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Design of Extremum Seeking Control Adding an Accelerator to Krstić Approach

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Abstract—In this paper we present an extremum seeking control law for nonlinear systems. This is a modification of Krstić type approach. It is equipped with an accelerator to the original one aimed at achieving the maximum operating point more rapidly. This accelerator is designed by making use of a polynomial identification of an uncertain output map and Butterworth filter to smoothen the control. Numerical experiments show how this modified approach can be well in control of Monod model of bioreactors.

1. Introduction

An extremum seeking control problem is classified in a category of adaptive control problems. Mainstream methods of adaptive control deal only with regulation to known set points or reference trajectories. In many applications, however, the set point should be selected to achieve a maximum of an uncertain output map for nonlinear systems. This is one of typical problems of the extremum seeking control[1–6,9]. This kind of control problem can be shown in maximizing the yield of a desired product in chemical engineering and biotechnology[5,6,9], adjusting the spark ignition angle of an automotive engine[2], controlling a chip refiner motor[3], and so on. In all applications, it is desirable to have rapid response to the maximum operating point.

For this extremum seeking control problem, Krstić et al.[4,5,9] developed a feedback mechanism without requiring the knowledge of a plant dynamic equation from the concept of frequency domain. This Krstić approach is easy to implement to practical systems, but needs a longer time to reach the best operating point.

Another approach was presented by Takata et al.[6] from the concept of time domain using the modern control theories. This has the strongpoint of quick response, but needs the knowledge of a dynamic equation.

In this paper we consider a modification of Krstić approach which is equipped with an accelerator for the extremum seeking control problems. It is aimed at shortening a period until the optimal operating point. This accelerator is designed by making use of Chebyshev polynomial identification to estimate the uncertain output map, and Butterworth filter to smooth down the violent movement of control amounts. The proposed approach is applied to Monod model of bioreactors. Simulation results show that this enables to regulate the object around the best operating point speedily.

2. Problem Statement

We consider single-input-single-output systems of the form:

$$\dot{x}(t) = f(x(t), \alpha, u(t)) \tag{1}$$

$$y(t) = h(x(t), u(t)) \tag{2}$$

where $\bullet = d/dt$, $x \in R^n$ is the state, $u \in R$ is the control, $y \in R$ is the output, $\alpha \in R^L$ is the unknown parameter, and $f : R^{n+L+1} \longrightarrow R^n$ and $h : R^{n+1} \longrightarrow R$ are the unknown nonlinear smooth functions.

The performance function J is assumed to be the output equilibrium map such that

$$J(u) = h(z, u) \tag{3}$$

where $\dot{z} = f(z, \alpha, u) = 0, z \in R^n$.

The aim of this problem is to develop a feedback mechanism which enables to operate around the maximum point of the performance swiftly.

3. Extremum Seeking Control by Krstić Approach

Krstić approach could be designed from the following basic idea and its feedback scheme is shown in Fig.1 (see[4][5][9]).

It is impossible to conclude that a certain point is a maximum without visiting the neighborhood on both sides of the maximum. For this reason, this scheme employs a slow perturbation $\beta sin\omega t$ which is added to the control signal $\hat{u}$. The persistent nature of $\beta sin\omega t$ may be undesirable but is necessary to maintain a maximum in the face of changes in functions f and h.

The perturbation $\beta sin\omega t$ will create a periodic response of y. The high-pass filter $s/(s + \omega_h)$ would eliminate the DC component of y. Then, the product of the sinusoids $\beta sin\omega t$ produces $\beta^2/2(1 + cos2\omega t)$, and its DC component $\xi \propto \beta^2/2$ is extracted by the low-pass filter $\omega_l/(s + \omega_l)$. The sign of this ξ provides the direction to the integrator $\hat{u} = K\xi/s$ moving $\hat{u}$ towards the optimal operating point

u^*. By this way, the output y gradually approaches to the maximum output value $y^* = J(u^*)$.

Although it has the merit of easy implementation to practical systems, this Krstić approach usually needs a longer time to reach the optimal point u^*, namely, the maximum output y^*. We will consider a modification of this controller to shorten a reaching time in the next section.

4. Controller with Accelerator

Our feedback scheme is added an accelerator to the original structure and is shown in Fig.2.

Let Δ be a sampling period, so $y(k\Delta)$ and $u(k\Delta)$ are sampled values at time $t = k\Delta$ $(k = 1, 2, 3, \cdots)$. The search for the optimal point u^* is repeatedly executed at renewal times $t = \ell T = \ell M\Delta$ $(\ell = 1, 2, 3, \cdots)$ where T is a renewal cycle period and $M = T/\Delta$ is a natural number.

Let us collect the data $\{y(k\Delta)\}$ on the assumption that the output $y(k\Delta)$ is near to the performance value $J(u(k\Delta))$, because the state x approaches to the stable equilibrium point z as the control progresses:

$$y(k\Delta) = h(x, u(k\Delta)) \approx h(z, u(k\Delta)) = J(u(k\Delta)).$$

Thus it follows that

$$y(k\Delta) = J(u(k\Delta)) + w_1(k\Delta)$$

where w_1 is error.

At $t = \ell T = \ell M\Delta$, the following procedure shall be executed.

4.1. Polynomial Identification

We interpolate the performance function curve via Chebyshev polynomials [6][7] up to the N-th order using the data $\{y(k\Delta), u(k\Delta) : k = k_{\ell(start)}, \cdots, \ell M\}$.
Let the control domain be $D = [u_{min}, u_{max}]$. To transform into a standard domain $D_0 = [-1, 1]$, introduce a normalizing function:

$$\eta(u) = \frac{(u-m)}{p} \tag{4}$$

where $\eta : D \to D_0$, $m = (u_{max}+u_{min})/2$, $p = (u_{max}-u_{min})/2$. The Chebyshev polynomials are then defined by

$$\Phi_r(u) = \cos(r \cdot \cos^{-1}\eta(u)) \quad (r = 0, 1, 2, \cdots)$$

or

$$\begin{aligned} \Phi_0(u) &= 1 \\ \Phi_1(u) &= \eta(u) \\ \Phi_2(u) &= 2\eta^2(u) - 1 \\ \Phi_3(u) &= 4\eta^3(u) - 3\eta(u) \\ \Phi_4(u) &= 8\eta^4(u) - 8\eta^2(u) + 1 \\ \Phi_5(u) &= 16\eta^5(u) - 20\eta^3(u) + 5\eta(u) \\ &\vdots \end{aligned} \tag{5}$$

Assume that the performance function is described at $t = \ell T$ by

$$\begin{aligned} J(u) &= \boldsymbol{\Phi}(u)^T \boldsymbol{C}_\ell + w_2 \\ &= C_{\ell 0} + C_{\ell 1}\Phi_1(u) + C_{\ell 2}\Phi_2(u) + \cdots + C_{\ell N}\Phi_N(u) \\ &\quad + w_2 \end{aligned}$$

so that

$$\begin{aligned} y &= J(u) + w_1 \\ &= \boldsymbol{\Phi}(u)^T \boldsymbol{C}_\ell + w \end{aligned}$$

where

$$\begin{aligned} &\boldsymbol{C}_\ell = [C_{\ell 0}, C_{\ell 1}, C_{\ell 2}, \cdots, C_{\ell N}]^T \\ &\boldsymbol{\Phi}(u) = [1, \Phi_1(u), \Phi_2(u), \cdots, \Phi_N(u)]^T \\ &w = w_1 + w_2 \\ &w_2 \text{ is error.} \end{aligned}$$

By applying the least squares method, the coefficient $\boldsymbol{C}_\ell$ is estimated as

$$\hat{\boldsymbol{C}}_\ell = \left[\sum_{k=k_{\ell(start)}}^{\ell M} \boldsymbol{\Phi}(u(k\Delta))\boldsymbol{\Phi}(u(k\Delta))^T\right]^{-1}\left[\sum_{k=k_{\ell(start)}}^{\ell M} \boldsymbol{\Phi}(u(k\Delta))y(k\Delta)\right] \tag{6}$$

in which the squares error:

$$\sum_{k=k_{\ell(start)}}^{\ell M} w(k\Delta)^2 = \sum_{k=k_{\ell(start)}}^{\ell M} (y(k\Delta) - \boldsymbol{\Phi}(u(k\Delta))^T \boldsymbol{C}_\ell)^2$$

is minimized.

Therefore, we approximate the performance function at $t = \ell T$ as

$$\begin{aligned} \hat{J}_\ell(u) &= \boldsymbol{\Phi}(u)^T \hat{\boldsymbol{C}}_\ell \\ &= \hat{C}_{\ell 0} + \hat{C}_{\ell 1}\Phi_1(u) + \hat{C}_{\ell 2}\Phi_2(u) + \cdots + \hat{C}_{\ell N}\Phi_N(u). \end{aligned} \tag{7}$$

4.2. Estimation of Maximum Point

Let u_ℓ^* be estimate of the optimal operating point u^* at $t = \ell T$.

We search for the maximum point of the performance function $\hat{J}_\ell(u)$ by a step-by-step method as follows.

$$\hat{J}_\ell(u_\ell^*) = \max_u\{\hat{J}_\ell(u) : u = p(2j/L - 1) + m, \; j = 0, 1, 2, \cdots, L\} \tag{8}$$

where L is the number of division of $D_0 = [-1, 1]$.

In a special case of $N = 2$, the u_ℓ^* is analytically solved as follows.

From Eqs.(4) and (5), Eq.(7) becomes

$$\hat{J}_\ell(u) = \hat{C}_{\ell 0} + \hat{C}_{\ell 1}(u-m)/p + \hat{C}_{\ell 2}(2(u-m)^2/p^2 - 1) \tag{9}$$

so that $\partial\hat{J}_\ell(u)/\partial u = 0$ derives

$$u_\ell^* = m - \frac{p\hat{C}_{\ell 1}}{4\hat{C}_{\ell 2}}. \tag{10}$$

4.3. Smoothing by Butterworth Filter

Note that $\hat{u} = K\xi/s$ is the principal control of Krstić type in section 3.
Let $\hat{\mu}(\ell T)$ be the difference between u_ℓ^* and this principal control $\hat{u}(\ell T)$:

$$\hat{\mu}(\ell T) = u_\ell^* - \hat{u}(\ell T) \qquad (\text{at } t = \ell T). \tag{11}$$

To smooth down its violent movement, we introduce Butterworth filter[8] reprersented by

$$H(s) = \prod_{i=1}^{q} 1/(s/\omega_c - \exp(j\pi[1 + (2i-1)/q]/2)) \tag{12}$$

with

$$s \approx \frac{2}{T}\frac{1 - z^{-1}}{1 + z^{-1}} \tag{13}$$

which is bilinear transformation, where ω_c is cutoff frequency, and z^{-1} is lag operator.

By using this Butterworth filter of $q = 2$, the $\hat{\mu}(\ell T)$ of Eq.(11) can be changed to

$$\begin{aligned} \mu^B(\ell T) \;=\; & ((2/T)^2 + \sqrt{2}(2/T)\omega_c + {\omega_c}^2)^{-1} \\ & \times\Big\{ -2({\omega_c}^2 - (2/T)^2)\mu((\ell-1)T) \\ & -((2/T)^2 - \sqrt{2}(2/T)\omega_c + {\omega_c}^2)\mu((\ell-2)T) \\ & +{\omega_c}^2\Big(\hat{\mu}(\ell T) + 2\hat{\mu}((\ell-1)T) + \hat{\mu}((\ell-2)T)\Big)\Big\} \end{aligned} \tag{14}$$

where μ is defined in Eq.(16).

It may be better to avoid adding small increments after the control u has approached to the optimal operating point. We shall check whether a variation of the principal control $\hat{u}$:

$$\delta\hat{U}_\ell = \sum_{i=0}^{\lambda} |\hat{u}((\ell M - i)\Delta) - \hat{u}((\ell M - i - 1)\Delta)| \tag{15}$$

is small, where λ is a small natural number. In our method, by using Eqs.(14) and (15), the output of the accelerator at $t = \ell T$ is set as

$$\mu(\ell T) = \begin{cases} \mu^B(\ell T) & \text{if } \delta\hat{U}_\ell \geq \varepsilon \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where $\varepsilon > 0$ is a given small value.

4.4. Converter

The accelerator based on Eq.(16) is added to the Krstić type scheme as shown in Fig.2, which has A/D and D/A. The A/D is an analog to digital converter by a sampler with the sampling period Δ. The D/A is a digital to analog converter by the zero-th holder, in which $\mu(t) = \mu(\ell T) = constant$ $(\ell T \leq t < (\ell+1)T)$ during the renewal cycle period T.

5. Simulations

Consider the problem of optimizing the yield for a bioreactor which is described by Monod model[5,6,9]:

$$\begin{cases} \dot{x}_1 = f_1(x,\alpha,u) = x_1\left(\dfrac{x_2}{(\alpha + x_2)} - u\right) \\ \dot{x}_2 = f_2(x,\alpha,u) = u(1 - x_2) - \dfrac{x_1 x_2}{(\alpha + x_2)} \\ y = h(x,u) = x_1 \cdot u \end{cases}$$

where $x = [x_1, x_2]^T$ and $0 \leq u \leq 1$. The unknown α is initially set to $\alpha = 0.02$, but it is changed to $\alpha = 0.1$ at $t = 600(sec)$. The optimal operating value and the maximum output are $u^* = 0.860$ and $y^* = 0.754$ when $\alpha = 0.02$, and $u^* = 0.698$ and $y^* = 0.537$ when $\alpha = 0.1$, though they are unknown during the experiments. We set $\Delta = 0.06(sec)$, $T = 3(sec)$, $\beta = 0.03$, $\omega = 0.08$, $\omega_h = 0.2$, $\omega_l = 0.02$, $\omega_c = 0.5$, $k = 5$, $N = 2$, $q = 2$, $\lambda = 2$, $\varepsilon = 0.004$. The experiment starts at $u(0) = 0.6$.

Figure 3 shows a comparison between the Krstić approach(OLD) and our proposed approach(NEW) for the time response of the extremum seeking control u. Figure 4 dose for the time response of the output y.

These results indicate that this new extremum seeking control approach enables the system to regulate to the optimal operating point swiftly.

6. Conclusions

This paper has proposed a modification of Krstić type extremum seeking control, which speedily regulates to an unknown extremum point for nonlinear systems. It is equipped with an accelerator which consists of A/D, polynomial identification, search of extremum point, Butterworth filter, and D/A.

Simulation results indicate that this extremum seeking control approach enables the system of Monod model to regulate to the optimal operating point more rapidly than the Krstić original approach. This new approach shall be studied at the Monod model in more detail, and be tested by applying to another systems such as Haldane model, in future works.

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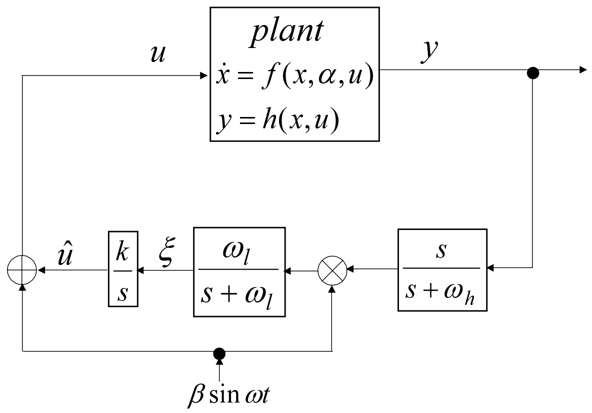

Figure 1: Krstić type control scheme.

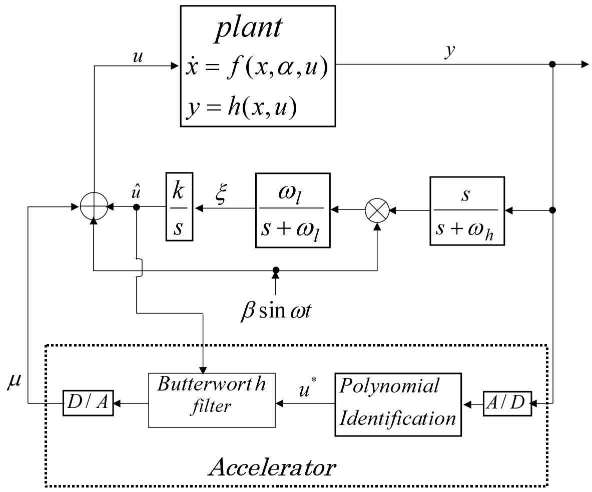

Figure 2: Extremum seeking control scheme.

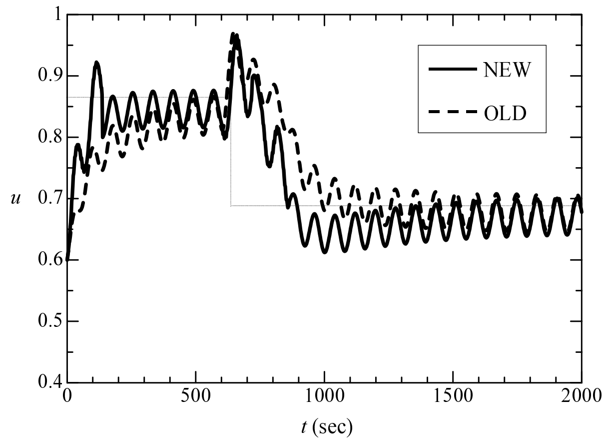

Figure 3: Time responses of the control u.

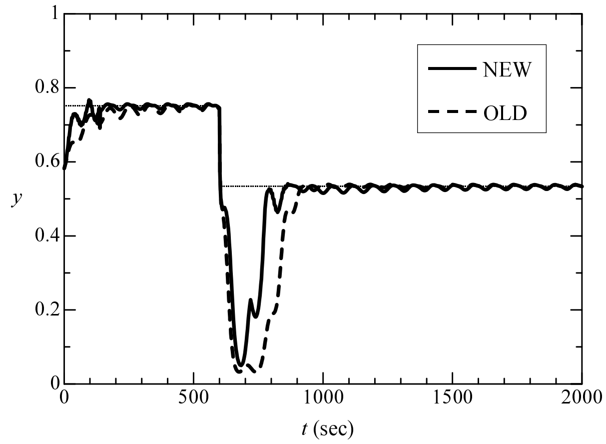

Figure 4: Time responses of the output y.