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A Noise Reduction Method in Oscillators and PLL's, Analyzing Correlation between Low Frequency Phase and Amplitude Fluctuations in Distributed Network Model

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Abstract - The low frequency phase and average current fluctuations, and the correlation coefficient between them are studied in distributed network model and in a transistor. We find both fluctuations correlate each other in the low frequency. The phase stabilization technique is discussed and new methods to reduce the noise are proposed in transistor oscillator and PLL's, using the correlation.

1. Introduction

The phase noise and amplitude noise(or collector average current fluctuation) correlate each other in bipolar transistors[1]. This means both noises are from the same origin; diffusion coefficient fluctuation in the base of a transistor[1]. This is applicable to reduce the noises in a semiconductor circuits[2] and the frequency fluctuation in an oscillator[3]. In this paper, we will discuss the correlation coefficient between the phase noise and amplitude or average current fluctuation in distributed network model and in a transistor, and propose a method to reduce the noise, using the correlation. The phase measurements were carried out at 10MHz.

2. Evaluation of Correlation Coefficient between Low Frequency Phase and Amplitude Fluctuations in Distributed Network and for the Carrier Transmission in the Base of a Transistor

The partial correlation between amplitude and phase fluctuation has been experimentally reported in radio wave propagation[4] and the strong correlation has been observed in minority carrier transmission in the base of a transistor[1]. These correlations could be discussed in transmission line model.

The differential equations for a one-dimensional distributed line are

$$\frac{\partial V}{\partial x} = -RI - L\frac{\partial I}{\partial t} \quad (1)$$

$$\frac{\partial I}{\partial x} = -GV - C\frac{\partial V}{\partial t} \quad (2)$$

where R, G, L and C are the resistance, conductance, inductance and capacitance per unit length, respectively.

In a bipolar transistor, one of the most possible low frequency noise generating mechanism will be the diffusion coefficient fluctuation in low frequency[1]. If this fluctuates in the base, the attenuation and phase constants will fluctuate simultaneously.

The diffusion and recombination of the minority carriers can be represented by the distributed transmission line as follows. We will consider the fluctuation of the diffusion coefficient in the diffusion region of a N^+-P-N transistor. In the p-type base layer, the minority carriers are electrons. Introducing the excess electron density $n'(x)=n(x)-n_p$, the fundamental equations are

$$\frac{\partial n'(x)}{\partial x} = \frac{1}{eD_n} i_n(x) \quad (3)$$

$$\frac{\partial i_n(x)}{\partial x} = \frac{e}{\tau_n} n'(x) + e\frac{\partial n'(x)}{\partial t} \quad (4)$$

where D_n is the electron diffusion coefficient, τ_n the electron lifetime, e the electronic charge and $i_n(x)$ the electron current density; n_p is the equilibrium electron density and n(x) is the actual electron density.

Comparing (3) and (4) with the pair of the differential equations (1) and (2) of the distributed line with negligible distributed inductance, we obtain the following correspondences: [5]

V $\longleftrightarrow$ n'(x)
I $\longleftrightarrow$ $i_n(x)$
R $\longleftrightarrow$ $-(1/eD_n)$
G $\longleftrightarrow$ $-e/\tau_n$

Thus the diffusion coefficient fluctuation of electrons corresponds to the resistance fluctuation in the R-C-G line. From the corresponding relationship of R and D_n, we have

$$\Delta R \leftrightarrow \frac{1}{eD_n}\frac{\Delta D_n}{D_n} \qquad (5)$$

In the R-C-G line, the high frequency propagation constant fluctuation for resistance fluctuation will be expressed as follows, ignoring the conductance term:

$$\gamma + \Delta\gamma \approx \sqrt{j\omega CR(1+\frac{\Delta R}{R})}$$

$$\approx \sqrt{\omega CR}(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}})(1+\frac{\Delta R}{2R}) \qquad (6)$$

In low frequency, the propagation constant fluctuation for resistance will be expressed in (7).

$$\gamma + \Delta\gamma \approx \sqrt{GR(1+\frac{\Delta R}{R})} \approx \sqrt{GR}(1+\frac{\Delta R}{2R}) \qquad (7)$$

It is noted from (6) and (7) that the low frequency attenuation and high frequency phase constants fluctuate simultaneously. Hence, the low frequency amplitude and high frequency phase fluctuations will correlate each other. For short R-C-G line or base of a transistor, the correlation coefficient is calculated as follows.

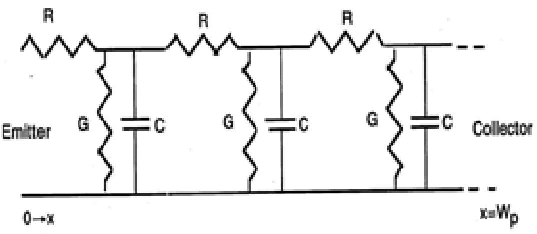

Fig.1 Distributed constant circuit.

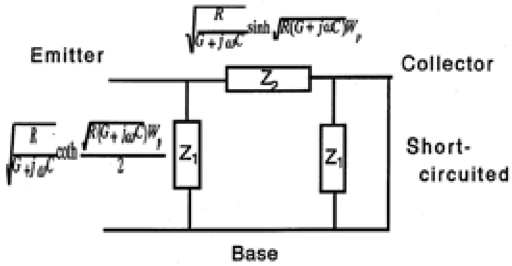

Fig.2. Lumped constant equivalent circuit.

The R-C-G line is described in Fig. 1. The lumped constant equivalent circuit of Fig.1 is shown in Fig.2 and we obtain the transconductance g_m as follows for moderately high frequency($G \ll \omega C$). Here the distribution of R is uniform.

$$g_m = \frac{i_c}{v_e} = \frac{1}{z_2} = \frac{1}{\sqrt{\frac{R}{j\omega C}}\sinh\sqrt{j\omega CR}W_p}$$

$$\approx \frac{1}{RW_p + j\frac{\omega CR^2W_p^3}{6}} \qquad (8)$$

Assuming that R $\rightarrow$R+ΔR at[x_1, $x_1+\Delta x_1$][2], the phase angle of the transconductance is obtained as in (9).

$$\Theta(g_m) + \Delta\Theta(g_m)$$

$$= -\tan^{-1}[\frac{\omega CW_p}{6}(RW_p + \Delta R\Delta x_1)] \qquad (9)$$

Hence, the phase fluctuation is obtained as follows, assuming $\omega CRWp^2 \ll 1$,

$$\frac{\Delta\Theta}{\Theta} \approx \frac{\Delta R\Delta x_1}{RW_p} \qquad (10)$$

On the other hand, current fluctuation due to the resistance fluctuation is described as follows[2].

$$\frac{\Delta I}{I} = -\frac{\Delta R\Delta x_1}{RW_p} \qquad (11)$$

In the entire region of the base layer, the phase noise, current noise and correlation noise between them are obtained by integration through the whole base region respectively. The results are shown as follows.

$$\overline{\theta^2} = \int_0^{W_p} \overline{\Delta\Theta^2} = \int_0^{W_p} \Theta^2 \left(\frac{\Delta R}{R}\right)^2 \left(\frac{\Delta x_1}{W_p}\right)^2 \quad (12)$$

$$\overline{i^2} = \int_0^{W_p} \overline{\Delta I^2} = \int_0^{W_p} I^2 \left(\frac{\Delta R}{R}\right)^2 \left(\frac{\Delta x_1}{W_p}\right)^2 \quad (13)$$

$$\overline{i \times \theta} = \int_0^{W_p} \overline{\Delta i \Delta\Theta} = \int_0^{W_p} I\Theta \left(\frac{\Delta R}{R}\right)^2 \left(\frac{\Delta x_1}{W_p}\right)^2 \quad (14)$$

The magnitude of the correlation coefficient is

$$C = \frac{\left|\overline{i \times \theta}\right|}{\sqrt{\overline{i^2} \times \overline{\theta^2}}} = 1.00 \quad (15)$$

From the above result, the strong correlation is noted between the phase and current fluctuations for the short R-C line and consequently for the carrier transmission in the base of a transistor.

3. Noise Measurements and Noise Reduction

Figure 3 shows a typical measured correlation coefficient between the amplitude and phase noises in a transistor. It is noted that both noises are correlated in low frequency 1/f regime. In another transistor with impulsive or burst noise, we could not find the correlation. Hence, it is presumed that the impulse noise is not from the diffusion process. This correlation will be applicable to reduce the noise by subtracting the correlated component in the noise.

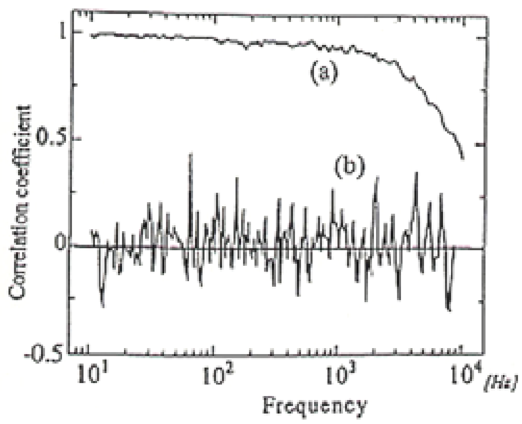

Fig. 3 The measured correlation coefficient between the phase and amplitude noises in collector current. (a) in 1/f noise. (b) in impulse noise.

Figure 4 shows the measured amplitude and phase noise spectra in a bipolar transistor. The experimental result on the amplitude noise reduction is also shown in Fig. 4. We had about 10 dB noise reduction in this experiment. We can also reduce the phase noise[2].

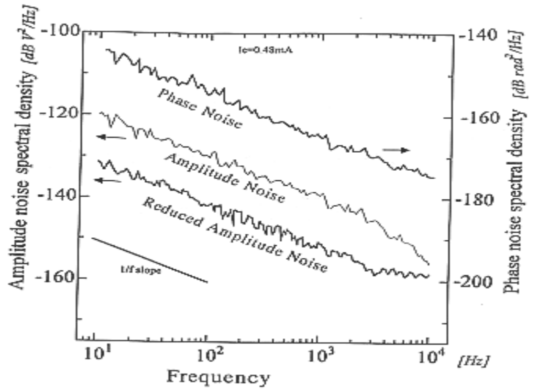

Fig. 4 The measured noise spectra in a bipolar transistor. Also is shown the reduced amplitude noise by subtracting the correlated component in the amplitude noise.

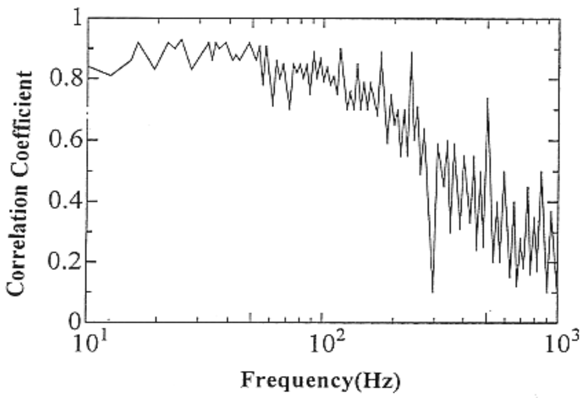

Fig. 5 Measured correlation coefficient between amplitude and frequency fluctuations in Colpitts oscillator.

We prepared a Colpitts type oscillator, for characterizing the noises. Figure 5 shows the measured result of the correlation coefficient between frequency and amplitude(or average collector current) fluctuations[2]. We have also reduced the noise, using a variable capacitor as a phase shifter controlled with the correlated average current noise[3]. We can stabilize an oscillator.

Finally, in order to subtract the phase noise component correlated with the amplitude noise in VCO in a phase locked loop system, we employed a differential amplifier in the output terminal of the PLL[6]. We can subtract the correlated component in the phase noise with adjusting the attenuator in the system and obtained the result for Colpitts VCO as shown in Fig. 6. We find the noise reduction; about 7 dB in maximum reduction condition in our experiment. This method will be a possible method to reduce the output noise in PLL.

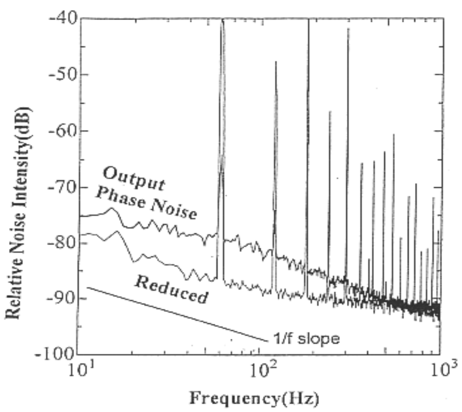

Fig. 6 A result of phase noise reduction in PLL system with Colpitts VCO, using a differential amplifier.

4. Conclusion

In this paper, we propose a method to reduce the noise. The fundamental technique is based on the subtraction of the correlated noise component. This method will be a possible method to reduce the noise in transistor circuit, oscillator, PLL[6] and other transmission systems. Further study should be carried out, including Kramers-Kronig relations[7].

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