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Lossless Image Coding based on Nonlinear Lifting Scheme by Discrete Time Cellular Neural Networks

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Abstract—This paper proposes a new lossless image coding technique based on separable 2D lifting using discrete-time cellular neural networks (DT-CNN). The main feature of the lifting scheme is that it provides a spatial domain interpolation of the transform, as opposed to the conventional construction based on a frequency domain. The advantage of our method is that the nonlinear output function of the DT-CNN is exploited to optimize the image interpolation considering the quantization error. The simulation results show a better coding performance as compared with the conventional methods.

1. Introduction

The discrete-time cellular neural networks (DT-CNN) [1] has been applied to many image processing applications such as image compression. In [2], the nonlinear interpolative dynamics by feedback A-template of the DT-CNN were used for image compression, and the DT-CNN dynamics were introduced to solve the optimal problem by minimizing its Lyapunov's energy function. For lack of the multi resolution analysis, however, the better coding performance was not obtained. In this paper, we propose the multi resolution interpolation based on the lifting scheme using the DT-CNN to obtain the better lossless image coding efficiency.

The lifting scheme [3] is a very general and highly flexible method for constructing biorthogonal wavelets and also provides reversible wavelet transforms for lossless image compression. It provides a spatial domain interpolation of the transform and it can be expanded into hierarchical structure easily. The performance of the lifting scheme depends on the ability of the filters to interpolate the images. In lossless image coding based on the conventional lifting scheme using the linear filters, the degradations are caused by the use of the integer wavelet transform instead of the discrete wavelet transform. For efficient interpolation, the quantization noises propagated by the rounding operations should be considered. In our proposed method, the nonlinear output function of the DT-CNN is exploited to optimize the image interpolation considering the quantization error.

2. Separable 2D Lifting

The lifting scheme is one of popular image coding methods which include the progressive transmission property. Figure 1(a) shows a typical lifting stage which is comprised of three steps: Split, Predict, and Update. In the first step, the original signal $x[n]$ is divided into its even polyphase component $x_e[n]$ and odd polyphase component $x_o[n]$. In the case of the image decomposition, two types of the split process are selected depending on the application of the vertical and horizontal direction, alternatively. In the vertical split step, the signal is divided into the even row coefficients $x_{e1}[n]$ and the odd row coefficients $x_{o1}[n]$ (Figure 2(a)). In the horizontal split step, the signal is divided into the even column coefficients $x_{e2}[n]$ and the odd column coefficients $x_{o2}[n]$ (Figure 2(b)). Generally, the even and odd polyphase components are highly correlated. In the second step, we predict $x_o[n]$ with a linear combination of neighboring $x_e[n]$, and the prediction residuals $d[n]$ is generated,

$$d[n] = x_o[n] - \lfloor P(x_e[n]) \rfloor, \quad (1)$$

where $P(x_e[n])$ indicates a linear combination of neighboring $x_e[n]$, and $\lfloor\ \rfloor$ is the round-off operator. In the third step, we update $x_e[n]$ with a linear combination of $d[n]$,

$$c[n] = x_e[n] + \lfloor U(d[n]) \rfloor, \quad (2)$$

where $U(d[n])$ indicates a linear combination of neighboring $d[n]$. By applying these three processes to $c[n]$ repeatedly, multi-resolution analysis is performed.

The lifting decomposition formulas (1) and (2) are invertible and the inverse is given by

$$x_e[n] = c[n] - \lfloor U(d[n]) \rfloor, \quad (3)$$

$$x_o[n] = d[n] + \lfloor P(x_e[n]) \rfloor. \quad (4)$$

These are called reconstruction formulas. Figure 1(b) shows a reconstruction lifting stage.

Figure 3 shows the band-splitting characteristic of the separable 2D lifting, which is similar to that of the wavelet transform based on a frequency domain.

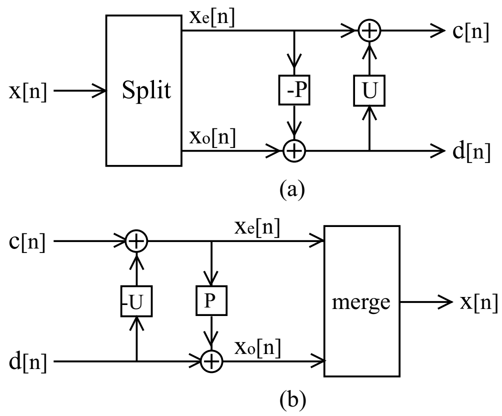

Figure 1: Lifting stages: (a) Decomposition and (b) Reconstruction.

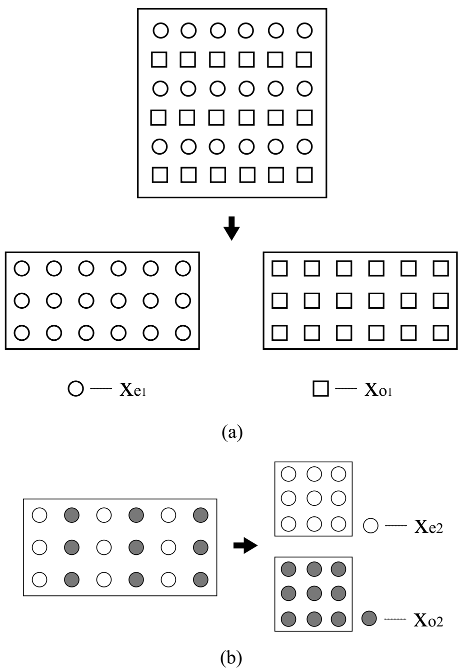

Figure 2: Split steps: (a) Vertical split and (b) Horizontal split.

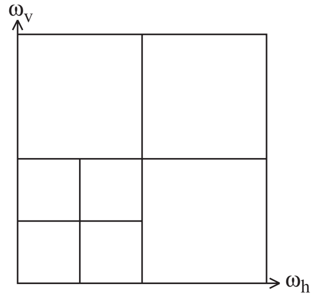

Figure 3: Band-splitting characteristic of separable 2D lifting.

3. Separable 2D Lifting using Two-Layered DT-CNN

3.1. DT-CNN

The block diagram in Figure 4 shows the DT-CNN. The state equation of the DT-CNN is described in matrix form as

$$\mathbf{x_{n+1}} = \mathbf{Af(x_n)} + \mathbf{Bu} + \mathbf{T}, \quad (5)$$

$$\mathbf{y_{n+1}} = \mathbf{f(x_{n+1})}, \quad (6)$$

where $\mathbf{u}$ is an input vector, $\mathbf{y}$ is an output vector, $\mathbf{x}$ is a state variable, $\mathbf{f}()$ is a multilevel quantizing function, $\mathbf{T}$ is a constant vector, and $\mathbf{A}$ and $\mathbf{B}$ are feedback and feed-forward template matrices respectively. The output function $\mathbf{f}()$ corresponds to the rounding operation for the lifting scheme. The output function plays an important role that the interpolation is optimized considering the nonlinearity caused by the quantization noises. The Lyapunov's energy function E_t of the DT-CNN is defined by

$$E_t = -\frac{1}{2}\mathbf{y}^t(\mathbf{A} - \xi\mathbf{I})\mathbf{y} - \mathbf{y}^t\mathbf{Bu} - \mathbf{T}^t\mathbf{y}, \quad (7)$$

where ξ is a positive constant value to determine the quantization region. It is proved that the Lyapunov's energy function is a monotone decreasing function if the $\mathbf{A}$ matrix is symmetric and its diagonal elements are larger than 0. In order to obtain the high accuracy prediction of the odd polyphase image, it is necessary that the even polyphase image can be reconstructed based on the distortion defined by

$$dist(y, u) = \left\| \frac{1}{2}\mathbf{y}^t(\mathbf{Gy} - \mathbf{u}) \right\|, \quad (8)$$

where $\mathbf{G}$ is a Gaussian filter. This distortion means that not only output value $\|\mathbf{y}\|$ but also the difference $\|\mathbf{Gy} - \mathbf{u}\|$ between the interpolative predicted image and the input even polyphase image should be small. By the comparison between (7) and (8), the template and the parameter of the first layer DT-CNN equations can be determined.

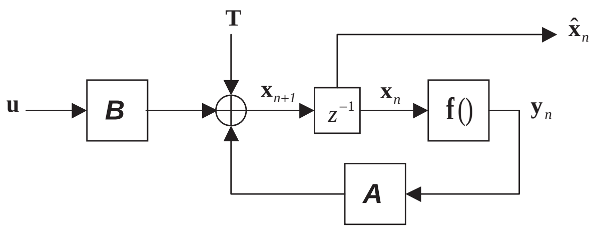

Figure 4: Block diagram of DT-CNN.

3.2. Lifting Scheme using Two-Layered DT-CNN

As shown in Figure 5, the even polyphase image is set to the input of the cell and the equilibrium output y^e_{ij} is obtained after the transition of the network. The template of the first layer DT-CNN equations are determined as

$$\mathbf{A} = A(i,j;k,l), \qquad C(k,l) \in N_r(i,j) \tag{9}$$
$$= \begin{cases} -(1+\lambda) \\ \qquad \text{if } k=i \text{ and } l=j, \\ -\dfrac{1}{2\pi\sigma^2}\exp\left(-\dfrac{\{(k-i)^2+(l-j)^2\}d_m^2}{2\sigma^2}\right) \\ \qquad \text{otherwise,} \end{cases}$$

$$\mathbf{B} = B(i,j;k,l), \qquad C(k,l) \in N_r(i,j) \tag{10}$$
$$= \begin{cases} 1 & \text{if } k=i \text{ and } l=j\ , \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathbf{T} = \mathbf{0}, \tag{11}$$

where $N_r(i,j)$ is the r-neighborhood of cell $C(i,j)$ as $N_r(i,j) = \{C(k,l)|\max\{|k-i|,|l-j|\} \le r\}$, λ is a regularization parameter, σ is the standard derivative of the Gaussian function, and d_m is a sampling interval.

Next, the output y^e_{ij} of the first layer DT-CNN becomes the input of the second layer DT-CNN which has no dynamics. In the second layer DT-CNN, the predicted value $\hat{y}_{ij}$ of the odd polyphase image is acquired from the $\hat{B}$ template which is obtained by extending A template of the first layer DT-CNN spatially (Figure 6).

$$\hat{y}_{ij} = \sum_{y_{kl} \in N'_r(i,j)} \hat{B}(i,j;k,l)y^e_{ij}. \tag{12}$$

At the odd stages, the $\hat{B}$ template which is obtained by extending A template vertically is used,

$$\hat{B}(i,j;k,l)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{((k-0.5)d_m-i)^2+(l-j)^2}{2\sigma^2}\right),$$
$$N'_r(i,j)=\{C(k,l)|\ |(k-0.5)d_m-i| \le rd_m,\ |l-j| \le r\}, \tag{13}$$

and at the even stages, the $\hat{B}$ template which is obtained by extending A template horizontally is used,

$$\hat{B}(i,j;k,l)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{(k-i)^2+((l-0.5)d_m-j)^2}{2\sigma^2}\right),$$
$$N'_r(i,j)=\{C(k,l)|\ |k-i| \le r,\ |(l-0.5)d_m-j| \le rd_m\}. \tag{14}$$

The block diagram of the lossless coding system based on the lifting scheme using DT-CNN interpolation is shown in Figure 7. At the split stage, the original image u_1 is divided into even polyphase components u_{e1} and odd polyphase components u_{o1}. The prediction for each u_{on} is designed by the two-layered DT-CNN. Then we obtain the prediction residual e_n, which is transmitted to the decoder. In the encoder, these lifting processes using the DT-CNN are applied to the even polyphase image iteratively. In the decoder, the same processes are applied, and the reconstruction image is gradually improved by adding the difference image to the interpolated components.

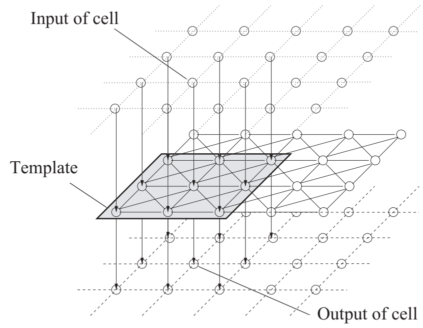

Figure 5: First layer DT-CNN.

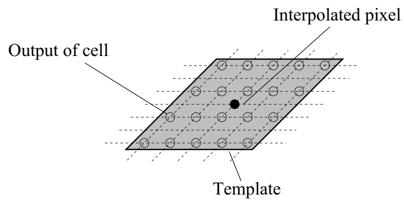

Figure 6: Second layer DT-CNN.

4. Simulation Results

We implemented the coder and decoder of our proposed lossless coding algorithm based on the lifting scheme using the DT-CNN. We present the results of our algorithm, using the following gray-scale standard test images: " Lena ", " Aerial ", " Boat ", " Barbara. "A size of all test images is 512×512. According

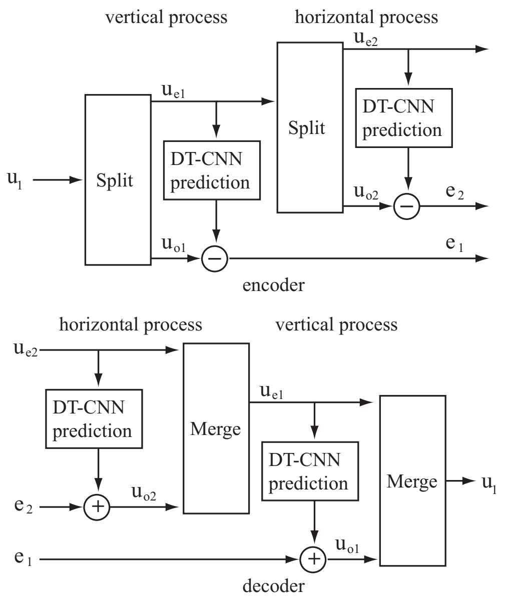

Figure 7: Proposed lossless image coding system.

to the simulations, the coding factor is decided as follows; the depth of the lifting L=4, the r-neighborhood is 2, the regularization parameter λ = -1 and the standard deviation of Gaussian $\sigma = 0.5$. Moreover, in consideration of the influence of the quantization error propagated by the rounding operations, the horizontal interpolation is applied to only the divided even components. The band splitting characteristic of this case is shown in Figure 8. In order to evaluate the performance of filters, it is necessary to compare each method under the same band-splitting characteristic. The performance of the proposed method was compared with the separable 2D lifting method using Le Gall 5-tap/3-tap filter and S+P transform [4]. Table 1 shows the energy of the difference images for each level, and Table 2 shows the entropy of the encoded image. The simulation results show that our method has a better coding performance compared with the conventional methods.

5. Conclusion

The lossless image coding based on separable 2D lifting using DT-CNN has been proposed. In our method, the optimal interpolation considering the round-off operator in the lifting scheme was realized by introducing the two-layered DT-CNN. The simulation results show that our method had good coding performance compared with the conventional methods. In the future, we will develop the optimal update process for accurate interpolation and implement our system in the hardware to accelerate the conversion of the DT-CNN.

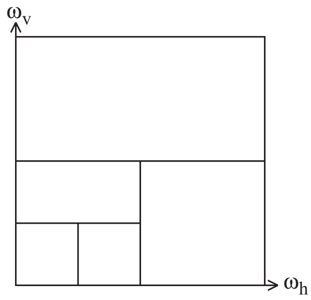

Figure 8: Band splitting characteristic of our method.

Table 1: Energy of the difference images for each level.

Image	method	1Level	2Level	3Level	4Level
Lena	Proposed	**26.79**	**109.7**	**287.7**	**503.3**
	5/3	28.11	131.0	394.5	693.5
Aerial	Proposed	127.4	**495.2**	**1034**	**1563**
	5/3	**119.2**	564.9	1273	1755
Boat	Proposed	**69.25**	**220.2**	**486.5**	787.0
	5/3	77.70	223.9	497.2	**630.7**
Barbara	Proposed	**190.2**	**533.5**	722.7	**916.7**
	5/3	212.5	586.0	**692.8**	969.0

Table 2: Entropy of the encoded image (bit/pixel).

Image	Lena	Aerial	Boat	Barbara
Proposed	**4.47**	**5.41**	**5.11**	**5.31**
5/3	4.50	5.44	5.15	5.35
S+P	4.50	5.54	5.12	**5.31**

References

[1] L. O. Chua and L. Yang," Cellular neural networks: theory, "*IEEE Trans. Circuits Syst.*, vol. 35, no. 10, pp. 1257-1272, 1988.

[2] N. Takahashi, T. Otake and M. Tanaka," Nonlinear interpolative effect of feedback template for image processing by DT-CNN, "*Journal of Circuits, Systems, and Computers, World Scientific Publishing Company*, Vol. 12, No. 4, Aug.2003.

[3] W. Sweldens," The lifting scheme: A custom-design construction of biorthogonal wavelets, " *Appl., Comput. Harmon. Anal.*, vol. 3, no. 2, pp. 186-200, 1996.

[4] A. Said and W. A. Pearlman," An image multiresolution representation for lossless and lossy compression, "*IEEE Trans. Image Processing*, vol. 5, no. 9, pp. 1303-1310, 1996.