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Deterministic Partial Discharge Model with Dissipation

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Abstract—The most simple model of partial discharge phenomena is called the three-capacitance equivalent circuit model. In the former studies, it has been shown by the authors that the behavior of the three-capacitance equivalent circuit model is complex, even though it is very simple and completely deterministic. However, it is too simple as a model of real partial discharge phenomena. In this paper, we investigate the behavior of the three-capacitance model with dissipation. We show that the behavior of the discharge rate of the model with dissipation can also be complex, resembling a devil's staircase.

1. Introduction

Experimental data observed from partial discharge phenomena is very complicated and seems stochastic. However, it has been reported that deterministic properties play important roles in partial discharge phenomena [1–6].

Accordingly, it is possible that stochastic behavior of partial discharge phenomena is deterministically produced from underlying simple physical process. In fact, we have shown [7] that such stochastic data can be generated from the most simple and completely deterministic partial discharge model, which is called the three-capacitance equivalent circuit model.

The three-capacitance equivalent circuit model was proposed more than fifty years ago [8–11]. Since it is very simple, we could analyze the behavior of the model in purely mathematical way, and we showed [7] that the model can be reduced to the map termed double rotation [12, 13], which forms a subclass of interval transformation mappings [14]. However, it is too simple as a model of partial discharge phenomena.

In real partial discharge phenomena, effects of dissipation and stochasticity cannot be ignored. Since we are interested in underlying deterministic structure of the model, it is important to know to what extent complicated characteristics of partial discharge phenomena can be produced without any stochastic effects.

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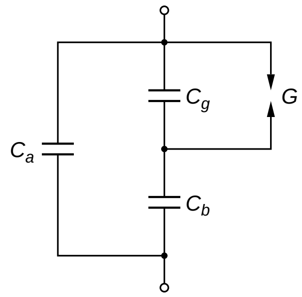

Figure 1: The three-capacitance equivalent circuit. The discharge gap G and the capacitor C_g represent the void, and the capacitors C_a and C_b represent the other part of the insulation.

Therefore, to elucidate deterministic properties of partial discharge model, in this paper, we investigate the three-capacitance equivalent circuit model with dissipation at the discharge site.

2. The Three-capacitance Model

As shown in Fig. 1, the three-capacitance equivalent circuit consists of three capacitors and a discharge gap. Capacitor C_g represents the capacitance of the void where partial discharges occur, and capacitors C_a and C_b represent the capacitance of the insulation in parallel and in series, respectively, with the void. Discharge gap G is the element such that when the voltage between the gap reaches *inception voltage* V_i^+, a discharge occurs, and the voltage is reset to *residual voltage* V_r^+ by the compensation caused by the discharge. Discharges also occur in the opposite direction: when the voltage between the gap reaches V_i^-, it is reset to V_r^- by a discharge in the opposite direction. A discharge is positive if its inception voltage is V_i^+, and is negative if its inception voltage is V_i^-.

Time evolution of the three-capacitance model is shown in Fig. 2, where a sinusoidal AC voltage is applied to the

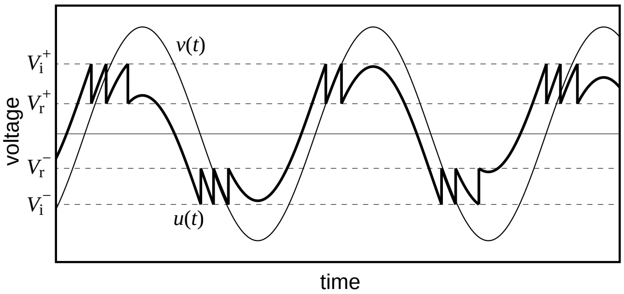

Figure 2: Time evolution of the three-capacitance equivalent circuit model. The thick solid line denotes the actual voltage $u(t)$ between the discharge gap. When it reaches the inception voltage $V_i^\pm$, a discharge occurs and the value changes discontinuously to $V_r^\pm$, respectively. The thin solid line denotes the applied voltage $v(t)$, which is the voltage between the gap, where no discharge is assumed to occur.

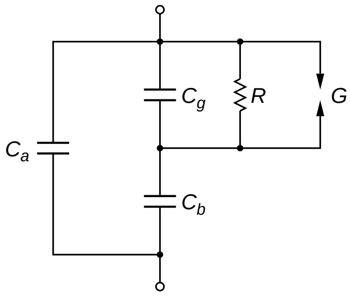

Figure 3: The three-capacitance equivalent circuit with dissipation. In comparison with Fig. 1, the resistance R is added in parallel with the discharge gap G.

circuit. We define *actual voltage* $u(t)$ as the voltage between the gap at time t and *applied voltage* $v(t)$ as the voltage between the gap where no discharges are assumed to occur.

This model is completely deterministic. We have shown that the behavior of the three-capacitance model is very complex, even though it is very simple and completely deterministic [7].

3. Model with Dissipation

Because the three-capacitance model is too simple as a model of real partial discharge phenomena, in this paper, we investigate the behavior of the three-capacitance model with dissipation, which is shown in Fig. 3.

This model is also completely deterministic. It is exactly same as the model proposed by Patsch et al. [15,16] except that their model has scatters in physical parameters.

The behavior of the model can be described by the following equation:

$$u(t) = V\sin\omega t + (V_r^\pm - V\sin\omega t_0)\exp\left(-\frac{t-t_0}{\tau}\right), \qquad (1)$$

where t_0 and $V_r^\pm$ respectively denote the time and the residual voltage of the previous discharge, and τ is the time constant of the dissipation. When τ is very large, the model can be considered as the original three-capacitance model. On the other hand, when τ is very small, the actual voltage $u(t)$ quickly follows the applied voltage $v(t)$, and therefore discharge bursts occur while $v(t)$ exceeds the inception voltages.

Table 1: Model parameters used in numerical simulations

	positive	negative
inception voltage	0.4325	−0.3932
residual voltage	0.1157	−0.1212

4. Numerical Simulation

We performed numerical simulation of the model with the parameters shown in Tab. 1 and $\omega = 1$. Figs. 4, 5, and 6 show the graph of discharge rate as a function of V for $\tau = 8\pi$, π, and $\pi/8$, respectively.

Self-similar structure of the devil's staircase can be observed in Figs. 4 and 5. On the other hand, staircase in Fig. 6 are periodic, which seems to be caused by the discharge bursts.

5. Conclusion

We have shown that complex behavior resembling a devil's staircase can also be observed in the three-capacitance model with dissipation. As decreasing time constant τ, we have seen the transition of the model's behavior from a devil's staircase to a periodic staircase. Investigation of mechanism of this transition is our important future problem.

Acknowledgments

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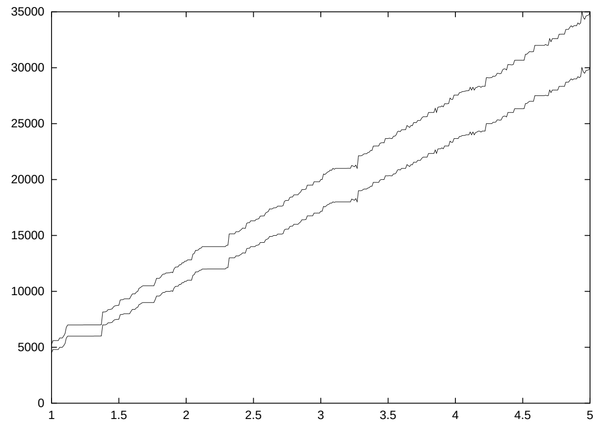

Figure 4: Discharge rate of the model with large time constant.

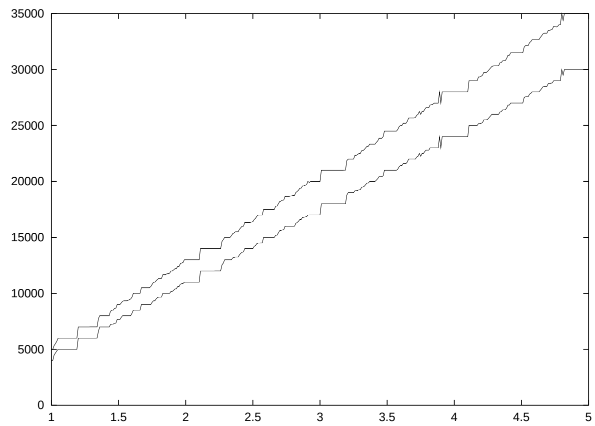

Figure 5: Discharge rate of the model with intermediate time constant.

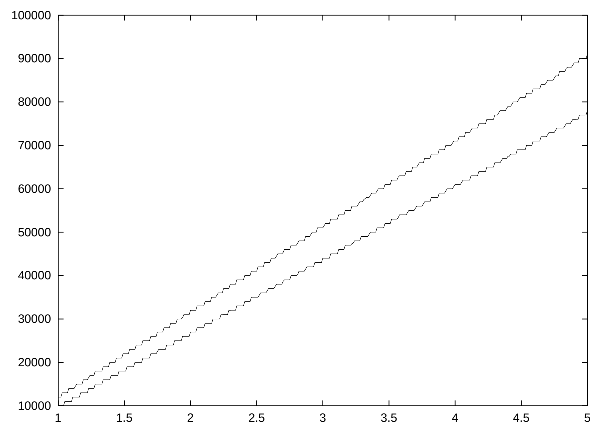

Figure 6: Discharge rate of the model with small time constant.

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