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Dynamical Properties of Associative Chaos Neural Network with Correlated Stored Patterns

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Abstract—Effects of the correlation among stored patterns on the associative dynamics in a chaos neural network model and their parameter dependences are numerically investigated. The model includes two kinds of parameters: One is a measure of the Hopfield like behavior in the retrieval process and another controls the chaotic behavior. The following results are obtained: i) Two dimensional parameter space is divided into two kinds of associative states by a distinct boundary, one is the Hopfield like retrieval state and another is the wandering state. As the degree of the correlation becomes larger, the area of the wandering state on two dimensional space becomes larger. ii) The recall ratio of correlated stored patterns is larger than that of uncorrelated stored patterns. iii) Whole region of the wandering state is not necessarily chaotic, but most of the wandering state is chaotic.

1. Introduction

Recently, complex phenomena have been studied in various fields. The brain nervous system is a typical example of complex system with large degree of freedom where chaotic responses are observed from EEG experiment and computational researches on the olfactory bulb by Skarda and Freemann[1]. The results suggest that chaos would play important roles in a recall process and a learning process. In the recall process, chaos could ensure rapid access to previously trained patterns. In the learning process, chaos could provide driving activity essential for memorizing novel inputs.

From the theoretical aspects, several workers have investigated functional possibilities of chaos related with information processing in biological systems including brain[2]-[9]. Tsuda *et al*[6, 7] have shown that asynchronous neural networks give chaotic wandering in memory space related with chaotic itinerancy. Nara *et al*[4, 5] have shown that the network consisting of simple binary neurons can give chaotic wandering in cycle memories, which is due to emergence of the complex dynamics occurring in systems with a finite but large degrees of freedom. Aihara *et al.*[3] have shown that a single neuron can give chaotic activity introducing effects of a relative refractoriness. Adachi and Aihara[2] have investigated the chaotic dynamics in the network composed of neurons which can give chaotic activity as a single neuron. Kuroiwa *et al.*[8, 9] have shown the common functionality in the chaotic wandering dynamics, which emerges from different mechanism of Aihara's model and Nara's model.

Based on the results[4, 8, 9], in neural network models, the wandering dynamics among memories is strongly intermittent, and there is a dynamical structure with a hierarchical linking and merging of memories. The results suggest that the wandering dynamics would depend on the correlation among memory inputs.

Our purposes of this work are to investigate effects of the correlation among stored patterns on the associative dynamics of a chaotic neural network.

2. Model

Aihara and co-workers proposed an associative chaotic neural network model based on the chaotic neuron[3]. The dynamics of the associative chaotic neural network model is described by the following equations[2],

$$x_i(t+1) = f(\eta_i(t+1) + \zeta_i(t+1)), \tag{1}$$

$$\eta_i(t+1) = k_f \eta_i(t) + \alpha \sum_{j=1}^{N} w_{ij} x_j(t), \tag{2}$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \theta x_i(t) + a, \tag{3}$$

where $x_i(t)$ denotes output from i-th neuron at time t. $\eta_i(t)$ is a part of an internal potential which represents feedback inputs from other neurons through synaptic couplings, w_{ij}. $\zeta_i(t)$ is another part of an internal potential which represents the relative refractoriness of the i-th neuron. Parameters k_f and k_r are decay constants. θ is a scaling parameter of the refractoriness. A parameter a is a constant external input to each neuron. N is a number of neurons in the network. We choose $N = 32$. Output function of the neuron is defined by

$$f(x) = 1/\{1 + \exp(-2\beta x)\}, \tag{4}$$

with the steepness parameter β. As shown in Eqs.(1) and (4), the value of $x_i(t)$ takes continuous value between 0 and 1. When $k_r = k_f = \theta = 0$, the network corresponds to the conventional discrete-time Hopfield network. Thus, the parameter α scales the degree of the Hopfield like behavior of the network. The Hopfield like behavior is called "retrieval state" in this paper. On the other hand, the refractoriness brings the chaotic behavior to the dynamics of the network, in other words, parameters k_r, θ and a, are control parameters of the chaotic behavior. In order to investigate the parameter dependence of the associative dynamics, control parameters α and a take various values. Values of other parameters are chosen as $k_r = 0.9$, $k_f = 0.2$, $\theta = 1$ and $\beta = 20$. Synaptic couplings are defined as the following form:

$$w_{ij} = \frac{1}{P}\sum_{\mu=1}^{P}(2\xi_i^{(\mu)} - 1)(2\xi_j^{(\mu)} - 1), \tag{5}$$

where $\xi_i^{(\mu)}$ takes 0 or 1 and is the i-th element of μ-th stored pattern vector, $\xi^{(\mu)}$. P is the number of stored patterns. In numerical experiments, we choose $P = 4$.

Our purpose of this work is to analyse effects of the correlation among stored patterns on the associative dynamics. We used five sets of stored patterns with different degree of the correlation. In this paper, we present results from three sets. In each set, three stored patterns, $\{\xi^{(A)}, \xi^{(B)}, \xi^{(C)}\}$, are mutually orthogonal and another pattern, $\xi^{(\mu)}$, correlates with $\xi^{(A)}$ ($\mu = 0, 1, 2$). Stored pattern vectors in three sets are defined below.

$$\begin{aligned}\xi^{(A)} =&(1,1,1,1,\ 1,1,1,1,\ 1,1,1,1,\ 1,1,1,1,\\ &0,0,0,0,\ 0,0,0,0,\ 0,0,0,0,\ 0,0,0,0), \end{aligned}\tag{6}$$

$$\begin{aligned}\xi^{(B)} =&(1,1,1,1,\ 1,1,1,1,\ 0,0,0,0,\ 0,0,0,0,\\ &1,1,1,1,\ 1,1,1,1,\ 0,0,0,0,\ 0,0,0,0), \end{aligned}\tag{7}$$

$$\begin{aligned}\xi^{(C)} =&(1,1,1,1,\ 0,0,0,0,\ 1,1,1,1,\ 0,0,0,0,\\ &1,1,1,1,\ 0,0,0,0,\ 1,1,1,1,\ 0,0,0,0), \end{aligned}\tag{8}$$

$$\begin{aligned}\xi^{(0)} =&(1,1,0,0,\ 1,1,0,0,\ 1,1,0,0,\ 1,1,0,0,\\ &1,1,0,0,\ 1,1,0,0,\ 1,1,0,0,\ 1,1,0,0), \end{aligned}\tag{9}$$

$$\begin{aligned}\xi^{(1)} =&(1,1,1,0,\ 1,1,0,0,\ 1,1,0,0,\ 1,1,1,0,\\ &1,0,0,0,\ 1,1,0,0,\ 1,1,0,0,\ 1,0,0,0), \end{aligned}\tag{10}$$

$$\begin{aligned}\xi^{(2)} =&(1,1,1,1,\ 1,1,0,0,\ 1,1,0,0,\ 1,1,1,1,\\ &0,0,0,0,\ 1,1,0,0,\ 1,1,0,0,\ 0,0,0,0). \end{aligned}\tag{11}$$

In this paper, the inner product is defined by

$$\mathbf{a}\cdot\mathbf{b} = \sum_{i=1}^{N}(2a_i - 1)(2b_i - 1), \tag{12}$$

where $\mathbf{a}$ and $\mathbf{b}$ are pattern vectors. The first set is referred as "orthogonal pattern set", $\{\xi^{(A)}, \xi^{(B)}, \xi^{(C)}, \xi^{(0)}\}$. The stored patterns in the set are mutually orthogonal under the above definition of the inner product. The second set of stored patterns is called "correlation pattern set I", $\{\xi^{(A)}, \xi^{(B)}, \xi^{(C)}, \xi^{(1)}\}$. Inner products between the pattern vector $\xi^{(1)}$ and others are as follows:

$$\xi^{(1)}\cdot\xi^{(\mu)} = \begin{cases} 8 & \text{in the case of } \mu = A, \\ 0 & \text{in the case of } \mu = B, C. \end{cases} \tag{13}$$

There is a correlation in only a pair of pattern vectors, $\xi^{(1)}$ and $\xi^{(A)}$, in the set. The third set is called "correlation pattern set II", $\{\xi^{(A)}, \xi^{(B)}, \xi^{(C)}, \xi^{(2)}\}$. Inner products between the pattern vector $\xi^{(2)}$ and others are as follows:

$$\xi^{(2)}\cdot\xi^{(\mu)} = \begin{cases} 16 & \text{in the case of } \mu = A, \\ 0 & \text{in the case of } \mu = B, C. \end{cases} \tag{14}$$

In the correlation pattern set II, there is also a correlation in only a pair of pattern vectors, $\xi^{(2)}$ and $\xi^{(A)}$. The degree of the correlation between $\xi^{(2)}$ and $\xi^{(A)}$ is larger than that between $\xi^{(1)}$ and $\xi^{(A)}$.

3. Measured Quantities and Method of Numerical Experiments

In this section, quantities which characterize the associative dynamics and the method of numerical experiments are explained. At first, we define a recall of a stored pattern in the network during the numerical experiments. When a pattern vector constituted of outputs of all neuron at t step closely resembles a stored pattern or its reversed pattern, it is assumed that the network successfully recalls the stored pattern at t step. Thus, the recall of μ-th stored pattern is defined by the following inequality in this work.

$$|D_\mu(t) - 0.5| > 0.45, \tag{15}$$

$$D_\mu(t) = \sum_{i=1}^{N}[x_i(t)(1 - \xi_i^{(\mu)}) + (1 - x_i(t))\xi_i^{(\mu)}]/N, \tag{16}$$

where $D_\mu(t)$ is the normalized Hamming distance between a pattern vector constituted of outputs of neuron at t step, $\mathbf{x}(t)$, and μ-th stored pattern vector, $\xi^{(\mu)}$. By use of above definition, we calculate the successful recall ratio of μ-th pattern vector within a certain time duration of measurements, T, given by the following equation.

$$R_\mu = T_\mu/T, \tag{17}$$

Both the time duration, T, and the transient time steps are chosen 5.0×10^5 in numerical experiments.

Secondly, both the Lyapnov dimension and the information dimension are evaluated to analyze the orbital instability of the network dynamics. The information dimension in this work is derived from calculating two kinds of return plots on two dimensional phase space, (η_i, η_j) and (ζ_i, ζ_j). i and j are chosen 1 and 2, respectively. The result does not statistically depend on the choice of the pair of internal potential, because all neurons are connected each other. The two dimensional phase space is divided into a large number of square regions to calculate the information dimension. The information dimension is defined in the zero limit of the size of the square region. In numerical experiments, we chose the size, Δy, as 5×10^{-3}, 2.5×10^{-3} and 1.25×10^{-3} for the orthogonal pattern set and the correlation pattern set I. Although it seems that the numerical calculated information dimension does not converge at the true value as Δy becomes smallest in three cases, the qualitative feature of the parameter dependence of the information dimension is found.

In numerical experiments, a randomly selected neuron is asynchronously updated. As one numerical step, we refer to the duration while N neurons have been updated. It notes that N neurons do not necessarily correspond to all neurons in the network. The initial value of $x_i(0)$ is $\xi_i^{(A)}$ for all numerical experiments. Those of $\{\eta_i(0), \zeta_i(0)\}$ are fixed to zero.

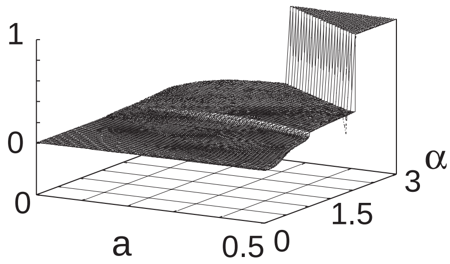

Fig.1: Bird's-eye view of rate of recall of $\xi^{(A)}$ on (α, a) space for the correlation pattern set II.

4. Results

The dependence of parameters a and α on the associative dynamics are illustrated in Fig. 1. Figure 1 shows a bird's-eye view of the recall ratio of a pattern $\xi^{(A)}$ on a parameter space (a, α) for the correlation pattern set II. $\xi^{(A)}$ is the initial output vector $\mathbf{x}(0)$. A distinct line on the space (a, α) can be seen in the figure which divides the space into two kinds of associative states. When α is smaller than that on the boundary, the recall ratio is smaller than one. On the other hand, when α becomes larger than that on the boundary, it is exactly one. This means that only the initially presented pattern is recalled in this region. In this paper, the former and the latter regions are called "wandering state" and "retrieval state", respectively. The same tendency is found for both the orthogonal pattern set and the correlation pattern set I. It is found that the boundary is shifted to the region for larger α as the correlation among stored patterns larger. Furthermore, the region of the wandering state is divided into two areas by a boundary which is a small singular jump of the recall ratio in Fig. 1. Such a singular jump of the recall ratio is not seen for the orthogonal pattern set, but seen for the correlation pattern set I.

In order to investigate the wandering state in detail, the α dependence of recall ratios of all stored patterns in the orthogonal pattern set at $a = 0.5$ is illustrated in Fig. 2. Figures 3 and 4 show the same as in Fig. 2 for the correlation pattern set I and II, respectively. In these figures, symbols of 'plus', 'circle' and 'cross' represent recall ratios of $\xi^{(A)}$, $\xi^{(B)}$ and $\xi^{(C)}$, respectively. Symbols of 'triangle' in Figs. 2 - 4 represent recall ratios of $\xi^{(0)}$, $\xi^{(1)}$ and $\xi^{(2)}$, respectively. In each figure, a jump in 'pluses' is observed which corresponds to the boundary between two associative states as seen in Fig. 1. In Figs. 2-4, it is found that the value of α at the boundary becomes larger as the degree of the correlation larger. In Fig.4, a small singular jump in the wandering state is seen at $\alpha \sim 1$ which is also observed in Fig. 1. Figure 2 shows that recall ratios of all stored pattern take same value in the wandering state. On the other hand, Fig. 3 shows that recall ratios of correlated patterns, which marked by 'plus' and 'triangle', take large value, when α is smaller than 0.7. For $0.7 < \alpha < 1.4$, those for uncorrelated patterns increase and reaches to the value of one intermittently. For $\alpha > 1.4$, the retrieval state emerges. Figure 4 shows that recall ratios of correlated patterns take non-zero value in the wandering state and ratios for uncorrelated patterns, $\xi^{(B)}$ and $\xi^{(C)}$, take nearly equal to zero. Thus, as the degree of the correlation among stored patterns becomes larger, recalls of correlated patterns dominate those of uncorrelated patterns.

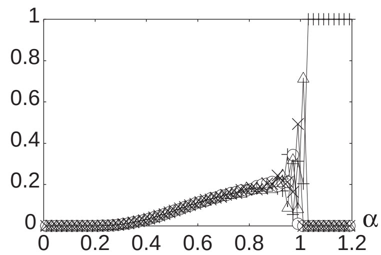

Fig.2: Recall ratios of stored patterns versus α at $a = 0.5$ for the orthogonal pattern set.

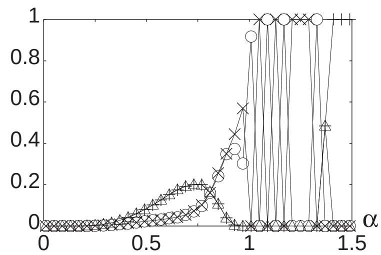

Fig.3: Recall ratios of stored patterns versus α at $a = 0.5$ for the correlation pattern set I.

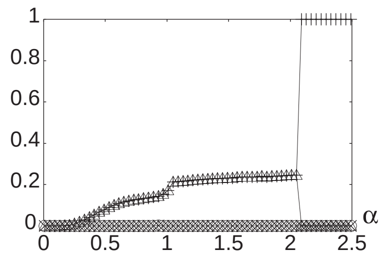

Fig.4: Recall ratios of stored patterns versus α at $a = 0.5$ for the correlation pattern set II.

We also calculate the Lyapnov dimension and the information dimension to examine the relation between the wandering state and chaos. Figure 5 shows the α dependence of the Lyapnov dimension at $a = 0.5$ for three sets of stored patterns. In the figure, 'plus', 'triangle' and 'square' represent the Lyapnov dimension for the orthogonal pattern set, that for the correlation pattern set I and II, respectively. We find no qualitative dependencies of the Lyapnov dimension on the degree of the correlation. The figure shows that the dynamics of the network is chaotic in α roughly smaller than 0.7 for all sets. On the other hand, as shown in Figs. 2-4, the boundary between the wandering state and the retrieval state depends on the degree of the correlation. Positions of the boundary between two associative states for three sets are larger than $\alpha = 1$. Thus, the whole region in the wandering state is not necessarily chaotic, but most of the region in the wandering state is chaotic.

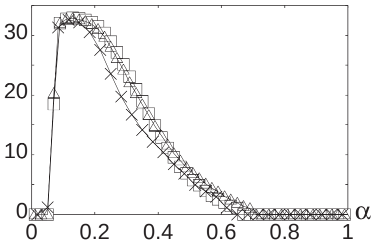

Fig.5: The α dependence of Lyapnov dimension of network dynamics at $a = 0.5$ for three pattern sets.

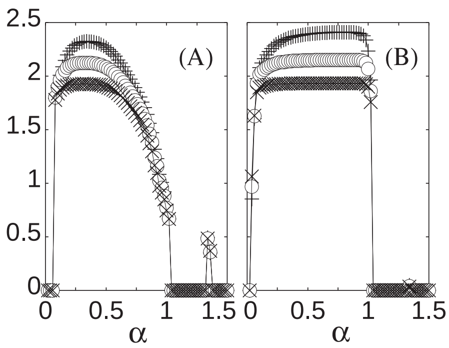

Fig.6: Information dimensions derived from return plots (A) on (η_1, η_2) for the correlation pattern set I and (B) on (ζ_1, ζ_2) for the same pattern set.

The α dependence of the information dimension at $a = 0.5$ derived from return plots on (η_1, η_2) for the correlation pattern set I is shown in Fig. 6(A). The α dependence of the information dimension from return plots on (ζ_1, ζ_2) for the correlation set I is shown in Fig. 6(B). In these figures, 'cross', 'circle' and 'plus' show the information dimension with the size of mesh $\Delta y = 5 \times 10^{-3}$, 2.5×10^{-3} and 1.25×10^{-3}, respectively. The information dimension measures the orbital instability on the phase space. By comparing Fig. 3 and Figs. 6, it is shown that the information dimension is zero when the recall ratio of only a stored pattern is equal to one. In the region of chaotic state as shown in Fig. 5, both the information dimension for (η_1, η_2) and that for (ζ_1, ζ_2) take large value. The information dimension for (ζ_1, ζ_2) is larger than that for (η_1, η_2) for all α, because internal potentials for the refractoriness, $\{\zeta_i\}$, bring chaotic behavior to the network. Thus, even in non-chaotic wandering state as shown in Fig. 5, the information dimension for (ζ_1, ζ_2) takes large value. In order to investigate the orbital instability in detail we calculate the local largest Lyapnov exponent at $(a, \alpha) = (0.5, 0.8)$ where the non-chaotic wandering state is observed in three sets. Although the time average of local largest Lyapnov exponents for three sets seems to take negative value, the sign of the local largest Lyapnov exponent takes occasionally positive. This means the dynamics of network is locally chaotic. Moreover, a singular jump for $\alpha \sim 1.35$ is observed which corresponds to the intermittency of the recall ratio in Fig. 3.

5. Conclusions and Discussions

Effects of the correlation among stored patterns on the dynamics of the chaotic neural network are numerically investigated. It is found that properties of the dynamics depend on the degree of the correlation, that is, the recall ratio of correlated patterns becomes larger as the degree of the correlation larger. Investigating closely the behavior of the racall ratios, we observe the various dynamical behavior depending on the degree of correlation. Thus, one future problem is to investigate relation between the degree of correlation and dynamical states in detail. The Lyapnov dimension does not qualitatively depend on the degree of the correlation. To investigate the orbital instability in wandering state in detail, we have to evaluate the parameter dependences of local largest Lyapnov exponent in detail. This is another future problem.

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