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The Application of Fuzzy Automata to Medical Signal Processing

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Abstract– This paper presents a model of finite fuzzy automata with adaptive input membership functions that could be used for evaluating time sequences of events. In our case an event is a certain amplitude of the patient's blood pressure signal, which was acquired from their wrist. A time of occurrence of this event is also important. The automaton accepts the sequence of events that occur in definite time instants, where some predefined deviations are allowed. This is achieved through defining the shape and position of the membership functions. With them the designer determines what differences between given and expected input sequences can be tolerated.

1. Introduction

Studies have showed that abnormalities in the peripheral circulation can be detected through arterial blood velocity analysis [6], [8]. We hypothesize that such early detection can be based on spotting certain events present in the blood pressure signal of the patient. In our case an event is represented by a certain amplitude of the patient's blood pressure signal, which was acquired from their wrist. The biggest drawback is that blood pressure signals show great variability between individuals. As there are great differences between signals, standard methods of signal processing are difficult to apply. Manual screening of signals is slow and laborious as the standard measuring period is up to 30 minutes (i.e. approximately 1800 heartbeats). These reasons lead us to develop a method based on fuzzy automata for the isolation of waves that differ from an ideal one. The rules, as well as the allowed tolerances are based on the ideal wave. The output of the automata is the degree of similarity of the observed wave and the ideal one. When the output is low, the physician should be notified about this abnormal wave. By using this method, the time spent by the physician for screening signals, can be reduced significantly.

In many application domains it is necessary to handle time dependant data. Our knowledge about time and the course of events is rarely complete, but rather pervaded with vagueness and uncertainty. Because of this, the fuzzy approach to handling time dependant data seems appropriate [7]. Dubois and Prade [1] proposed a model for the representation and processing of fuzzy temporal knowledge based on possibility logic, but handling time dependant data is also possible by using fuzzy automata [2].

In this paper we apply the fuzzy automaton [2] to the problem of recognition of time sequences of events. In the first part of the paper we discuss fuzzy time, we continue by presenting our fuzzy automata and conclude by applying it to medical signal processing. We prefere to use Yager's notation for representing membership functions [8], so throughout the paper $A(x)$ is the degree of membership of x in A, thus $A(x)=\mu_A(x)$.

2. The Fuzzy Temporal Primitive

Following the definition in Dubois and Prade [1], the fuzzy temporal primitive date a is any fuzzy instant of time represented by the possibility distribution π_A over T, where T is a continuous linear scale that models time. The possibility distribution is a mapping from T to [0,1], where for each $t \in T$, is the possibility that date a is precisely the time instant t. By means of π_A, we can define the fuzzy set A over T, which represents all possible values of a. If A is the membership function associated with, A we have $\forall t \in T, \pi_A(t) = A(t)$.

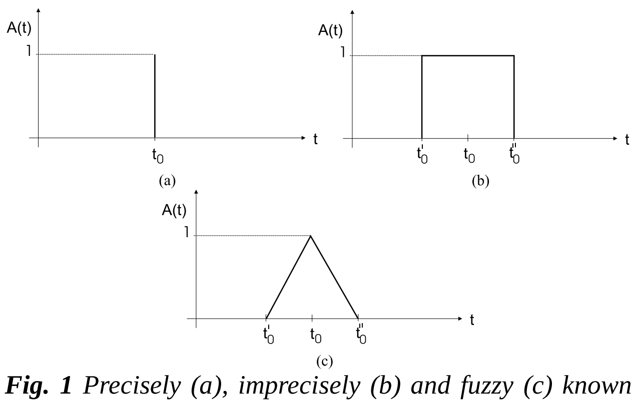

Fig. 1 *Precisely (a), imprecisely (b) and fuzzy (c) known dates.*

The fuzzy set associated with date a is assumed to be convex and normalized. Date a can be precise (Fig. 1a), imprecise (Fig. 1b) or fuzzy (Fig. 1c). In the first case, there exists only one time instant t_0 such that $\pi_A(t_0) = 1$. In the second case all possible values of date a are between two time points t_0' and t_0'' such that $\forall t \in [t_0', t_0''], \pi_A(t) = 1$. In the third case the possible values of date *a* are defined by the possibility distribution that takes values from the interval [0,1]. The degree of membership is in this case, of a triangular shape and varies from 0 to 1 with the maximum value at t_0.

2.1. The Time Sequence of Dates

Dates are usually connected with the presence of some phenomena or some event from the physical world. Natural phenomena usually do not start with full intensity, therefore it is sometimes difficult to say if and when the phenomenon starts at all. For example, when considering the progress of an illness, there is usually known only an approximate sequence of events and these can not be described precisely.

A time sequence of events can be represented on the time axis as a sequence of dates which are precisely, imprecisely or fuzzy known, depending of our knowledge about them.

Let us study the sequence of two events, where the occurrence of the first one is imprecisely known and can be represented by date *a* with nonzero possibility between points t_0' and t_0'' and the occurrence of the second is precisely known and can be represented by date *b* with value 1 at t_2 . When the first event relating to date *a* occurs exactly at the expected time instant t_1, the second event should occur at time instant t_2 (Fig . 2a).

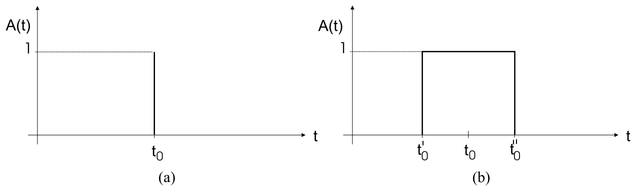

Fig. 2 *Sequence of a imprecisely and precisely known events (a) and the modification of the second event caused by the time difference of the actual time of occurrence of the first event (b).*

However, if the first event occurs at time instant $t \neq t_1$, where $t_1' \le t \le t_1''$, then the second event represented by date *b* is shifted along the time axis for the difference between the expected (t_1) and actual occurrence (*t*) of the first event (Fig. 2b). The new expected occurrence of the second event represented by date *b* now coincides with t_2 , where $t_2' = t_2 - \Delta$ and $t_1' = t_1 - \Delta$.

The event related to date *b* therefore usually occurs at the same time distance from the event related to date *a*. As date *a* is imprecisely known, the maximum shifting of date *b* is determined by the degree of imprecision of date *a* (the interval $[t_1', t_1'']$).

Let us study the sequence of two events, where the first can be represented by a fuzzy known date *a* and the second by a precisely known date *b* with value 1 at t_2 (Fig. 3a).

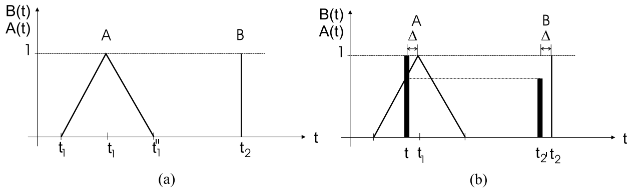

Fig. 3 *Sequence of a fuzzy and precisely known events (a) and the modification of the second event caused by the time difference of the acctual time of occurence of the first event (b).*

The modification of date *b* caused by the discrepancy between the expected time of occurrence (t_1) and the actual time of occurrence (*t*) of the first event that is related to the date *a*, is shown in Fig.3b. In this case, the modification takes the form of shifting date *b* for Δ and reducing its membership degree as well. The reduction is the result of the fact that the membership in date *a* at the actual time of occurrence of the related event is less than 1.

Figure 4 depicts the case when the first event related to date *a* does not occur at a crisp time instant *t*, but is rather given as a fuzzy set $\tilde{t}$. In this case the modified date *b* takes the form of the fuzzy intersection between date *a* and the fuzzy set $\tilde{t}$ and is shifted in time for Δ. In this way the originally precisely known date *b* related to the second event becomes a fuzzy date.

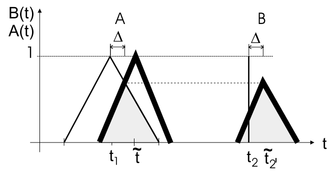

Fig. 4 *Modification of the second event caused by a fuzzy known time of occurrence of the first event.*

2.2. The Fuzzy Automaton with an Adaptive Input Membership Function Accepting 2-Dimensional Inputs

The fuzzy automaton with adaptive input membership functions, explained in detail in [2]-[4] , is able to adapt to variations between the processed unknown pattern and a template. The fuzzy automaton can be defined as:

Definition 1:Let $Q = \{q_1, ..., q_n\}$ be a set of all possible states of the automaton and $E = \{e_1, ..., e_m\}$ be a finite set of input symbols defined as fuzzy sets on the Cartesian product $X \times Y = [x_1, x_2] \times [y_1, y_2] \subset \mathbb{R}^2$ where $E_i(x, y)$ is the membership function of fuzzy set *E* and $E_i(e_i)$. Let δ be the mapping that defines transitions between states $(\delta : Q \times E \times Q \to \{0,1\})$, $\hat{s}^0$ the fuzzy initial state and

$Q_F \subset Q$ the set of final states. Then the five-tuple $A = \left\langle Q, E, \delta, \hat{s}^0, Q_F \right\rangle$ is a fuzzy automaton.

The state of automata A at step k, denoted $\hat{s}^k$ is the n-tuple $\hat{s}^k = \left\langle S_1^k, ..., S_n^k \right\rangle$, where S_i^k is the fuzzy set on Cartesian product $S_X \times S_Y = [s_{x_1}, s_{x_2}] \times [s_{y_1}, s_{y_2}] \subset \mathbb{R}^2$ at step k that is assigned to state q_i. The input to the automaton in step k, denoted X^k, is the fuzzy set on Cartesian product $X \times Y \subset \mathbb{R}^2$

2.3 Medical Signal Processing

The blood pressure signals, acquired from the patients' wrists, when compared, show considerable variability between individuals as well as between different measurements on the same patient. The latter case is usually caused by the patient's breathing or movement, which both lead to movements of the measuring equipment. This reduces the possibility of the application of standard signal processing methods. The automaton that was presented in the previous section can cope with this variability and as such represents a possible solution to the problem of medical signal processing.

The signal is acquired by means of a heart rate monitor Colin BP-508 that simultaneously acquires the Echocardiograph (ECG) and blood pressure signals. The blood pressure signal is acquired from the patient's wrist. In Fig. 5a we can see an example of the two signals acquired from a healthy patient. The ECG signal (Fig. 5a top) measures electrical pulses, which directly effect the contractions of the heart muscle. A standard part of the ECG signal is called R peak (short and high pulse). Between two R peaks there is a complete heart muscle contraction period (systole and diastole). Therefore we can use R peaks as precise time markers that enable us to extract blood pressure signal sections from the complete blood pressure signal (Fig. 5a bottom). A set of signal sections can be found in Fig. 5b. The figure presents a set of 60 consequent signal sections. It can be seen that they can be judged as equal with a certain amount of variability in time and amplitude.

The automaton that we presented in the previous section uses a two-valued input, where the first value is time and the second is the amplitude of the signal. For the evaluation of the similarity of the signals we defined four fuzzy sets that describe four characteristic events in the signal. The membership functions representing the dates related to these events can be found in Fig. 6a. From all of the membership functions the first one allows for the greatest variability in both time and amplitude and as such also covers the greatest area when displayed as a contour plot. For the rest of the membership functions we used low variation in time but high in amplitude, thus their plots are narrow and long.

Figure 6b shows an example of one processing cycle. It can be seen that the first membership function remains unchanged compared to the original, while all the following membership functions change in accordance with the degree of dissimilarity of the time of occurrence of the previous event. The last membership function is thus affected by all of the changes caused by the discrepancies of all previous events. A detailed description of the algorithm can be found in [2].

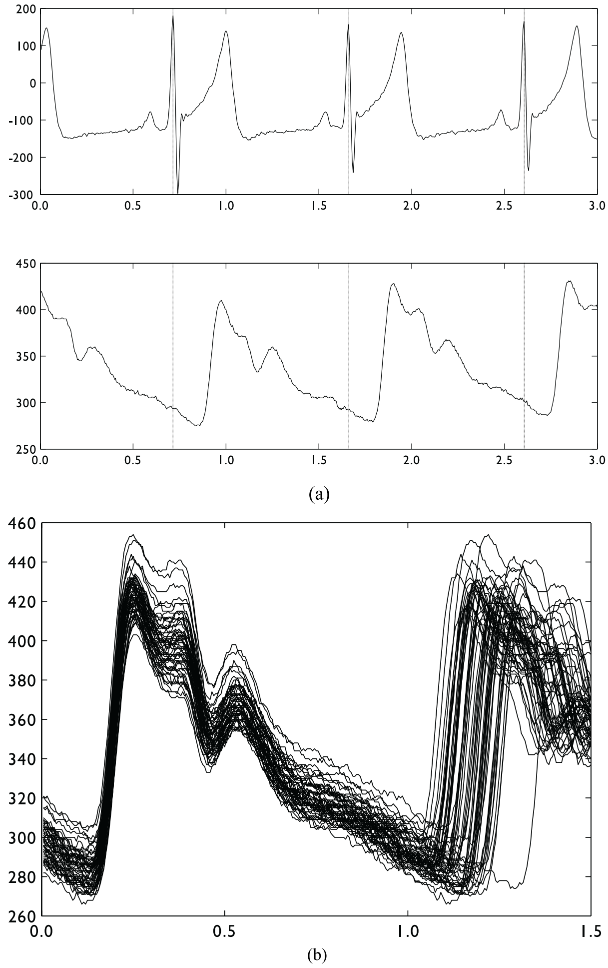

Fig. 5 *ECG and blood pressure signals (a) and a set of 60 successive blood pressure signal sections (b).*

The output of the automaton is the degree of similarity of our four characteristic events of the current signal wave when compared to the four membership functions. The number of membership functions is optional, but in our case the four were defined. For a greater precision a higher number of membership functions can be defined, but in our case four proved as enough.

Figure 7 shows the degree of similarity of the signal sections as a function of time, where time is the time of occurrence of the section in the complete blood pressure signal (in our case 1 minute). The sample time in this case was relatively short, the patient was lying perfectly still and thus the signal sections are noise-free. The latter fact can be deduced from the degrees of similarity that are all higher than 0.5. Regardless to that the degree of similarity fluctuates. This can be assigned to the fact that the pressure in the vein depends on the intensity of the contractions of the heart muscle that varies constantly, the dilation of the thorax, the movements of the patient, etc.

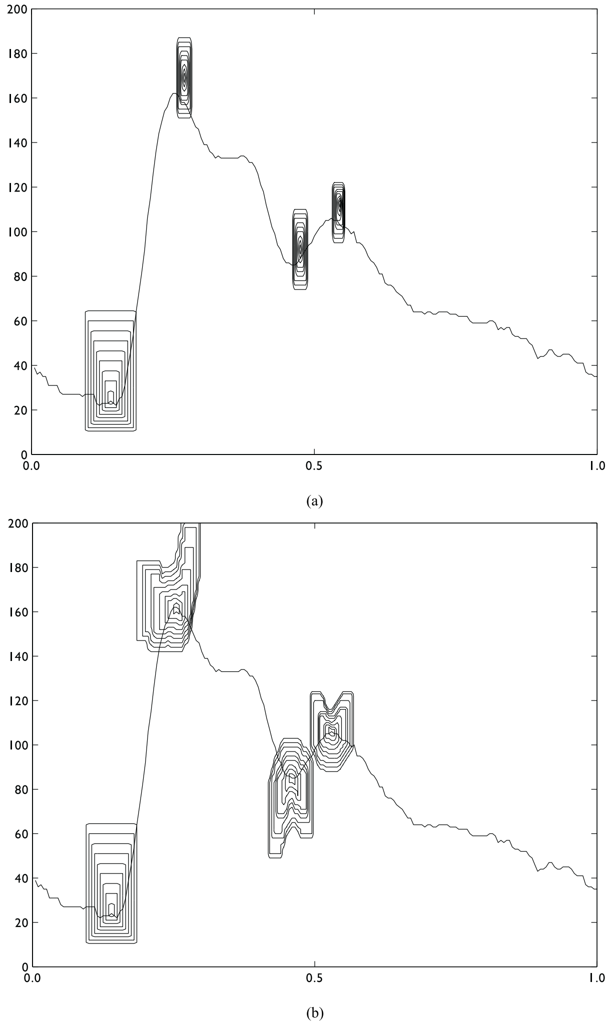

Fig. 6 *Characteristic events of the blood pressure signal sections and their membership functions (a) and their modifications caused by the time difference of their actual time of occurrence (b).*

The usefulness of the presented algorithm becomes apparent when longer monitoring periods are taken into account. In these cases big similarity variations become instantly apparent. The physician can read this as a notification that a thorough analysis of the signal is needed and thus can save valuable time for not analysing normal data.

3. Conclusion

The usefulness of the presented algorithm becomes apparent when longer monitoring periods are taken into In this article we presented a novel approach to medical signal analysis that is based on fuzzy automata. The most interesting part of our approach is that it allows physicians set reference events and their allowed variations in both amplitude and time. When longer monitoring periods have to be analysed the algorithm becomes very useful since it notifies the physicians that a thorough analysis is needed and as such can save them valuable time for not analysing normal data.

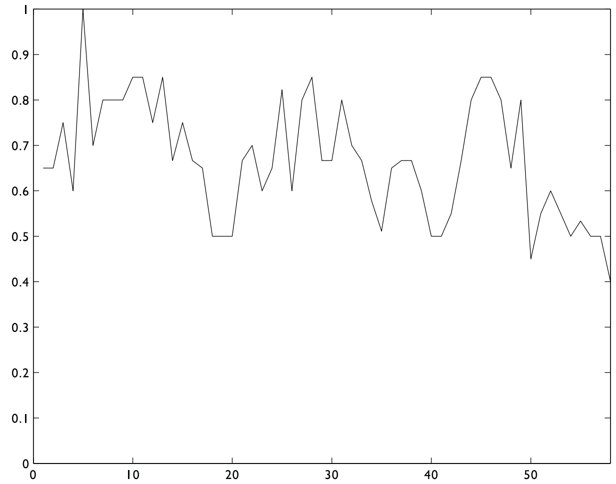

Fig. 7 *Plot of the degree of similarity as a function of time.*

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