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Limit cycle oscillation of induction motor drive and control

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Abstract– Open loop V/f controlled induction motor drive suffers an unfavorable sustained oscillation problem usually under light load and at low frequency, and therefore limits the applications of these drives. In this paper, a nonlinear mathematical model of IM system is set up to study the problem, so that the limit cycle oscillation of rotor speed caused by dead time of inverter is evident. Moreover, based on the aforementioned results, a discontinuous space vector DPWM3 modulate strategy is proposed through software implementation. The control of limit cycle oscillation is numerically discussed.

1. Introduction

With the introduction of solid state inverters, three phase induction motor (IM) drive systems fed by V/f PWM voltage source inverter become popular. The great majority of operating variable speed drives are of this type today, they have often experienced unexpected sustained oscillations in low speed range and in light load conditions [1]. Although oscillations have been observed and analyzed [2-5], few papers deal with the improvement of stability of the V/f controlled IM drive system.

In this paper, a nonlinear mathematical model of IM is set up, allowing for the effect of main magnetic circuit saturation. The study shows the oscillatory states depend on the dead time of PWM inverter, and the rotor speed motion has limit cycles. Finally, DPWM3 modulate strategies are presented possibility to control the limit cycles of rotor speed without any additional hardware modification. Simulated results demonstrate the effectiveness of the control.

2. Mathematical model of IM system

2.1. Model of IM

Assuming magnetization curve of IM is described by the following formula:

$$I_m = K_1\lambda_m + K_3\lambda_m^3 + K_5\lambda_m^5 \ , \tag{1}$$

where I_m denotes magnetizing current and λ_m magnetizing linkage.

The voltage matrix equation of IM, fixed in stator d-q reference frame, is given by:

$$\begin{bmatrix} U_{q1} \\ U_{d1} \\ U_{q2} \\ U_{d2} \end{bmatrix} = [Z_1]\begin{bmatrix} I_{q1} \\ I_{d1} \\ \lambda_{mq} \\ \lambda_{md} \end{bmatrix} + [Z_2]P\begin{bmatrix} I_{q1} \\ I_{d1} \\ \lambda_{mq} \\ \lambda_{md} \end{bmatrix}, \tag{2}$$

$$[Z_1] = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & r_1 & 0 & 0 \\ -r_2 & L_{2\sigma}\omega_r & K_1 r_2 + r_2 Y & -K_1 L_{2\sigma}\omega_r - \omega_r - L_{2\sigma}\omega_r Y \\ -L_{2\sigma}\omega_r & -r_2 & K_1 L_{2\sigma}\omega_r + \omega_r + L_{2\sigma}\omega_r Y & K_1 r_2 + r_2 Y \end{bmatrix},$$

$$[Z_2] = \begin{bmatrix} L_{1\sigma} & 0 & 1 & 0 \\ 0 & L_{1\sigma} & 0 & 1 \\ -L_{2\sigma} & 0 & K_1 L_{2\sigma} + 1 + L_{2\sigma}A & L_{2\sigma}B \\ 0 & -L_{2\sigma} & L_{2\sigma}B & K_1 L_{2\sigma} + 1 + L_{2\sigma}C \end{bmatrix},$$

where r denotes resistor, L_σ leakage impedance, ω rotor electrical angular velocity, and λ_m magnetic flux linkage. Subscripts 1 and 2 correspond to stator and rotor. P depicts differential operator d/dt. Moreover,

$$A = 2K_3\lambda_{mq}^2 + 4K_5\lambda_{mq}^4 + 4K_5\lambda_{mq}^2\lambda_{md}^2 + K_3\lambda_m^2 + K_5\lambda_m^4,$$

$$B = 2K_3\lambda_{mq} + 4K_5\lambda_{md}\lambda_{mq}^3 + 4K_5\lambda_{md}^3\lambda_{mq} \ ,$$

$$C = 2K_3\lambda_{md}^2 + 4K_5\lambda_{mq}^4 + 4K_5\lambda_{mq}^2\lambda_{md}^2 + K_3\lambda_m^2 + K_5\lambda_m^4,$$

$$Y = K_3\lambda_m^2\lambda_m^4 \ ,$$

$$\lambda_m^2 = \lambda_{md}^2 + \lambda_{mq}^2,$$

$$\lambda_m^4 = \lambda_{md}^4 + 2\lambda_{md}^2\lambda_{mq}^2 + \lambda_{mq}^4 \ ,$$

are satisfied.

Allowing for the effect of main magnetic circuit saturation, the mathematical model of IM is in the form [5]:

$$\begin{cases} P\begin{bmatrix} I_1 \\ \lambda \end{bmatrix} = -[Z_2]^{-1}[Z_1]\begin{bmatrix} I_1 \\ \lambda \end{bmatrix} + [Z_2]^{-1}[U], \\ P\omega_r = n(T_e - T_l)/J, \\ T_e = \frac{3}{2}n(I_{qs}\lambda_{md} - I_{ds}\lambda_{mq}), \end{cases} \tag{3}$$

where n denotes pole pairs, T_e electromagnetic torque, J moment of inertia of motor, and T_l load torque respectively.

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2.2. Model of inverter

We assume switch functions of inverter are $S_1(\omega t)$, $S_3(\omega t)$, and $S_5(\omega t)$. The phase voltages of inverter (output phase of inverter with respect to the fictions center point of the dc supply) are figured by U_{ao}, U_{bo} and U_{co}, and U_D denotes the DC-link voltage. They are derived as follows [6]:

$$U_{ao}(\omega t) = U_D S_1(\omega t),$$
$$U_{bo}(\omega t) = U_D S_3(\omega t),$$
$$U_{co}(\omega t) = U_D S_5(\omega t),$$

Taking the dead time into consideration, the deviation of U_a from the exact potential U_{ao} becomes pulse-wise voltage. Then the deviation pulse $U_{a\varepsilon} = U_{ao} - U_a$ has the following features:

1) Constant height is equal to dc source voltage U_D
2) Pulse width is always kept at t_d
3) Polarity of pulse depends on the polarity of motor current.

The voltage matrix U is expressed as:

$$[U] = \begin{bmatrix} U_{q1} \\ U_{d1} \\ U_{q2} \\ U_{d2} \end{bmatrix} = \begin{bmatrix} \frac{1}{3}(U_{ab} - U_{ca}) \\ -\frac{\sqrt{3}}{3} U_{bc} \\ 0 \\ 0 \end{bmatrix}, \qquad (4)$$

Taking main magnetic circuit saturation of IM and dead time of inverter into consideration, formulas (3) and (4) address nonlinear mathematical model of IM drive system controlled by SPWM modulate strategy.

3. Simulation results

The parameters of system are as follows: r_1=0.2687Ω, r_2=0.2294Ω, $L_{1\sigma}$=1.7316 mH, $L_{2\sigma}$ = 3.059 mH, J=0.125 $\mathrm{kg.m^2}$, $T_l = 0$, Inverter output frequency $f = 10\mathrm{Hz}$, dead time $t_d = 20\mu s$, and carrier frequency $f_c = 900\mathrm{Hz}$.

Figure 1 shows results from acceleration response of IM system without dead time, the system is stable, such as a little pulsation of rotor speed, inexistence of distorted stator current and a little torque pulsation.

Figure 2 shows results from acceleration response of IM system with dead time, the system is instable, such as the sustained rotor speed oscillation, distorted stator current, and large-scale torque pulsation. Moreover, limit cycles of rotor speed are evident (see Fig.3).

As we know, existence of the dead time will bring dead time effect; dead time effect corresponds to increasing resistance of stator windings, and the unstable range of open loop V/f controlled induction motor drive will expand with the stator resistance increasing [7]. So, a conclusion can be reached that limit cycle oscillation of rotor speed is caused by dead time of inverter.

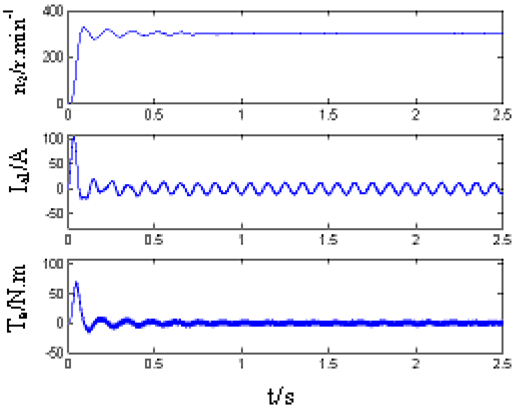

Figure.1 Acceleration responce without dead time

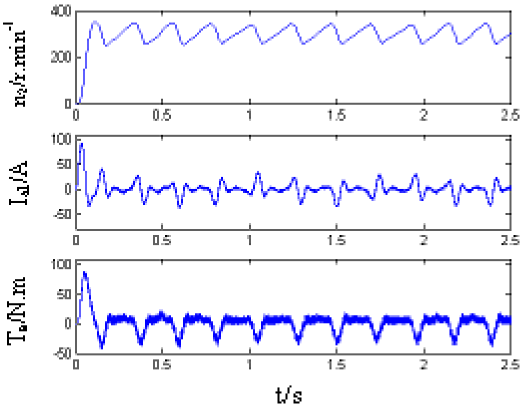

Figure.2 Acceleration responce with dead time

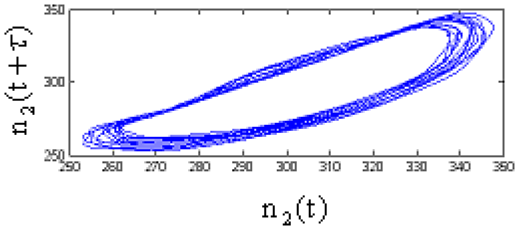

Figure.3 Limit cycles of rotor speed

4. Discontinuous DPWM3 modulate strategy

We assume that the three phase sinusoidal modulate waves are expressed as:

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$$\begin{cases} U_a^* = M\cos(\omega t), \\ U_b^* = M\cos(\omega t - 120^\circ), \\ U_c^* = M\cos(\omega t + 120^\circ), \end{cases}$$

where M denotes modulate factor.

Rotary transform is implemented to the three phase sinusoidal modulate waves. According to power factor angle ϕ of load, the following is derived.

$$U = U^* e^{j\phi'} \tag{6}$$

The zero-sequence signal of DPWM3 modulate strategy is given as follows:

$$\begin{aligned} &U_{min} + U_{max} \ge 0 \Rightarrow U_o = U_t/2 - U^*_{max}, \\ &U_{min} + U_{max} < 0 \Rightarrow U_o = -U_t/2 - U^*_{min}, \end{aligned} \tag{7}$$

where

$$U^*_{max} = \max(U^*_a, U^*_b, U^*_c),$$

$$U^*_{min} = \min(U^*_a, U^*_b, U^*_c),$$

$$U_{max} = \max(U_a, U_b, U_c),$$

Here, $U_{min} = \min(U_a, U_b, U_c)$, U_t corresponds to triangular carrier amplitude.

When the IM system has no load, power factor $\phi = 90^\circ$, we select $\phi' = 60^\circ$, then DPWM3 modulate strategy is brought (see Fig. 4).

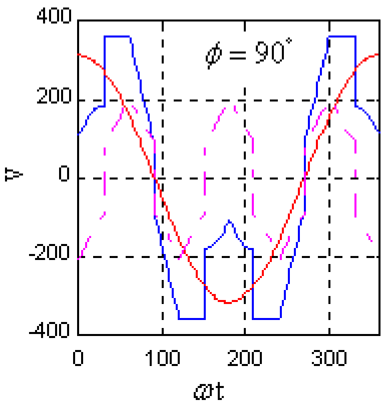

Figure.4 DPWM3 modulate strategy

5. Control limit cycles of rotor speed

When an inverter operates in condition of discontinuous PWM modulate strategy, sectors of two 60° times have not switch in per phase and per cycle, therefore have not dead time effect, DPWM modulate strategy can effectively improve stability of the V/f controlled IM drive system. Figure 5 shows results for acceleration response of IM system using DPWM3 modulate strategy. Figure 6 shows the trajectory of rotor speed. A conclusion can be reached that, under no load operating condition, limit cycle oscillation of rotor speed caused by the dead time of inverter can be effectively suppress using DPWM3 modulate strategy.

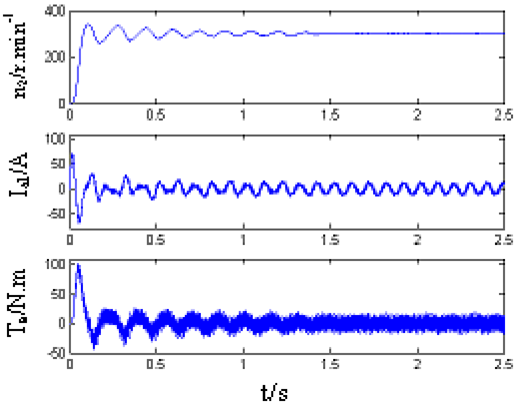

Figure.5 Acceleration response using DPWM3 modulate strategy

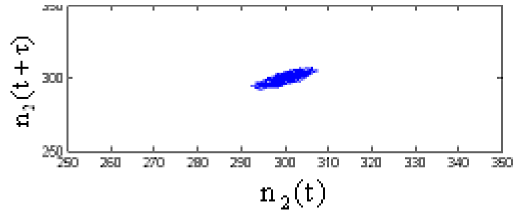

Figure.6 Embedded map using DPWM3 modulate strategy

6. Conclusion

Limit cycle oscillations of rotor speed are substantially caused by the dead time, when an IM drive operates in low frequency condition. In this paper, without any hardware modification, DPWM3 modulate strategy possibly controls the unfavorable rotor speed limit cycles under no load operation condition. Simulated results are presented to demonstrate the effectiveness.

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