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Basins of Attraction of Phase Patterns in Pulse-Driven Star-Coupled Wien-Bridge Oscillators with Parameter Deviations

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Abstract—In this study, we investigate the basins of attraction of the phase patterns and clarify the effect of parameter deviations in pulse-driven star-coupled Wien-bridge oscillators with parameter deviations. From the simulation results, it is shown that some phase pattern can be seen easily and the others can be hardly seen because of the deviations. Such phenomena can separate the preferred patterns from undesirable patterns and it is convenient for the use of the system as some kinds of neural networks.

1. Introduction

There have been many investigations of mutual synchronization and multimode oscillation in coupled oscillators [1]–[4]. In particular, we have reported synchronization phenomena observed from N oscillators with the same natural frequency mutually coupled by one resistor [3, 4]. In LC oscillators systems, we have con rmed that N-phase oscillation can be stably excited when each oscillator has strong nonlinearity [3]. In this case, there exist $(N-1)!$ stable phase states according to the initial states. Moreover, we have investigated the coupled system with RC Wien-bridge oscillators. This system is suitable for VLSI implementation because the system does not include any inductors. They also exhibit the "phase-shift synchronization" and we can get 3^{N-1} different stable phase patterns [4]. Because these "star-coupled" oscillators exhibit a large number of different steady states, they would be used as a structural element of large scale memories and neural networks.

When we use the coupled oscillators systems as neural networks and large scale memories, it should be an important problem how to control the systems to get the appropriate phase patterns. To achieve the phase pattern control, we have proposed the star-coupled system of Wien-bridge oscillators driven by the periodic pulse train and con rmed that the stimulation of the pulse train can cause the phase pattern switching [5]. In this system, however, only the phase of the oscillator where the pulse train is directly added switches. Moreover, we have proposed two types of star-coupled Wien-bridge oscillators whose driving methods with pulse train are different [6]. In these systems, though multiple oscillators' phases can be switched by pulse train, there are some disadvantages in each system. To avoid these problems, we have proposed the star-coupled systems with some parameter deviations [7]. In these systems, the phase pattern switching of the successive multiple oscillators can be achieved due to the deviations. In such systems, it is considered that the symmetry of the system is collapsed by the parameter deviation. In this study, we calculate the basins of attraction of the phase patterns in the star-coupled Wien-bridge oscillators system stimulated by pulse using SPICE and clarify the effect of parameter deviations. To derive such basin structures can be the preparation for controlling the phase patterns in coupled oscillators systems by using these parameters as the control parameters.

2. Circuit Models

The circuit models are shown in Fig. 1. In this study, we propose the following two models.

Model 1 The switch unit is connected to *Osc* 4.

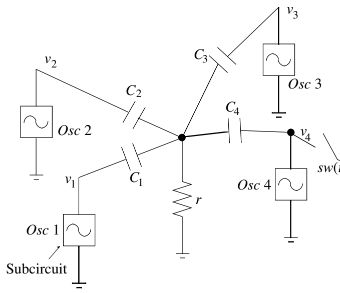
Model 2 The switch unit is connected to the coupling resistor r.

In each model, the switch unit stimulates the star-coupled Wien-bridge oscillators. In this case, the switch closes Δt seconds in every T seconds, and the periodic pulse stimulation with period T is added to the system. T should be sufficiently large to achieve the synchronization within the period. The construction of the subcircuits is shown in Fig. 1 (c).

In both systems, the parameter deviations are provided by the different capacitance in each $C_1 \sim C_4$. The capacitance C_k is described as follows,

$$C_k = C + (k-1)\Delta C \qquad (1)$$

where C is the capacitance of the capacitor in subcircuit and ΔC is the deviation parameter. If ΔC is larger, the difference of the natural frequency of each oscillator becomes larger.



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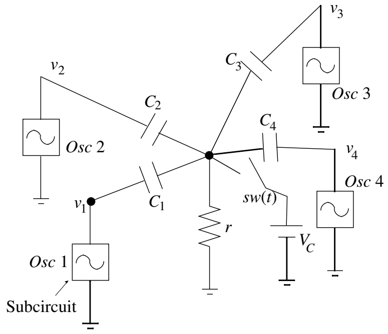

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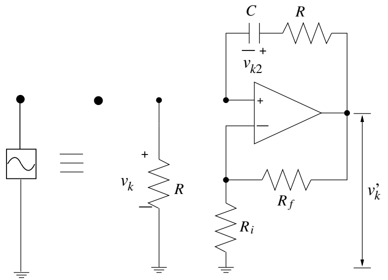

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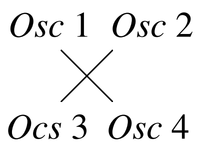
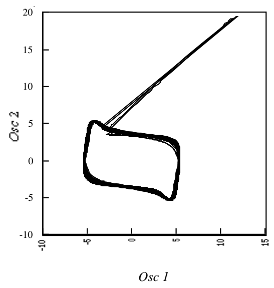


(d)

Figure 1: Circuit models. (a) Switch unit is connected to an oscillator (Model 1). (b) Switch unit is connected to the coupling resistor (Model 2). (c) Construction of subcircuit. (d) Schematic of the system.



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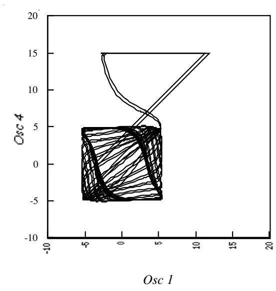

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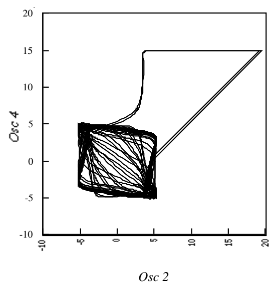

(c)

Figure 2: Lissajours' gures for $\Delta C = 10^{-3}\mu$F for Model 1.

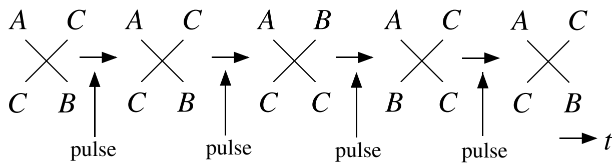

Figure 3: An example of the phase pattern transition when $\Delta C = 10^{-3}\mu$F for Model 1.

3. Simulation Results

In this section, we show the simulation results in proposed models by standard circuit simulator package SPICE. In this study, we use the following circuit parameters: R = 10kΩ, C = 0.015μF, r = 200Ω, R_f = 14.7kΩ, R_i = 4.7kΩ, Δt = 50μsec, T = 100msec. In the following results, A, B and C indicate in-phase, $+120°$ and $-120°$ phase shift with respect to the phase of *Osc* 1, respectively.

Figures 2 and 3 show the results for Model 1 and Figures 4 and 5 show the results for Model 2 when ΔC = $10^{-3}\mu$F. In these models, we can see successive phase pattern switching of the multiple oscillators due to the parameter deviations. From the results, it is shown that the in-phase synchronization of *Osc* 1 and *Osc* 4 is hardly seen. It is considered that this is because the natural frequencies of these oscillators are more different than the other combinations of the oscillators. Such parameter deviations make the system asymmetric and they affect the system dynamics and the derived phase patterns.

Next, we show the precise results of phase pattern

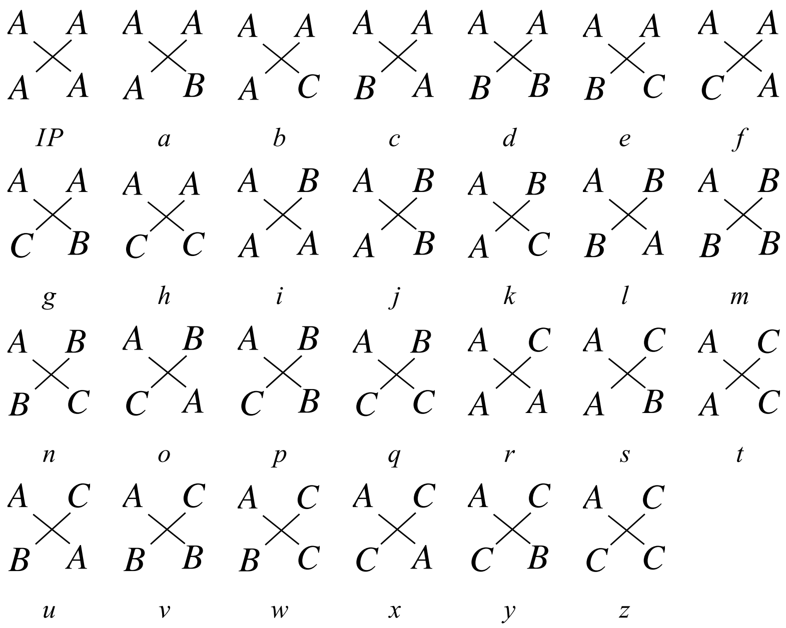

Figure 6: The notation of the phase patterns.

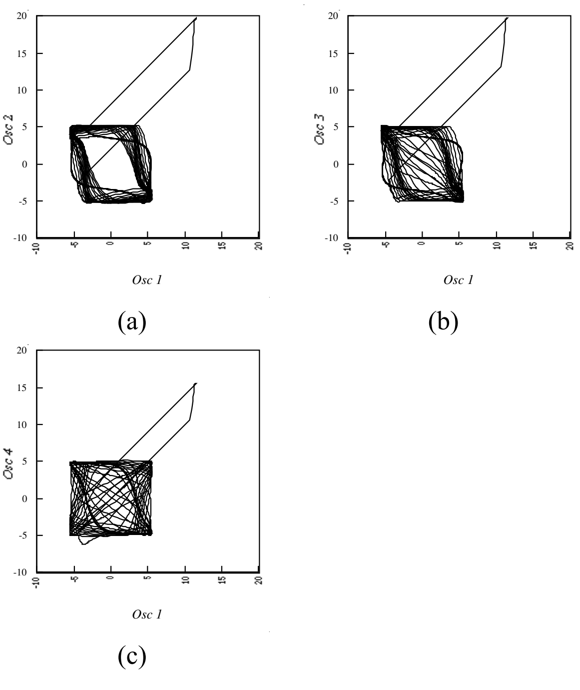

Figure 4: Lissajours’ gures for $\Delta C = 10^{-3}\mu$F for Model 2.

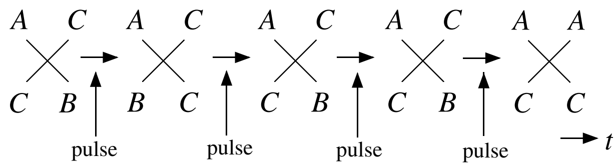

Figure 5: An example of the phase pattern transition when $\Delta C = 10^{-3}\mu$F for Model 2.

switching. In the following results, the possible $27 = 3^{4-1}$ phase patterns are indicated as the notation from *a* to *z* and *IP* as shown in Fig. 6. In this case, $\Delta C = 10^{-3}\mu$F and the single pulse (i.e., not periodic) is added to the system. Tables 1 and 2 show the phase patterns when the timing of the pulse and pulse voltage are changed. In these cases, the phase pattern before adding the pulse is *w*. In both models, not so many patterns appear after the pulses are added. As stated in Table 1, for Model 1, only three patterns *v*, *w* and *y* are seen. In particular, pattern *y* is more frequently seen after the switching from *w* than the other patterns. In this case, note that only the phases of *Osc* 3 and *Osc* 4 change after stimulation.

On the other hand, in Model 2, the phase pattern diagram is different from one of Model 1. In particular, there are some cases where the phase of *Osc* 2 is changed to *A* or *C*. It suggest that the pulses affect the system more globally in Model 2. Note that the pattern *y* is much more frequently seen than the other patterns. From these results, the pattern *y* is more stable than the other patterns because of the parameter deviations. Therefore, it can be considered that the

Table 1: 2 parameter phase pattern diagram for **Model 1**.

	Phase									
V_c	0	$\frac{1}{5}\pi$	$\frac{2}{5}\pi$	$\frac{3}{5}\pi$	$\frac{4}{5}\pi$	π	$\frac{6}{5}\pi$	$\frac{7}{5}\pi$	$\frac{8}{5}\pi$	$\frac{9}{5}\pi$
15	y	y	v	v	w	w	w	y	y	y
14	y	y	v	v	w	w	w	y	y	y
13	y	y	y	v	w	w	w	y	y	y
12	y	y	y	v	w	w	w	y	y	y
11	y	y	y	v	v	w	w	y	y	y
10	y	y	y	y	v	w	w	y	y	y

Table 2: 2 parameter phase pattern diagram for **Model 2**.

	Phase									
V_c	0	$\frac{1}{5}\pi$	$\frac{2}{5}\pi$	$\frac{3}{5}\pi$	$\frac{4}{5}\pi$	π	$\frac{6}{5}\pi$	$\frac{7}{5}\pi$	$\frac{8}{5}\pi$	$\frac{9}{5}\pi$
15	y	y	y	y	y	y	y	y	h	q
14	y	y	y	y	y	y	y	y	w	w
13	y	y	y	y	w	y	y	y	y	w
12	y	y	y	y	h	y	y	y	y	w
11	y	y	y	y	h	y	y	y	y	w
10	y	y	y	w	h	y	y	y	w	y

deviation of the parameter can be the control parameter of the phase patterns.

4. Conclusions

In this paper, we show the frequency of appearance of the phase patterns in pulse-driven star-coupled Wien-bridge oscillators with the parameter deviations. The symmetry of the system is collapsed by such parameter deviations, and they affect the system dynamics and the phase patterns. From the results, it is shown that some phase pattern can be seen easily and that some phase patterns can be hardly seen because of the deviations. Such phenomena can separate the preferred patterns from undesirable patterns and it is convenient for the use of the system as some kinds of neural networks and associative memories.

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