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Control of Hyperchaos Using State Feedback

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Abstract—In this paper we present a simple method for controlling hyperchaos with $n \times m$ scrolls into an aimed-equilibrium by utilizing feedback of state. Using the method we can stabilize any fixed point among $n \times m$ unstable embedded-fixed-points.

1. Introduction

In continuous-time systems, when the system order is higher than 3, it is possible to have more than one positive Lyapunov exponent. If this is the case, the corresponding behavior is known as *hyperchaos* [1] . The $n \times m$-scroll attractor presented by D. Cafagna and G.Grassi [2] is one of typical hyperchaos. The attractor is generated by combining the chaotic n-scroll attractor related to the first Chua's circuit with the chaotic m-scroll attractor related to the second Chua's circuit. The $n \times m$-scroll hyperchaos behaves in very complicated manner. Systems with such hyperchaotic behavior are considered to be a good candidate for fundamental cells for dynamic associative memories. For this application, a simple and powerful controlling method for hyperchaos is required.

In this paper, we present a simple method for controlling $n \times m$-scroll hyperchaos into an equilibrium by utilizing a state-feedback approach. Observing the nonlinear function terms in the hyperchaotic system, we stabilize the system into an aimed-fixed point through feedback of time-derivative of the states related to the nonlinear functions. Location of the fixed point does not change from original one without feedback since feedback of time-derivative vanishes at an equilibrium. The proposed method can stabilize any one of $n \times m$ unstable fixed-points in the system.

2. Controlling Chaos by State-Feedback

Chaotic systems are characterized by its nonlinear terms. The feedback method to be proposed here observes the nonlinear terms. Stabilizing the states related to these nonlinear terms, all states in this system are controlled into an equilibrium. Although there are chaos control approaches based on conventional feed back method [4], the proposed method is different from them.

As an example of controlling chaos by the present method, we consider n-scroll chaotic system:

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - f(x_1)) \\ \dot{x}_2 &= x_1 - \gamma\dot{x}_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \tag{1}$$

where f is a nonlinear function of a modified-sine with n zeros with positive-slope . When $\gamma = 0$, the system (1) coincides with the conventional n-scroll system [3]. The equilibria $\mathbf{x}_{eq}$ satisfy $\dot{\mathbf{x}} = 0$, and there are $(2n-1)$ equilibria. Note that from (1), the conponent x_1 of each equilibria $\mathbf{x}_{eq}$ is determined by nonlinear function $f = 0$. Near an equilibrium, if x_1 is forced to satisfy $f = 0$, then the system (1) can be stabilized to the equilibrium. In order to accomplish this situation , in (1), $\gamma\dot{x}_1$ is fed back to the right hand side of the second equation.This feedback acts such that it makes to decrease x_1 when x_1 increases, while it makes to increase x_1 when x_1 decreases. As a result x_1 converges to give $f = 0$ and simultaneously the other components of $\mathbf{x}$ also converge. The converged-equilibrium is the same point with the original equilibrium without feedback since $\dot{\mathbf{x}}_{eq} = 0$.

We are going to stabilize the chaotic system (1) by changing the control parameter γ. The Jacobian matrix $\mathbf{J}$ at an equilibrium is given by

$$\mathbf{J} = \begin{pmatrix} -\alpha\dot{f}(x_{eq}) & \alpha & 0 \\ 1 + \gamma\alpha\dot{f}(x_{eq}) & -1-\gamma\alpha & 1 \\ 0 & -\beta & 0 \end{pmatrix}. \tag{2}$$

Eigenvalues of $\mathbf{J}$ can be found as solutions of the characteristic equation $\det[\lambda\mathbf{I} - \mathbf{J}] = 0$:

$$\det[\lambda\mathbf{I} - \mathbf{J}] = \begin{vmatrix} \lambda + \alpha\dot{f}(x_{eq}) & \alpha & 0 \\ -1-\gamma\alpha\dot{f}(x_{eq}) & \lambda+1+\gamma\alpha & -1 \\ 0 & \beta & \lambda \end{vmatrix} \tag{3}$$

$$= a_0\lambda^3 + a_1\lambda^2 + a_2\lambda^1 + a_3\lambda^0 \tag{4}$$

$$= 0 \tag{5}$$

where

$$a_0 = 1 \tag{6}$$

$$a_1 = 1 + \gamma\alpha + \alpha\dot{f}(x_{eq}) \tag{7}$$

$$a_2 = \alpha\dot{f}(x_{eq}) - \alpha + \beta \tag{8}$$

$$a_3 = \alpha\beta\dot{f}(x_{eq}). \tag{9}$$

We classify characteristics of the equilibria by their eigenvalues as a number of eigenvalues with positive real parts, index r:

- Saddle-type I : Equilibrium for $\frac{df}{dx} < 0$
- Saddle-type II : Equilibrium for $\frac{df}{dx} > 0$

where I and II stand for $r = 1$ and $r = 2$ respectively. For these equilibria, we stabilize the system by suitably setting γ in such a way that all $\Re\lambda$ becomes negative. However, we can not stabilize all the equilibria. From Routh-Hurwitz's stability criterion we can show that only the type-II (with $\frac{df}{dx} > 0$) can be stabilized.

Figure 1 shows an attractor of 3-scrolls and types of equilibria with $\alpha = 10.814, \ \beta = 14, \ n = 3, \gamma = 0$. Figure 2 shows its nonlinear function f with $a = 1.3, \ b = 0.11, \ c = 2$ [3].

Figure 3 shows an attractor of 4-scrolls ($n = 4$) and Figure 4 shows its nonlinear function f .

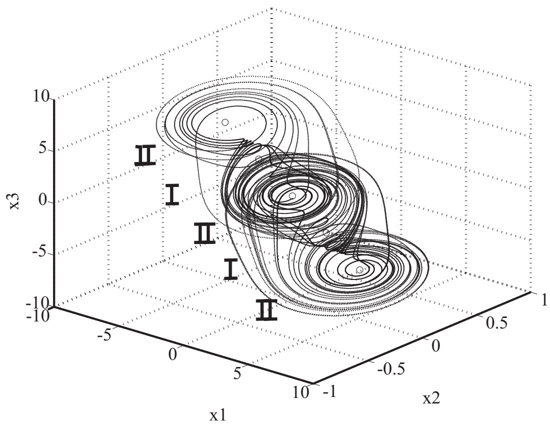


Figure 1: 3-Scroll attractor

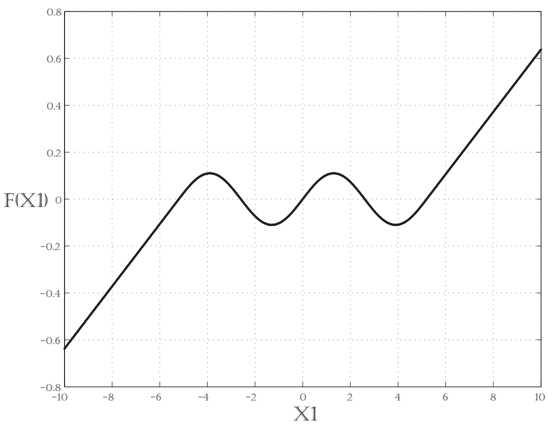


Figure 2: Nonlinear function f for 3-scrolls

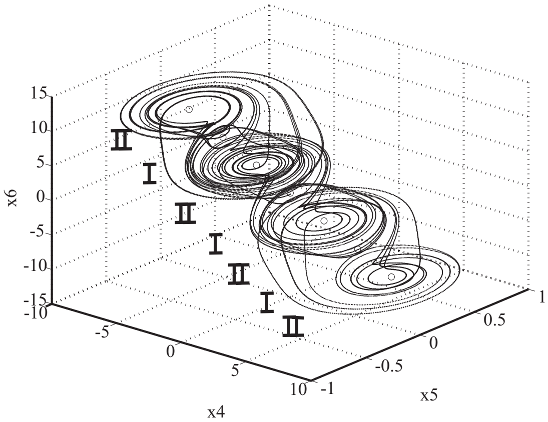


Figure 3: 4-Scroll attractor

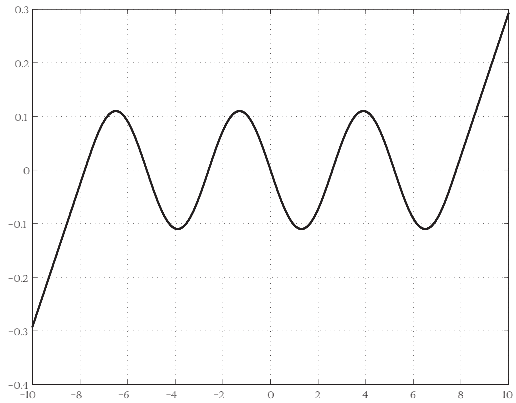


Figure 4: Nonlinear function f for 4-scrolls

We express characteristics of each equilibrium as in the Table 1 where n denotes the number of scrolls and k denotes the sequential number of each equilibrium from origin in the coordinates x_1 , $(k = 0, \pm 1, \pm 2, ...)$.

Table 1: Characteristics of Each Equilibrium

n	k	index r
odd	odd	index 1
	even	index 2
even	odd	index 2
	even	index 1

Table 2 and 3 shows characteristics of each equilibrium for 3-scrolls and 4-scrolls respectively.

Table 2: Characteristics of unstable equilibria for 3-scrolls

k	equilibria $\mathbf{x}_{eq}$	index r
-2	$(-5.2, 0, 5.2)$	index 2
-1	$(-2.6, 0, 2.6)$	index 1
0	$(0, 0, 0)$	index 2
1	$(2.6, 0, -2.6)$	index 1
2	$(5.2, 0, -5.2)$	index 2

Table 3: Characteristics of unstable equilibria for 4-scrolls

k	Equilibria $\mathbf{x}_{eq}$	index r
-3	$(-7.8, 0, 7.8)$	index 2
-2	$(-5.2, 0, 5.2)$	index 1
-1	$(-2.6, 0, 2.6)$	index 2
0	$(0, 0, 0)$	index 1
1	$(2.6, 0, -2.6)$	index 2
2	$(5.2, 0, -5.2)$	index 1
3	$(7.8, 0, -7.8)$	index 2

3. Control of $n \times m$ Scroll Attractors

Hyperchaos with $n \times m$-scrolls[2] can be generated by combining n-scrolls (system 1) with m-scrolls (system 2). Characteristics of equilibria in this system is classified into 3 types as in the following:

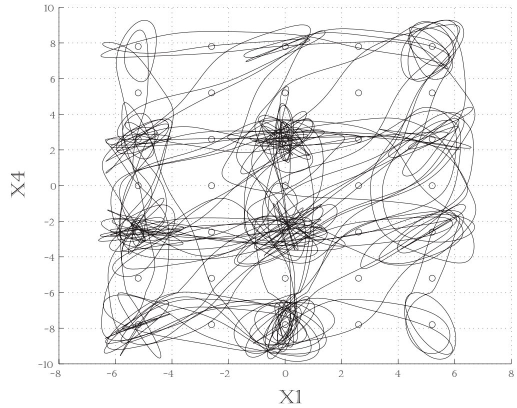
- Saddle-type II : Combination of saddle-type I with saddle-type I
- Saddle-type III : Combination of saddle-type I with saddle-type II
- Saddle-type VI : Combination of saddle-type II with saddle-type II

where each saddle-type II, III, IV has index $r = 2, 3, 4$ respectively.

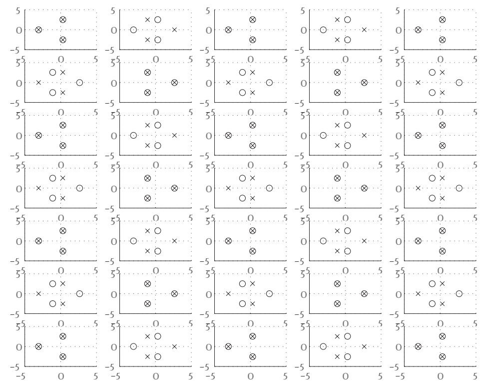
Now we consider to stabilize $n \times m$ scrolls into an equilibrium by applying feedback of state as shown in section 2 for each $n(m)$-scrolls. Referring (1), the system equation is described by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - f(x_1)) \\ \dot{x}_2 &= x_1 - \gamma_1\dot{x}_1 - x_2 + x_3 + H(x_5 - x_2) \\ \dot{x}_3 &= -\beta x_2 \\ \dot{x}_4 &= \alpha(x_5 - g(x_4)) \\ \dot{x}_5 &= x_4 - \gamma_2\dot{x}_4 - x_5 + x_6 + H(x_2 - x_5) \\ \dot{x}_6 &= -\beta x_5. \end{aligned} \tag{10}$$

Figure 5 shows (a) attractors and (b) allocation of eigenvalues for conventional $n{\times}m$ scrolls (without feedback $\gamma_1\dot{x}_1$ and $\gamma_2\dot{x}_2$). Figure 6 shows types of equilibria shown in Figure 5-(b).



(a) Attractor of hyperchaos



(b) Allocation of eigenvalues at equilibria. Abscissa: $\Re\lambda$; Ordinate: $\Im\lambda$; ∘:eigenvalues of system 1; ×:eigenvalues of system 2

Figure 5: Attractor of hyperchaos with 4×3 scrolls

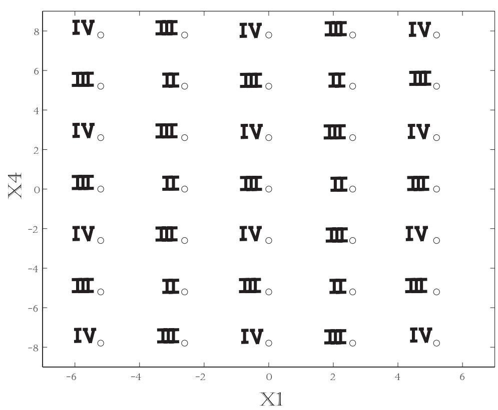
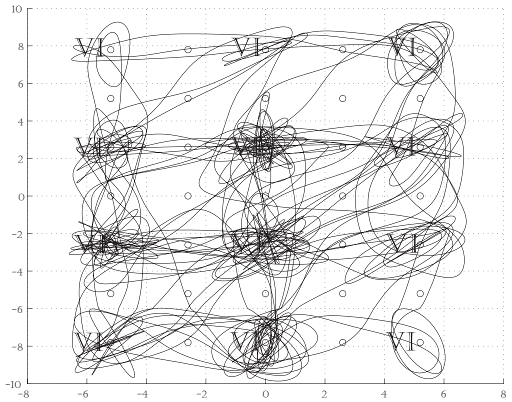
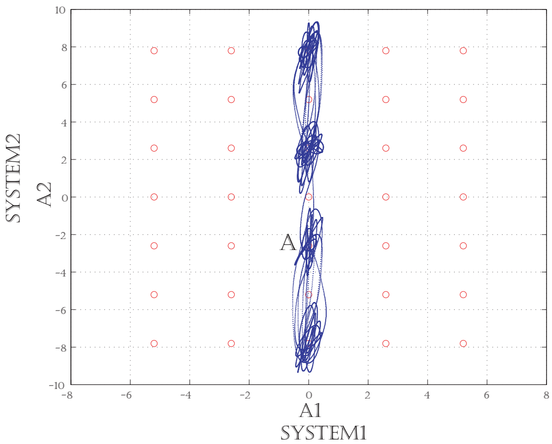


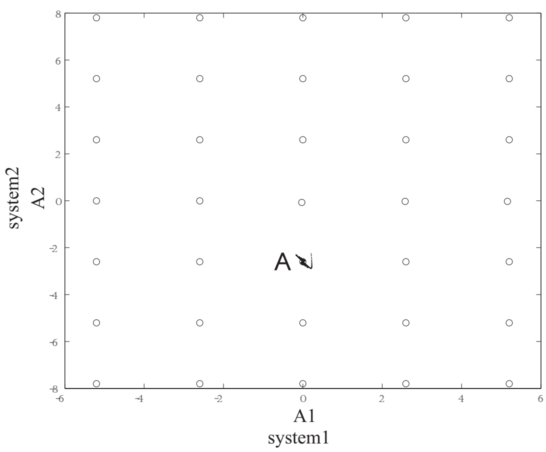
Figure 6: Type of each equilibrium



(a) step 1



(b) step 2



(c) step 3

Figure 7: Stabilzing the hyperchaos into an equilibrium A

In system (10), observing nonlinear functions $f, g,\ \ \gamma_1 \dot{x}_1$ and $\gamma_2 \dot{x}_2$ are fed back to stabilize states x_1 and x_2 related to nonlinear functions. Using the Routh-Hurwitz's stability criterion and near the targetted-equilibrium , control parameters γ_1 and γ_2 are set such that all the real parts of eigenvalues of the Jacobian matrix for system (10) are negative.

In this way the feedback-control is carried out for each subsystem 1 and 2 to stabilize whole system (10). However, in each subsystem, only saddles with type-II can be stabilized. Therefore, as a cobination of these type-II's, only saddles with type-IV are stabilized in the system (10).

Detailed steps for stabilizing into an equilibrium A are given as in the following. Figure 7-(a) shows trajectories of 4×3-scrolls and allocation of type-IV saddles (one of them is the target point A) on the $x_1 - x_4$ plain. The point A consists of conponents A_1 and A_2 which are corresponding equilibria in system 1 and system 2 respectively. When a trajectory of system 1 comes aroud A_1 we begin by changing γ_1 to stabilize A_1 in a way as mentioned above. Then the system (10) behaves as shown in Figure 7-(b). Next, keeping this state, when a trajectory of system 2 comes near A_2 we change γ_2 to stabilize A_2 in the same way above. Consequently, the system(10) is stabilized to the point A as shown in Figure 7-(c).

4. Conclusions

In this work, we have presented a new state-feedback method for controlling $n \times m$-scroll hyperchaos. Time derivative of states related to nonlinear functions are fed back to stabilize type-IV saddles. By simulations, we confirmed that any equilibrium among the $n \times m$ equilibria can be stabilized by sequentialy stabilizing each subsystems using the present method. And also, errors between actually stabilized equilibria and the true ones are convinced to be very small around 10^{-13}.

References

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