# A Design of Line Codes Based on Chaos Maps and Their Performance in Bandlimited Spread Spectrum Transmission

Kenshi Oobuchi, Daisaburo Yoshioka, Akio Tsuneda, and Takahiro Inoue

Department of Electrical Computer Engineering, Kumamoto University
2-39-1 Kurokami, Kumamoto 860-8555, Japan
Tel: +81-96-342-3853, Fax: +81-96-342-3630
Email: kenshi@st.eecs.kumamoto-u.ac.jp, tsuneda@eecs.kumamoto-u.ac.jp

**Abstract–** This paper gives a design of line codes with negative autocorrelations based on chaos maps with finite bits. Numerical results show that the proposed line codes are useful for bandlimited spread spectrum transmissions.

## 1. Introduction

In recent years, high data rate wireless communication technologies such as International Mobile Telecommunication-2000 (IMT-2000), wireless ATM (Asynchronous Transfer Mode) applications, and wireless LAN (Local Area Network) are extensively used. It is expected in near future that the bandwidth of digital data signals is required to be much broader for high data rate transmissions with enormous capacity such as UWB (Ultra Wideband) communications [1]. However, wideband communications make it difficult to design antennas and circuits which can meet such wideband signals.

On the other hand, some chaos-based communication systems are proposed in the last decade, for example chaos-based CDMA systems and chaos-shift keying (CSK) communication systems [2]. Such chaotic communication systems have some advantages, *e.g.*, security enhancement, many kinds of sequences, and so on.

In this paper, we employ chaotic binary sequences as line codes for baseband spread spectrum communication systems. The motivation of this work is that chaotic binary sequences with negative autocorrelations are useful as line codes because they have high density spectrum in high frequency band. We give some numerical results on properties of the proposed line codes.
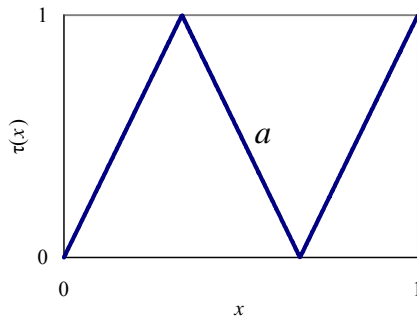


Figure 1: Piecewise linear onto map.

## 2. Generation Method of Chaotic Sequence by One-Dimensional Map

The simplest system generating chaotic sequences is a one-dimensional difference equation given by

$$x_{n+1} = \tau(x_n), \quad x_n \in I, \quad n = 0, 1, 2, \cdots \quad (1)$$

where $\tau(\cdot)$ is a nonlinear map defined on an interval $I$. In this paper, we use a piecewise linear chaotic map with $I = [0,1]$ as shown in Fig.1. The nonlinear function is defined by

$$\tau(x) = \begin{cases} \dfrac{2|a|}{|a|-1} x & \left(0 \le x < \dfrac{1}{2} - \dfrac{1}{2|a|}\right) \\ ax - \dfrac{a-1}{2} & \left(\dfrac{1}{2} - \dfrac{1}{2|a|} \le x < \dfrac{1}{2} + \dfrac{1}{2|a|}\right) \\ \dfrac{2|a|}{|a|-1}\left(x - \dfrac{|a|+1}{2|a|}\right) & \left(\dfrac{1}{2} + \dfrac{1}{2|a|} \le x \le 1\right) \end{cases} \quad (2)$$

where $a(|a|>1)$ is a slope of the linear function in' the center subinterval [3].

We transform a real-valued sequence $\{\tau^n(x)\}_{n=0}^{\infty}$ into a binary sequence by the threshold function defined by

$$\Theta_t(x) = \begin{cases} -1 & (x < t) \\ 1 & (x \ge t) \end{cases} \quad (3)$$

By adopting the above threshold function with $t = 0.5$, a balanced chaotic binary sequence $\{\Theta_{0.5}(\tau^n(x))\}_{n=0}^{\infty}$ is obtained. The autocorrelation function of such a chaotic binary sequence is theoretically derived as

$$\begin{aligned} R(\ell) &= E[\Theta_{0.5}(x)\Theta_{0.5}(\tau^\ell(x))] \\ &= \int_I \Theta_{0.5}(x)\Theta_{0.5}(\tau^\ell(x)) f^*(x)dx = a^{-\ell}, \end{aligned} \quad (4)$$

where $\ell$ is the time delay, $E[\cdot]$ denotes the ensemble average, and $f^*(x)$ is the invariant density function.

If $a$ is negative, the probability of bit changes from 1 to –1 or from –1 to 1 is higher than no correlation case. This implies that the binary sequences with negative autocorrelations have high power spectrum density in a high frequency band, which is suitable for baseband spread spectrum communications. Thus, we can easily design line codes with negative autocorrelations based on the above chaotic maps.
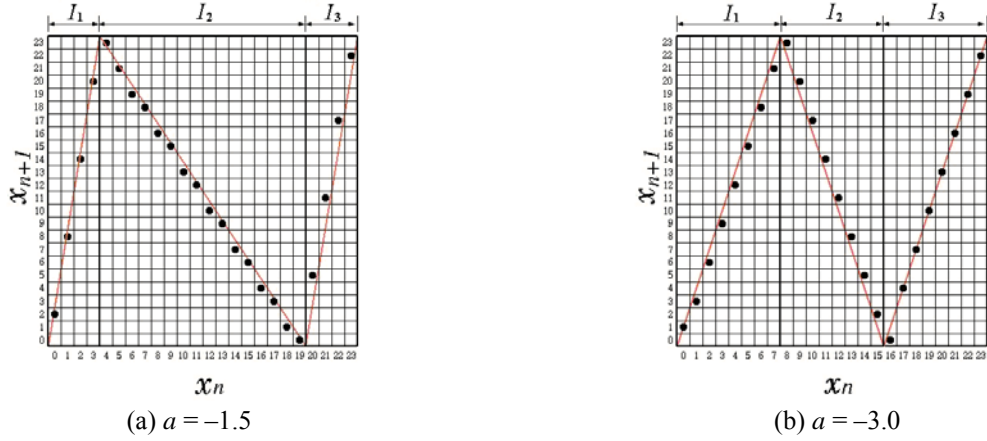
(a) $a = -1.5$            (b) $a = -3.0$

Figure 2: Examples of a one-dimensional map with finite bits ($N = 24$).



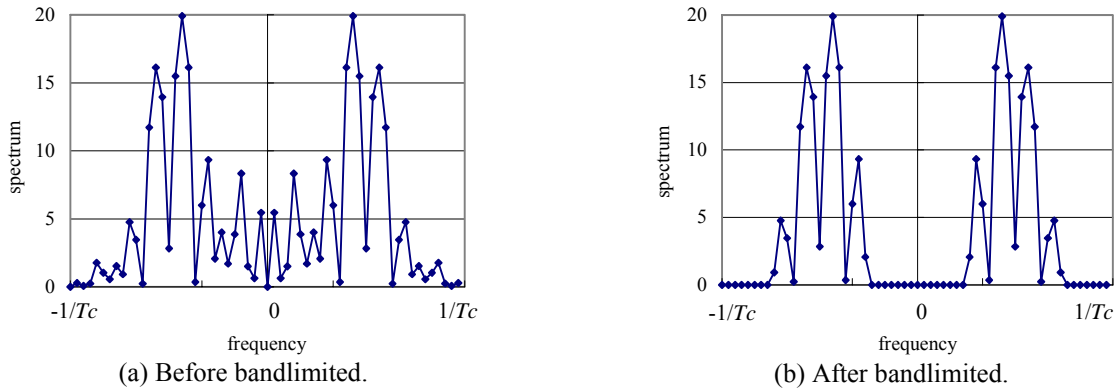(a) Before bandlimited.            (b) After bandlimited.

Figure 3: Examples of power spectrum.

## 3. Maximal-Period Sequences

The statistical properties of chaotic binary sequences of finite length have some deviations from their theoretical values. However we can design binary sequences with constant autocorrelation values corresponding to the theoretical ones by the use of quantized chaos maps with finite bits. Fig.2 (a) and (b) show examples of quantized one-to-one maps ($N = 24$) based on the chaos maps with the correlation parameters $a = -1.5$ and $-3.0$, respectively. $N$ black points indicate the mapping function $f(x)$ obtained by quantizing the original mapping function $\tau(x)$. A periodic integer sequence can be generated by the difference equation $x_{n+1} = f(x_n)$ from an initial value $x_0$, where $x_n \in \{0, 1, ..., N-1\}$. We use maximal-period sequences $\{x_n\}_{n=0}^{N-1}$ and convert them to binary sequences $\{\Theta_{N/2}(x_n)\}_{n=0}^{N-1}$ by the threshold $t = N/2$ corresponding to the real number $t = 0.5$ for the original chaos maps. It is confirmed by numerical experiments that the auto-correlation properties of such maximal-period binary sequences are similar to those of chaotic ones generated by the original maps [4]. In addition, maximal-period binary sequences have the following remarkable features.

- If $N$ is an even number, $\{\Theta_{N/2}(x_n)\}_{n=0}^{N-1}$ is completely balanced, which implies that such sequences have no DC component.

- Constant autocorrelation values of $\{\Theta_{N/2}(x_n)\}_{n=0}^{N-1}$ can be designed for some $N$ satisfying the conditions described in [5].

## 4. Bandlimited Sequences

The signal of the line codes employing a maximal-period binary sequence is given by

$$x(t) = \sum_{n=0}^{N-1} \Theta_{N/2}(x_n) P_{T_c}(t - nT_c), \qquad (5)$$

where $T_c$ is pulse duration and

$$P_{T_c}(t) = \begin{cases} 1 & (0 \le x < T_c) \\ 0 & (\text{elsewhere}) \end{cases} \qquad (6)$$

We evaluate the discrete Fourier transform (DFT) of $x(t)$, where the number of samples per pulse duration is equal to $S$. An example of power spectrum is shown in Fig.3 (a), where the type of 1-D maps in Fig.2 (a) is used. Let $\mathbf{X} = \{X_k\}_{k=0}^{SN-1}$ and $\hat{\mathbf{X}} = \{\hat{X}_k\}_{k=0}^{SN-1}$ be a set of DFT coefficients of $x(t)$ and its bandlimited version, respectively. In the transmission systems, we assume the signals are bandlimited as
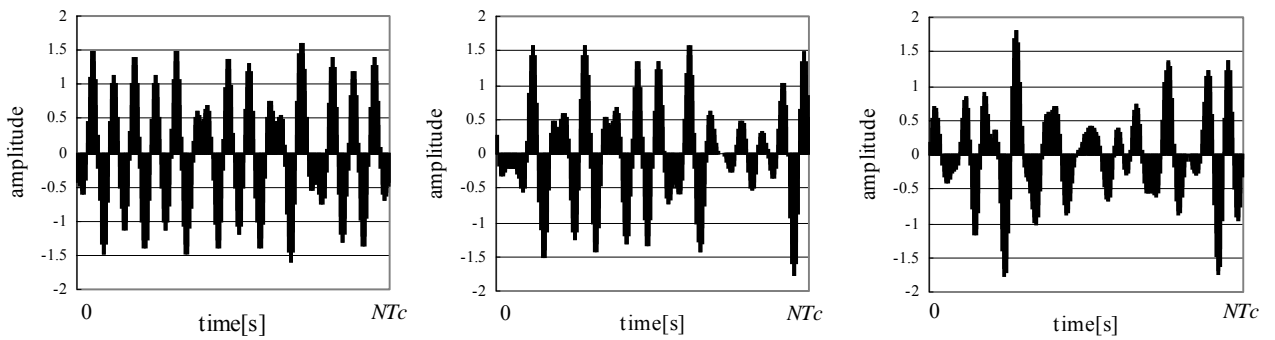
$$\hat{X}_k = \begin{cases} X_k & (m \le k \le N - m, \ (S-1)N + m \le k \le SN - m) \\ 0 & (\text{otherwise}) \end{cases} \qquad (7)$$

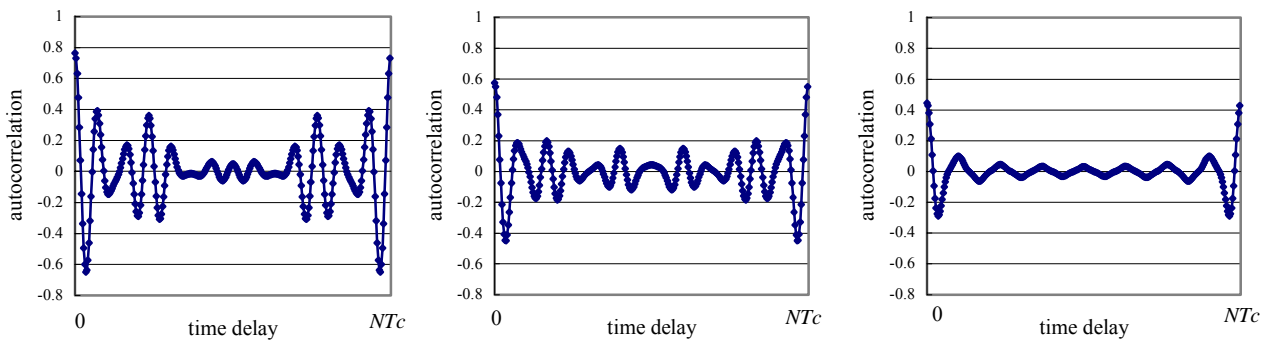where $m$ is the number of components to be eliminated.
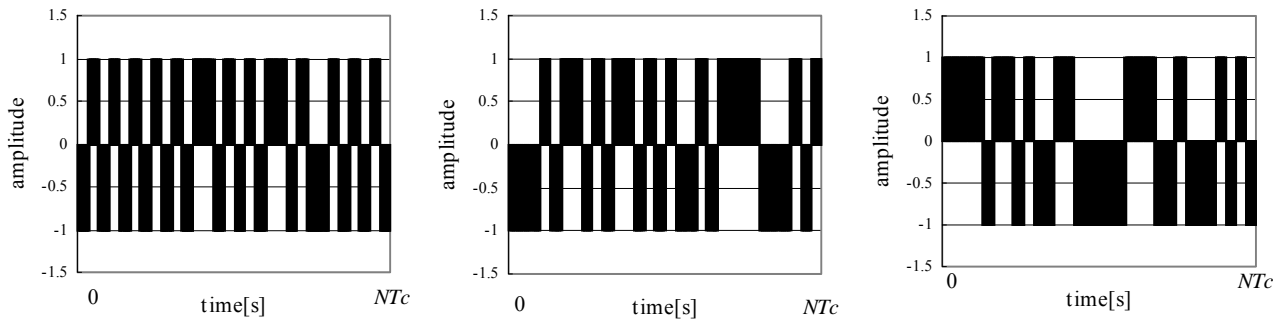
(a) Maximal-period sequence ($a = -1.5$)    (b) Maximal-period sequence ($a = -3.0$)    (c) M-sequence
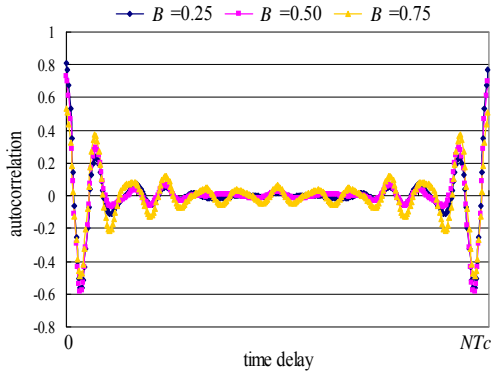
Figure 4: Original waveforms before bandlimited



(a) Maximal-period sequence ($a = -1.5$)    (b) Maximal-period sequence ($a = -3.0$)    (c) M-sequence

Figure 5: Bandlimited waveforms ($B = 0.5$)
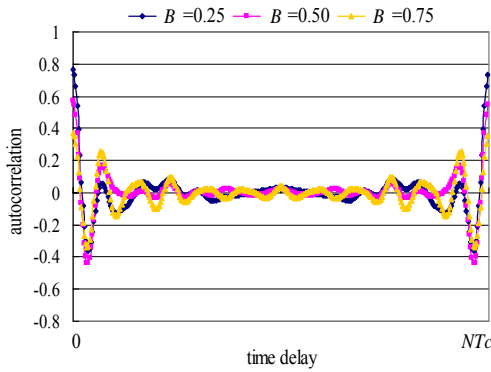


(a) Maximal-period sequence ($a = -1.5$)    (b) Maximal-period sequence ($a = -3.0$)    (c) M-sequence

Figure 6: Autocorrelations of bandlimited signals

(a) $a = -1.5$



(b) $a = -3.0$

Figure 7: Average autocorrelation functions of bandlimited signals of maximal-period sequences

Table 1: Average peak values of autocorrelation of bandlimited signals.

| $B$ | average peak value | | | |
|---|---|---|---|---|
| | $a = -1.5$ | $a = -2.0$ | $a = -3.0$ | $a = -4.0$ |
| 0.25 | 0.806 | 0.763 | 0.763 | 0.723 |
| 0.50 | 0.732 | 0.642 | 0.576 | 0.532 |
| 0.75 | 0.536 | 0.437 | 0.372 | 0.252 |

es are shown in Fig.7 (a) and (b), where the types of 1-D map in Fig.2 (a) and (b) are used, respectively. The peak average values at the zero time delay are summarized in Table 1. In this table, the results for the correlation parameters $a = -2.0$ and $a = -4.0$ are also included. From Table 1, we can find that the influence of bandlimitation becomes smaller as the negative autocorrelation becomes stronger (*i.e.*, $a \rightarrow -1$).

## 5. Conclusion

We have investigated the properties of bandlimited signals of maximal-period sequences with negative autocorrelations. As a result, we can find that maximal-period sequences with strongly negative autocorrelations have an enough peak value of autocorrelation even in a strongly bandlimited system. This implies that such sequences can be used as line codes for baseband spread spectrum communications.

**References**

[1] D. Porcino and W. Hirt, "Ultra-wideband radio technology: Potential and challenges ahead," IEEE Commum. Mag., pp.66–74, July 2003.

[2] M. P. Kennedy, R. Rovatti. G. Setti (Eds.), *Chaotic Electronics in Telecommunications*, CRC Press, 2000.

[3] A. Tsuneda, "Design of chaotic binary sequences with prescribed auto-correlation properties based on piecewise monotonic onto maps," Proc. NOLTA'99, vol.2, pp.605–608, 1999.

[4] D. Yoshioka, A. Tsuneda, and T. Inoue, "Maximal-period sequences with negative auto-correlation and their application to asynchronous DS-CDMA," IEICE Trans. Fundamentals, vol.E86-A, no.6, pp.1405–1413, June 2003.

[5] A. Tsuneda, D. Yoshioka, and T. Inoue, "Design of maximal-period sequences with prescribed auto-correlation properties based on one-dimensional maps with finite bits," Proc. ITC-CSCC2002, vol.3, pp.2019–2022, July 2002.

[6] D. Yoshioka, A. Tsuneda, and T. Inoue, "An algorithm for the generation of maximal-period sequences based on one-dimensional chaos maps with finite bits," IEICE Trans. Fundamentals, vol.E87-A, no.6, pp.1371–1376, June 2004.

Namely, the transfer function of this channel is modeled as an ideal band pass filter (BPF). Define the bandlimitation ratio by $B=2m/N$. Fig.3(b) shows an example of $\{\hat{X}_k\}_{k=0}^{SN-1}$, where the ratio $B$ is equal to 0.5.

First, Fig.4 (a) and (b) show examples of the original waveform of the line code $x(t)$, where $N=30$ and the types of 1-D maps in Fig.2 (a) and (b) are used, respectively. For comparison, Fig.4 (c) shows an example of the waveform of an M-sequence, where the primitive polynomial is $h(x) = x^5+x^3+1$ and $N = 31$. Fig.5 shows the waveforms of each bandlimited signal which is obtained by the inverse DFT of $\hat{\mathbf{X}} = \{\hat{X}_n\}_{n=0}^{SN-1}$, where $B =0.5$.

Next, we calculated the crosscorrelation values between the original signal and its bandlimited signal. Note that such a crosscorrelation function is the same as the autocorrelation of the bandlimited signal itself. Fig.6 (a)–(c) show the autocorrelation functions of such signals. It is found that the peak values of autocorrelation of both maximal-period sequences are larger than that of the M-sequence. Note that if the synchronization is achieved, then the peak value of the autocorrelation is desired to be large for detecting the information data in the presence of noise and interferences.

Furthermore, we calculated autocorrelation functions of 1,000 bandlimited signals of maximal-period sequences for each case of $B$=0.25, 0.5, and 0.75. Their average valu-