results in [9, 10] to validate numerical discontinuous solutions (external jumps) in the DAE system (1).

The discontinuous solutions have bean reported for power system transient analysis using structure-preserving models [9, 10]. Practical power systems do not hold such discontinuous states, and they therefore originate from modeling over-abstraction. However, since abstraction is inevitable for analyzing massively complex power networks, understanding how the solutions affect the transient stability is essential, e.g., for the development of controlling UEP method for the power system analysis [10].

4.2. Numerical simulations

Numerical simulations are performed for the DAE system (1). The parameters setting are identical to Tab. 1. They are obtained for the practical system [8]. We here adopt the 3rd-stage Radau-IIA implicit Runge-Kutta method [14] to integrate the DAE system numerically. Since the numerical scheme is one-step type, we possibly obtain numerical discontinuous solutions although they do not generally hold uniqueness properties.

Figure 2 shows the transient behavior of $\Delta\omega$, $V_{\rm r}$, and active power with setting the case-1 fault at $t_{\rm cl}/(120\pi\ {\rm s}^{-1}) = 140$ ms. In the figures, □ denotes

Table 1 Parameters setting

L_d	1.79	L_q	1.77	L'_d	0.34
$T'_{d0}/(120\pi\ {\rm s}^{-1})$	6.3 s	V_0	1.7	$p_{\rm m}$	0.5
$H/(120\pi\ {\rm s}^{-1})$	0.89 s	D	0.05	L_∞	0.883
V_∞	1.0	$L_{\rm dc}$	4.2	$R_{\rm dc}$	0.014
$V_{\rm i}$	1.0	K_V	1.19	K_I	1.19
$X_{\rm c}$	0.12	G_α	30.0	$I_{\rm dc(ref)}$	1.0

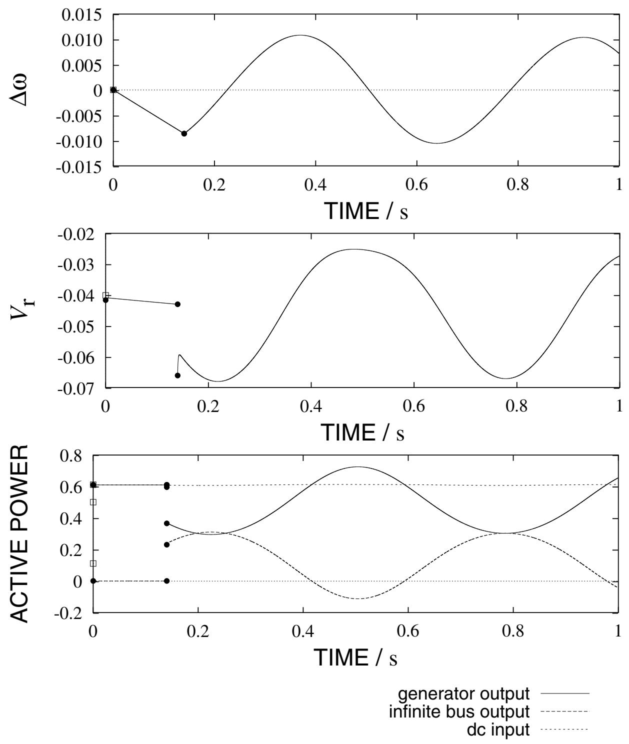

Figure 2 Discontinuous solution of the DAE system with case-1 fault setting. The fault-clearing time $t_{cl}/(120\pi\ s^{-1})$ is fixed at 140 ms.

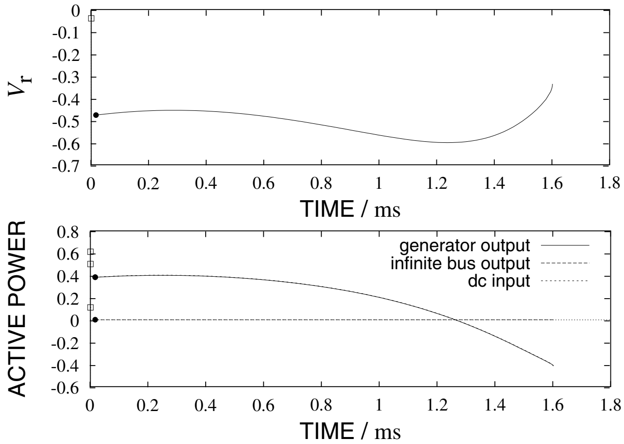

Figure 3 Discontinuous solution of the DAE system with case-2 fault setting

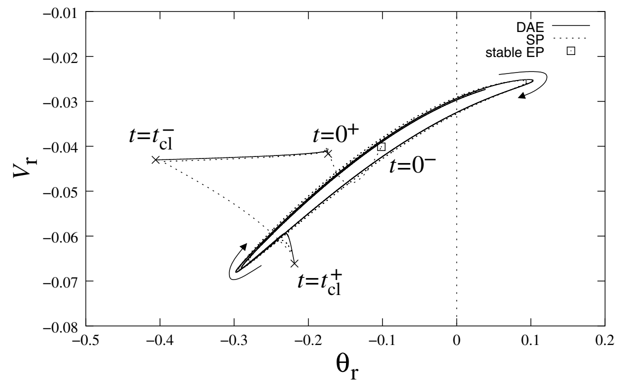

Figure 4 Projected trajectories of the DAE and SP systems with case-1 fault setting onto $\theta_r - V_r$ plane. The perturbation parameter ε is set at 0.5. The *solid* line denotes the trajectory of the DAE system (1) and the *dotted* line that of the SP system (2).

the initial condition, which coincides with a pre-fault steady state, and • the state at each $t = 0^+$, t_{cl}^-, or t_{cl}^+. The solution in the figure converges to a post-fault asymptotically stable EP. In Fig. 2, $\Delta\omega$ in $\boldsymbol{x}$ is continuous, while V_r in $\boldsymbol{y}$ is discontinuous at $t/(120\pi\ s^{-1}) = 0$ s and 140 ms. This property is discussed in the next subsection. Fig. 2 also shows the active power swing in the system. During the fault duration, the active power output of the infinite bus is zero, and the generator output is therefore identical to the dc power input. Since the generator output during the fault duration is greater than the mechanical power input $p_m = 0.5$, the generator is de-accelerated as shown in Fig. 2. After the fault is cleared at $t = t_{cl}$, both generator and infinite bus output show oscillatory motions and converge to the values at the steady state.

The case-2 fault solution is shown in Fig. 3. The solution here converges to a singular surface S of the fault-on DAE system: $S \triangleq \{(\boldsymbol{x}, \boldsymbol{y}) \in L;\ \det(D_y\boldsymbol{g})(\boldsymbol{x}, \boldsymbol{y}) = 0\}$. The solution which reaches S is also possibly discontinuous; the discontinuity here is qualitatively different from external jumps [9, 10]. This result implies an application limit of the DAE system (1) for the transient stability analysis. It is stated in [15] that the transient stability is determined by the dynamics of the *post-fault* DAE system. The transient behavior in Fig. 3 therefore suggests that the present DAE system (1) is not relevant to clarifying the transient stability problem relative to the case-1 fault. To overcome the limitation, we need to model the transient dynamics in detail with taking some equipments of the ac/dc system: AVRs, shunt capacitors at converter stations, and so on.

4.3. Discontinuous solution and boundary layer systems

Next we discuss the discontinuous solution of the case-1 fault setting through the associated SP system (2). As discussed before, the SP system plays a key role in validating numerical simulations for the DAE system (1). Fig. 4 shows the projected trajectories of the DAE and associated SP systems onto $\theta_r - V_r$ plane. The perturbation parameter ε is set at 0.5. The *solid* line denotes the trajectory of the DAE system (1) and the *dotted* line that of the SP system (2). The figure implies that the trajectory of the SP system (2) precisely traces the discontinuous solution of the DAE system (1).

The discontinuous solution of the case-1 fault is now analytically justified based on the BL system. Each

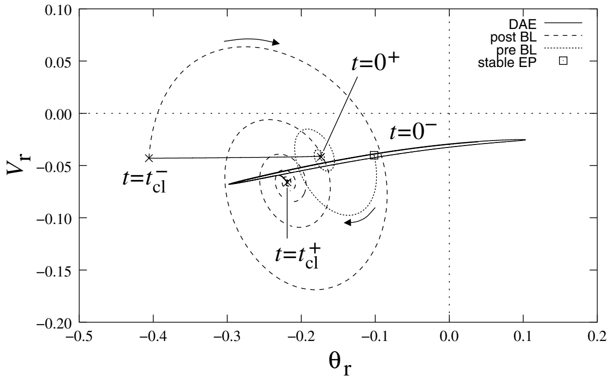

Figure 5 Projected discontinuous solution and trajectories of the pre- and post-BL systems with case-1 fault setting onto $\theta_r - V_r$ plane. The *solid* line denotes the trajectory of the DAE system (1) and the two *dotted* lines those of the pre- and post-fault BL systems.

point in a stable component $\mathit{\Gamma}_s$ becomes an asymptotically stable EP of the following post-fault BL system at $t = t_{cl}$:

$$\begin{cases} \dfrac{d\theta_r}{ds} &= -\dfrac{\partial \mathcal{U}_{ac}}{\partial \theta_r}(v_q'^*, \delta^*, \theta_r, V_r) \\ & \quad -\left(K_V e^{V_r}\cos\alpha^* - \dfrac{3}{\pi}X_c I_{dc}^*\right) I_{dc}^*, \\ \dfrac{dV_r}{ds} &= -\dfrac{\partial \mathcal{U}_{ac}}{\partial V_r}(v_q'^*, \delta^*, \theta_r, V_r) \\ -\sqrt{K_I^2 e^{2V_r} I_{dc}^{*2} - \left(K_V e^{V_r}\cos\alpha^* - \dfrac{3}{\pi}X_c I_{dc}^*\right)^2 I_{dc}^{*2}}, \end{cases} \tag{4}$$

where $\boldsymbol{x}(t_{cl}^+) \triangleq (v_q'^*, \delta^*, \Delta\omega^*, I_{dc}^*)^T$, and we assume that $I_{dc}^* > 0$ and $K_I e^{V_r} I_{dc}^* \sin\varphi_r > 0$; this is relevant to the transient stability analysis. From [12, 9] we here state that if the DAE system (1) admits of the discontinuous solution at $t = t_{cl}$, then the trajectory of the BL system (4) with the initial condition $\boldsymbol{y}(t_{cl}^-)$ converges to the point $\boldsymbol{y}(t_{cl}^+)$ in the stable component $\mathit{\Gamma}_s$ satisfying $\boldsymbol{x}(t_{cl}^-) = \boldsymbol{x}(t_{cl}^+)$; the coincident property holds in Fig. 2. This follows that the point $\boldsymbol{y}(t_{cl}^-)$ is on a stable manifold of the stable EP $\boldsymbol{y}(t_{cl}^+)$ in the BL system (4). Fig. 5 definitely describes the trajectories of the pre- and post-BL systems and the projected discontinuous solution of the DAE system. In the figure, all trajectories of the BL systems converge to asymptotically stable EPs of the BL systems, and these EPs coincide with the starting points of the discontinuous solution at $t = 0^+$ and t_{cl}^+. This implies that our numerical discontinuous solution is valid in the analytical sense.

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