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Pseudorandom Orthogonal Sequences Based on Hadamard Matrix and Their Correlation Properties

Tetsu Kondou, Daisaburo Yoshioka, Akio Tsuneda, Takahiro Inoue

Department of Electrical and Computer Engineering, Kumamoto University
2-39-1 Kurokami, Kumamoto 860-8555, Japan
Tel: +81-96-342-3853, Fax: +81-96-342-3630
Email: kondu@st.eecs.kumamoto-u.ac.jp, tsuneda@eecs.kumamoto-u.ac.jp

Abstract—Pseudorandom orthogonal codes based on Hadamard matrices for synchronous DS/CDMA systems in multipath environment are proposed. The important result is that the proposed method can improve correlation properties of Walsh codes while keeping their orthogonality and balanced property.

1. Introduction

A code division multiple access (CDMA) system based on spread spectrum (SS) techniques, which has already been used for Interim Standard 95 (IS-95) and International Mobile Telecommunications 2000 (IMT-2000), has various merits such as increase of its potential capacity, robustness to multi-path fading, anti-jamming, and so on. The performance of CDMA greatly depends on spreading sequences used for spreading the bandwidth of information signals [1].

In synchronous DS/CDMA systems, orthogonal sequences have been adopted for channel separation. The well-known orthogonal sequences are Walsh codes and orthogonal variable spreading factor codes (OVSF) [2]. However, the autocorrelations of their codes may be considerably large even when the delay time is nonzero due to their regular structure. This might greatly degrade the system performance in the presence of multipaths even though the system would acheive perfect synchronization. As in W-CDMA [2], the use of scramble codes (long codes) can improve autocorrelation properties of orthogonal codes. A similar method to such improvement for Walsh codes is also proposed in [3]. However, these methods cannot guarantee the balanced property of the orthogonal codes.

In this paper, a simple method to randomize the regular structure of Walsh codes is proposed while preserving their orthogonality and balanced property. We evaluate the correlation properties of the proposed sequences by numerical experiments, and compare them with those of Walsh codes.

2. DS/CDMA System

In this paper, we consider a baseband model with U active users and M propagation paths. The i-th user generates a baseband signal $d^i(t)$ given by

$$d^i(t) = \sum_{n=-\infty}^{\infty} d_{i,n} a^i(t - nNT_c), \tag{1}$$

$$a^i(t) = \sum_{k=0}^{N-1} a_k^{(i)} P_{T_c}(t - kT_c), \tag{2}$$

$$P_{T_c}(t) = \begin{cases} 1 & (t \in [0, T_c]) \\ 0 & (t \notin [0, T_c]), \end{cases} \tag{3}$$

where $d_{i,n} \in \{-1, 1\}$ is a data bit, $a^i(t)$ is a spreading signal, $a_k^{(i)} \in \{-1, 1\}$ is an element of the spreading code, T_c is chip pulse duration, and N is the spreading factor. The interference components at the i-th correlation receiver when $t = T_d = NT_c$ is given by

$$\begin{aligned} Z^i = \sum_{m=1}^{M} \Big\{ & d_{i,-1}\rho_m^i \int_0^{\tau_m^i} a^i(t - \tau_m^i) a^i(t) dt \\ & + d_{i,0}\rho_m^i \int_{\tau_m^i}^{T_d} a^i(t - \tau_m^i) a^i(t) dt \Big\} \\ + \sum_{j=1, j\neq i}^{U} \sum_{m=0}^{M} \Big\{ & d_{j,-1}\rho_m^j \int_0^{\tau_m^j} a^j(t - \tau_m^j) a^i(t) dt \\ & + d_{j,0}\rho_m^j \int_{\tau_m^j}^{T_d} a^j(t - \tau_m^j) a^i(t) dt \Big\}, \end{aligned} \tag{4}$$

where ρ_m^i and τ_m^i are the amplitude and the delay time of the m-th path of the i-th signals, respectively. In the right-hand side of eq.(4), the first and second integrals depend on autocorrelation functions (ACF) of $a^i(t)$, and the third and fourth ones depend on crosscorrelation functions (CCF) between $a^i(t)$ and $a^j(t)$.

Now we define the even and odd CCF respectively by [4]

$$R_{i,j}(l) = A_{i,j}(l) + A_{i,j}(l - N), \tag{5}$$

$$\hat{R}_{i,j}(l) = A_{i,j}(l) - A_{i,j}(l - N), \tag{6}$$

where $A_{i,j}(l)$ is the aperiodic CCF defined by

$$A_{i,j}(l) = \begin{cases} \sum_{k=0}^{N-1-l} a_k^{(i)} a_{k+l}^{(j)} & (0 \le l \le N-1) \\ \sum_{k=0}^{N-1+l} a_{k-l}^{(i)} a_k^{(j)} & (1 - N \le l < 0) \\ 0 & (|l| \ge N). \end{cases} \tag{7}$$

Note that they are ACFs when $i = j$.

The maximum value of bit error probability $P_{\max}$ is given by

$$P_{\max} = 1 - \Phi((1 - \mathrm{Max}\, Z^i/N)\sqrt{2E_b/N_0}) \qquad (8)$$

where E_b is the data bit energy, N_0 is the two-sided spectral density of additive white Gaussian noise, and $\Phi(\cdot)$ is cumulative Gaussian distribution function. Max Z^i depends on maximum values of the even/odd ACFs and CCFs.

3. Pseudorandom Orthogonal Sequences

We propose orthogonal sequences based on Hadamard matrices for synchronous DS/CDMA communications in multipath environment. An Hadamard matrix H_k of order k is a $2^k \times 2^k$ matrix recursively defined by

$$H_k = \begin{bmatrix} H_{k-1} & H_{k-1} \\ H_{k-1} & -H_{k-1} \end{bmatrix}, \quad H_0 = [1], \qquad (9)$$

which satisfies

$$H_k H_k^T = 2^k I_k, \qquad (10)$$

where the superscript T denotes the transpose and I_k is the $2^k \times 2^k$ unit matrix. Eq.(10) implies that the rows (or columns) are orthogonal to each other. Note that each row (or column) except the 1st one is completely balanced, that is, the number of 1 or -1 is exactly equal to 2^{k-1}. Thus, each row or column can be used as orthogonal codes of length $N = 2^k$, which are referred to as *Walsh codes*, for synchronous DS/CDMA systems.

However, the ACFs of Walsh codes have some high peak values even when the delay time is nonzero. The reason is that the construction of Walsh codes is regularly done. Hence, we interchange the columns of an Hadamard matrix at random in order to randomize their inherent regularity. After interchanging, we use the rows except the 1st one as orthogonal codes. It should be noted that such codes keep not only orthogonality but also balanced property. Figure 1 shows an example of a set of orthogonal sequences obtained by the proposed method, where $N = 8$.

$$\begin{array}{cc} & \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \mathrm{H}_3 = & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix} \end{array}$$

$$\Big\Downarrow \text{ interchange at random}$$

$$\begin{array}{cc} & \begin{array}{cccccccc} 5 & 1 & 8 & 3 & 2 & 7 & 4 & 6 \end{array} \\ \widehat{\mathrm{H}}_3 = & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$\Big\Downarrow$$

$$\widehat{\mathrm{H}}_3 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & -1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 \\ -1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 & 1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 \end{bmatrix}$$

randomized balanced orthogonal codes

Figure 1: An example of the proposed sequences.

4. Correlation Properties

4.1. Aperiodic ACF/CCF

First, we investigated aperiodic ACFs of the proposed orthogonal codes for $N = 32$ except the 1st row. Figures 2 and 3 show the average and maximum values of the aperiodic autocorrelation magnitude $|A_{i,i}(l)|$. We generate 100 sets of the proposed orthogonal sequences by changing an initial value for random number generation. Therefore, the average and maximum values are obtained from 31×100 sequences. For comparison, the results of Walsh codes are also shown in these figures, where the average and the maximum are obtained from the original 31 Walsh codes. From these figures, it is shown that both average and maximum aperiodic autocorrelation values of the proposed sequences are considerably smaller than those of Walsh codes.

Next, we investigated aperiodic CCFs of the proposed orthogonal sequences and Walsh codes for all possible pairs in each set. Figures 4 and 5 show the average and maximum values of the aperiodic corsscorrelation in each delay time. We use the same sequence sets as described above, that is, the average and the maximum is obtained from $100 \times \binom{31}{2}$ sequence pairs for the proposed sequences. From Figure 4, it is shown that the average aperiodic corsscorrelation properties of the proposed sequences become slightly worse than those of Walsh codes. However, from Figure 5, we can find that the maximum values are smaller than those of Walsh codes.

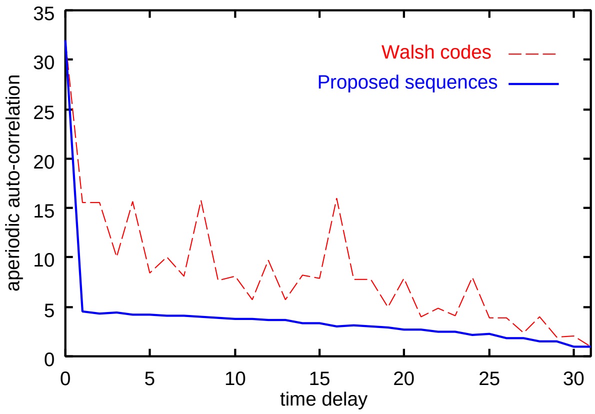

Figure 2: Average values of aperiodic autocorrelation.

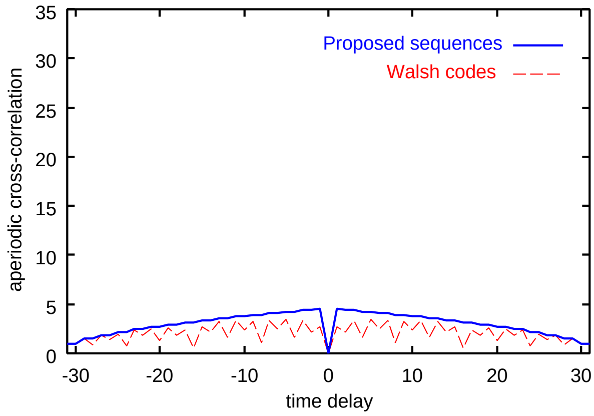

Figure 4: Average values of aperiodic crosscorrelation.

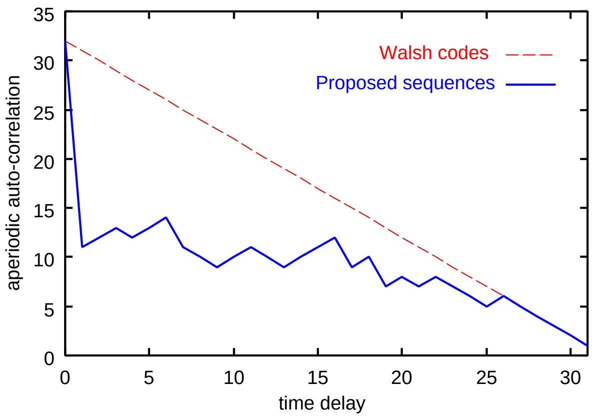

Figure 3: Maximum values of aperiodic autocorrelation.

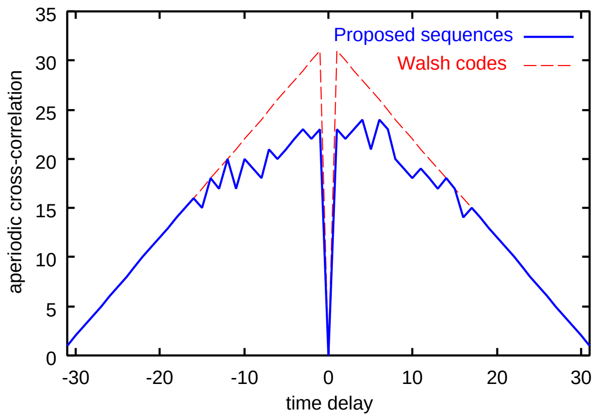

Figure 5: Maximum values of aperiodic crosscorrelation.

4.2. Even and Odd ACF/CCF

We also investigated even and odd ACFs/CCFs of the proposed sequences for the same conditions as the previous subsection. Figures 6–9 show the average and maximum values of even and odd ACFs, respectively. From these figures, we can find that the ACFs are improved, though the maximum values of odd ACFs are larger than those of Walsh codes for some time delays. Especially, the even ACFs are considerably improved.

Furthermore, we investigated even and odd CCFs for the same conditions as shown in Figures 10–13. From Figures 10 and 11, we can find that the average even and odd cross-correlation properties of the proposed sequences become slightly worse than those of Walsh codes. However, we find that the maximum values of even CCFs are improved, though the maximum values of odd CCFs are similar to those of Walsh codes.

5. Conclusion

We proposed pseudorandom orthogonal sequences based on Hadamard matrices. We investigated correlation properties of the proposed sequences by numerical experiments. It is shown that both average and maximum values of ACFs are superior to those of Walsh codes. We also find that the maximum values of CCFs of the proposed sequences are better than those of Walsh codes, though the average values of CCFs are slightly inferior to Walsh codes.

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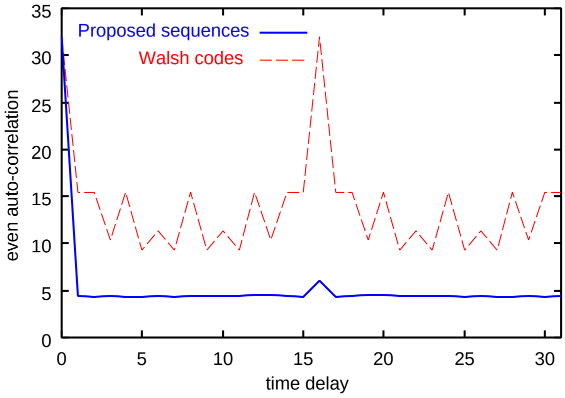

Figure 6: Average values of even autocorrelation.

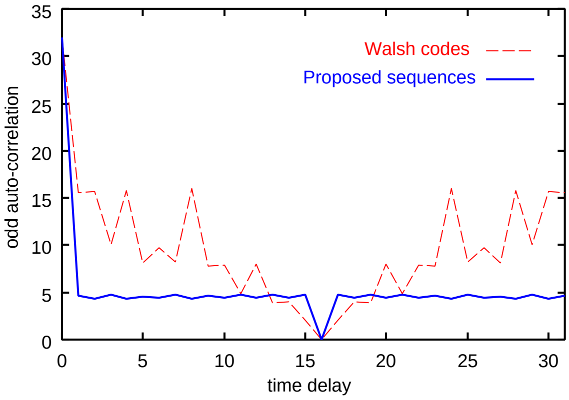

Figure 7: Average values of odd autocorrelation.

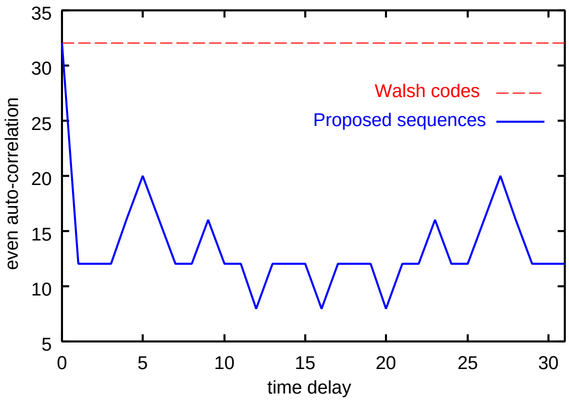

Figure 8: Maximum values of even autocorrelation.

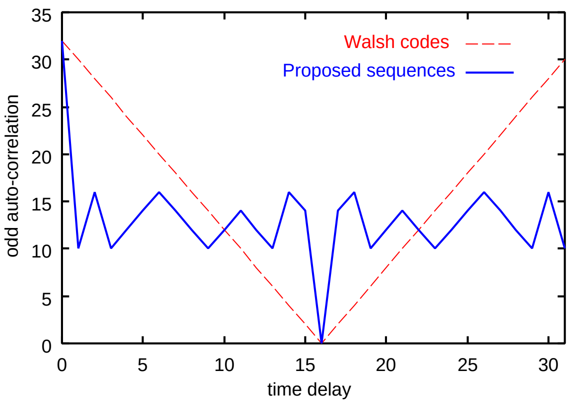

Figure 9: Maximum values of odd autocorrelation.

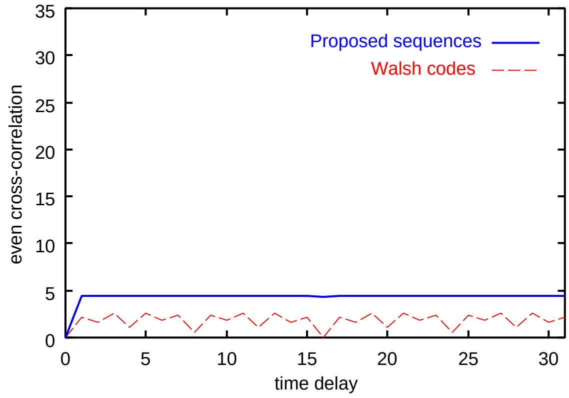

Figure 10: Average values of even corsscorrelation.

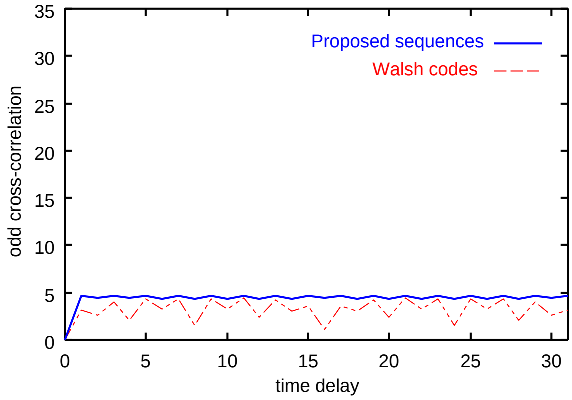

Figure 11: Average values of odd corsscorrelation.

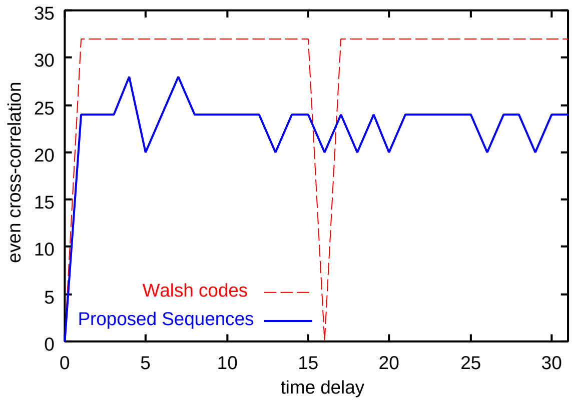

Figure 12: Maxmum values of even corsscorrelation.

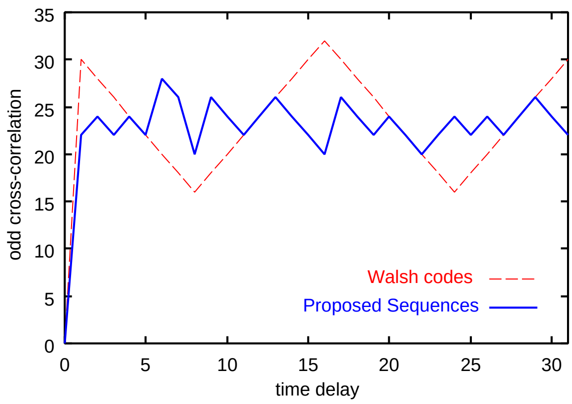

Figure 13: Maximum values of odd corsscorrelation.