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$\mathcal{H}_\infty$ Fuzzy Filter for Nonlinear Singularly Perturbed Systems

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Abstract—This paper addresses the problem of designing an $\mathcal{H}_\infty$ filter for a class of nonlinear singularly perturbed systems described by a Takagi-Sugeno (TS) fuzzy model. Based on a linear matrix inequality (LMI) approach, we develop a fuzzy $\mathcal{H}_\infty$ filter that guarantees the $\mathcal{L}_2$-gain from an exogenous input to a filter error to be less than or equal to a prescribed value. In order to alleviate the ill-conditioning resulting from the interaction of slow and fast dynamic modes, solutions to the problem are given in terms of linear matrix inequalities which are independent of the singular perturbation ε, when ε is sufficiently small. The proposed approach does not involve the separation of states into slow and fast ones and it can be applied not only to standard, but also to nonstandard singularly perturbed nonlinear systems.

1. Introduction

Over the past few decades, the problem of $\mathcal{H}_\infty$ filtering design for singularly perturbed system has been intensively studied by a number of researchers; see [1]-[13]. This is due not only to theoretical interest but also to the relevance of this topic in control engineering applications. Singularly perturbed systems are dynamical systems with multiple time-scales. Singularly perturbed systems often occur naturally due to the presence of small "parasitic" parameter, typically small time constants, masses, etc. Indeed multiple time-scales phenomena are almost unavoidable in "real-life" systems. Examples of such systems abound and include convection-diffusion systems, diffusion-drift motion systems, power systems, scheduling systems, economic models, telecommunication systems and bifurcations.

In the last few years, many researchers have studied the $\mathcal{H}_\infty$ filter design for a general class of linear singularly perturbed systems. In [6], the authors have investigated the decomposition solution of $\mathcal{H}_\infty$ filter gain for singularly perturbed systems. The reduced-order $\mathcal{H}_\infty$ optimal filtering for system with slow and fast modes has been considered in [7]. Although many researchers have studied linear singularly perturbed systems for many years, the $\mathcal{H}_\infty$ filtering design for nonlinear singularly perturbed systems remains as an open research area. This is because, in general, nonlinear singularly perturbed systems can not be easily separated into slow and fast subsystems.

Fuzzy system theory enables us to utilise qualitative, linguistic information about a highly complex nonlinear system to construct a mathematical model for it. Recent studies show that a fuzzy linear model can be used to approximate global behaviours of a highly complex nonlinear system; see for example, [14]-[19]. In this fuzzy linear model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" these linear models through nonlinear fuzzy membership functions. Unlike conventional modelling where a single model is used to describe the global behaviour of a system, the fuzzy modelling is essentially a multi-model approach in which simple sub-models (linear models) are combined to describe the global behaviour of the system. However, employing the existing fuzzy results [14]-[19] on the singularly perturbed system, one ends up with a family of ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes. In general, ill-conditioned linear matrix inequalities are very difficult to solve.

The aim of this paper is to design an $\mathcal{H}_\infty$ fuzzy filter for a class of nonlinear singularly perturbed systems. First, we approximate this class of systems by a Takagi-Sugeno fuzzy model. Then based on an LMI approach, we develop the $\mathcal{H}_\infty$ filter that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to a filter error to be less than or equal to a prescribed value for this class of fuzzy singularly perturbed systems. In order to alleviate the ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes, the ill-conditioned LMIs are decomposed into ε-independent and ε-dependent LMIs. The ε-independent LMIs are not ill-conditioned and the ε-dependent LMIs tend to zero when $\varepsilon > 0$ is small enough. It can be shown that when ε is sufficiently small, the original ill-conditioned LMIs are solvable if and only if the ε-independent LMIs are solvable.

This paper is organized as follows. In Section 2, system description and problem formulation are presented. Based

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on an LMI approach, we develop a technique in Section 3 for designing a fuzzy $\mathcal{H}_\infty$ filter that guarantees the $\mathcal{L}_2$-gain of the mapping from the exogenous input noise to the filter error to be less than or equal to a prescribed value. Finally, in Section 4, the conclusion is given.

2. System Description and Problem Formulation

The nonlinear singularly perturbed system under consideration is described by the following fuzzy system model:
Plant Rule i: IF $\nu_1(t)$ is M_{i1} and $\cdots$ and $\nu_\vartheta(t)$ is $M_{i\vartheta}$ THEN

$$\begin{array}{lcl} E(\varepsilon)\dot{x}(t) & = & A_i x(t) + B_{1_i} w(t), \quad x(0) = 0 \\ z(t) & = & C_{1_i} x(t) \\ y(t) & = & C_{2_i} x(t) + D_{21_i} w(t) \end{array} \tag{1}$$

where $E(\varepsilon) = \begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}$, $i = 1, 2, \cdots, r$, $M_{ij}(j = 1, 2, \cdots, \vartheta)$ are the fuzzy sets, $\nu_1(t), \cdots, \nu_\vartheta(t)$ are the premise variables, ϑ is the number of premise variables. $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the input, $w(t) \in \Re^p$ is the disturbance which belongs to $\mathcal{L}_2[0, \infty)$, $y(t) \in \Re^\ell$ is the measurement, $z(t) \in \Re^s$ is the controlled output, the matrices $A_i = \begin{bmatrix} A_{11_i} & A_{12_i} \\ A_{21_i} & A_{22_i} \end{bmatrix}$, $B_{1_i} = \begin{bmatrix} B_{11_i} \\ B_{12_i} \end{bmatrix}$, $C_{1_i} = \begin{bmatrix} C_{11_i} & C_{12_i} \end{bmatrix}$, $C_{2_i} = \begin{bmatrix} C_{21_i} & C_{22_i} \end{bmatrix}$, and $D_{21_i} = \begin{bmatrix} D_{211_i} \\ D_{212_i} \end{bmatrix}$ are of appropriate dimensions, r is the number of IF-THEN rules. Note that the system (1) is said to be in the standard form if the matrix A_{22_i} is nonsingular. Otherwise, it is called a nonstandard singularly perturbed system [3].

Let $\varpi_i(\nu(t)) = \prod_{k=1}^{\vartheta} M_{ik}(\nu_k(t))$ and $\mu_i(x(t)) = \frac{\varpi_i(\nu(t))}{\sum_{i=1}^r \varpi_i(\nu(t))}$ where $M_{ik}(\nu_k(t))$ is the grade of membership of $\nu_k(t)$ in M_{ik}. Without loss of generality, it is assumed in this paper that $\varpi_i(\nu(t)) \geq 0$, $i = 1, 2, ..., r$; $\sum_{i=1}^r \varpi_i(\nu(t)) > 0$ for all t. Therefore, $\mu_i(\nu(t)) \geq 0$, $i = 1, 2, ..., r$; $\sum_{i=1}^r \mu_i(\nu(t)) = 1$ for all t. For the convenience of notations, we let $\varpi_i = \varpi_i(\nu(t))$ and $\mu_i = \mu_i(\nu(t))$.

The resulting fuzzy system model is inferred as the weighted average of the local models of the form:

$$\begin{array}{lcl} E(\varepsilon)\dot{x}(t) & = & A(\mu)x(t) + B_1(\mu)w(t), \ x(0) = 0 \\ z(t) & = & C_1(\mu)x(t) \\ y(t) & = & C_2(\mu)x(t) + D_{21}(\mu)w(t) \end{array} \tag{2}$$

where $A(\mu) = \sum_{i=1}^r \mu_i A_i$, $B_1(\mu) = \sum_{i=1}^r \mu_i B_{1_i}$, $C_1(\mu) = \sum_{i=1}^r \mu_i C_{1_i}$, $C_2(\mu) = \sum_{i=1}^r \mu_i C_{2_i}$ and $D_{21}(\mu) = \sum_{i=1}^r \mu_i D_{21_i}$.

In this paper, we consider the following full order $\mathcal{H}_\infty$ fuzzy filter which is inferred as the weighted average of the local models of the form:

$$\begin{array}{lcl} E(\varepsilon)\dot{\hat{x}}(t) & = & \hat{A}(\mu)\hat{x}(t) + \hat{B}(\mu)y(t) \\ \hat{z}(t) & = & \hat{C}(\mu)\hat{x}(t). \end{array} \tag{3}$$

Note that $\hat{A}(\mu) \in \Re^{n \times n}$. Before ending this section, we describe the problem under our study as follows.

Problem Formulation: Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, design a fuzzy filter of the form (3) such that the $\mathcal{L}_2$-gain from the exogenous input to the filter error is less than or equal to γ, i.e.,

$$\int_0^{T_f} (z(t) - \hat{z}(t))^T (z(t) - \hat{z}(t))\, dt \leq \gamma^2 \left[\int_0^{T_f} w^T(t) w(t)\, dt \right]. \tag{4}$$

3. $\mathcal{H}_\infty$ Fuzzy Filter Design

In this section, we will present the main results of this paper. First, we select our filter as follows:

$$\begin{array}{lcl} E(\varepsilon)\dot{\hat{x}}(t) & = & \hat{A}(\mu)\hat{x}(t) + \hat{B}(\mu)y(t) \\ \hat{z}(t) & = & \hat{C}(\mu)\hat{x}(t). \end{array} \tag{5}$$

Define the filtering error as $\tilde{z}(t) = z(t) - \hat{z}(t)$. Then, the filtering error of the fuzzy system model (2) are given by

$$\begin{array}{lcl} E_e(\varepsilon)\dot{\tilde{x}}(t) & = & A_e(\mu)\tilde{x}(t) + B_e(\mu)w(t) \\ \tilde{z}(t) & = & C_e(\mu)\tilde{x}(t) \end{array} \tag{6}$$

where $\tilde{x} = \begin{bmatrix} x(t) \\ \hat{x}(t) \end{bmatrix}$, $E_e(\varepsilon) = \begin{bmatrix} E(\varepsilon) & 0 \\ 0 & E(\varepsilon) \end{bmatrix}$, $A_e(\mu) = \begin{bmatrix} A(\mu) & 0 \\ \hat{B}(\mu)C_2(\mu) & \hat{A}(\mu) \end{bmatrix}$, $B_e(\mu) = \begin{bmatrix} B_1(\mu) \\ \hat{B}(\mu)D_{21}(\mu) \end{bmatrix}$ and $C_e(\mu) = [C_1(\mu) \quad -\hat{C}(\mu)]$. Now, we have our first result in this paper.

Lemma 3.1 *Given a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, if there exist matrices $X(\varepsilon) = X^T(\varepsilon)$, $Y(\varepsilon) = Y^T(\varepsilon)$, $\mathcal{A}(\mu, \varepsilon)$, $\mathcal{B}(\mu, \varepsilon)$ and $\mathcal{C}(\mu, \varepsilon)$ satisfying the following nonlinear matrix inequalities:*

$$\begin{bmatrix} X(\varepsilon) & I \\ I & Y(\varepsilon) \end{bmatrix} > 0 \tag{7}$$

$$X(\varepsilon) > 0 \tag{8}$$

$$Y(\varepsilon) > 0 \tag{9}$$

$$\begin{bmatrix} \Psi_{11}(\mu,\varepsilon) & \Psi_{12}(\mu,\varepsilon) \\ \Psi_{12}^T(\mu,\varepsilon) & \Psi_{22}(\mu,\varepsilon) \end{bmatrix} < 0 \tag{10}$$

where

$$\Psi_{11}(\mu,\varepsilon) = \begin{bmatrix} \begin{pmatrix} E^{-1}(\varepsilon)A(\mu)X(\varepsilon) \\ +X(\varepsilon)(E^{-1}(\varepsilon)A(\mu))^T \end{pmatrix} & (*)^T \\ (E^{-1}(\varepsilon)B_1(\mu))^T & -\gamma I \end{bmatrix} \tag{11}$$

$$\Psi_{12}(\mu,\varepsilon) = \begin{bmatrix} \begin{pmatrix} \mathcal{A}(\mu,\varepsilon) \\ +(E^{-1}(\varepsilon)A(\mu))^T \end{pmatrix} & \beta(\mu,\varepsilon) \\ \begin{pmatrix} C_1(\mu)X(\varepsilon) \\ -\mathcal{C}(\mu,\varepsilon) \end{pmatrix} & 0 \end{bmatrix}^T \tag{12}$$

$$\Psi_{22}(\mu,\varepsilon) = \begin{bmatrix} \begin{pmatrix} (E^{-1}(\varepsilon)A(\mu))^T Y(\varepsilon) \\ +Y(\varepsilon)E^{-1}(\varepsilon)A(\mu) \\ +\mathcal{B}(\mu,\varepsilon)C_2(\mu) \\ +C_2^T(\mu)\mathcal{B}^T(\mu,\varepsilon) \end{pmatrix} & (*)^T \\ C_1(\mu) & -\gamma I \end{bmatrix} \tag{13}$$

and $\beta(\mu,\varepsilon) = Y(\varepsilon)E^{-1}(\varepsilon)B_1(\mu + \mathcal{B}(\mu,\varepsilon)D_{21}(\mu)$*, then the prescribed* $\mathcal{H}_\infty$ *performance* $\gamma > 0$ *is guaranteed. Furthermore, a suitable filter* $(\hat{A}(\mu,\varepsilon), \hat{B}(\mu,\varepsilon)$ *and* $\hat{C}(\mu,\varepsilon))$ *is given as follows:*

$$\begin{array}{rcl} \hat{B}(\mu,\varepsilon) & = & E(\varepsilon)N^{-1}(\varepsilon)\mathcal{B}(\mu,\varepsilon) \\ \hat{C}(\mu,\varepsilon) & = & C(\mu,\varepsilon)(M^T(\varepsilon)E(\varepsilon))^{-1} \\ \hat{A}(\mu,\varepsilon) & = & E(\varepsilon)N^{-1}(\varepsilon)[\mathcal{A}(\mu,\varepsilon) \\ & & -Y(\varepsilon)E^{-1}(\varepsilon)A(\mu)X(\varepsilon)E(\varepsilon) \\ & & -\mathcal{B}(\mu,\varepsilon)C_2(\mu)X(\varepsilon)E(\varepsilon)](M^T(\varepsilon)E(\varepsilon))^{-1} \end{array} \tag{14}$$

where $N(\varepsilon)M^T(\varepsilon) = I - Y(\varepsilon)X(\varepsilon)$*, and* $M(\varepsilon)$ *and* $N(\varepsilon)$ *are square and nonsingular matrices.*

Proof: The desired result can be carried out by a similar technique used in [20], which involves the changes of variable. The detail of the proof is omitted for brevity due to the page limited. ■

The linear matrix inequalities given in Lemma 3.1 becomes ill-conditioned when ε is sufficiently small, which is always the case for the singularly perturbed system. In general, these ill-conditioned linear matrix inequalities are very difficult to solve. Thus, to alleviate these ill-conditioned LMIs, we have the following theorem which does not depend on ε.

Theorem 3.1 *Given a prescribed* $\mathcal{H}_\infty$ *performance* $\gamma > 0$*, if there exist matrices* X*,* Y*,* $\mathcal{A}_{ij}$*,* $\mathcal{B}_i$ *and* C_i *satisfying the following matrix inequalities:*

$$\begin{bmatrix} XD + EX & I \\ I & YE + DY \end{bmatrix} > 0 \tag{15}$$

$$XD = DX^T, \quad EX = X^TE, \quad EX + XD > 0 \tag{16}$$

$$YE = EY^T, \quad DY = Y^TD, \quad YE + DY > 0 \tag{17}$$

$$\begin{bmatrix} \Psi_{11_{ii}} & \Psi_{21_{ii}}^T \\ \Psi_{21_{ii}} & \Psi_{22_{ii}} \end{bmatrix} < 0, \ i = 1, 2, \cdots, r \tag{18}$$

$$\begin{bmatrix} \Psi_{11_{ij}} & \Psi_{21_{ij}}^T \\ \Psi_{21_{ij}} & \Psi_{22_{ij}} \end{bmatrix} + \begin{bmatrix} \Psi_{11_{ji}} & \Psi_{21_{ji}}^T \\ \Psi_{21_{ji}} & \Psi_{22_{ji}} \end{bmatrix} < 0, \ i < j \le r \tag{19}$$

where $E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$,

$$\begin{array}{rcl} \Psi_{11_{ij}} & = & \begin{bmatrix} A_iX + X^TA_i^T & B_{1_i} \\ B_{1_i}^T & -\gamma I \end{bmatrix} \\ \Psi_{21_{ij}} & = & \begin{bmatrix} \mathcal{A}_{ij} + A_i^T & Y^TB_{1_i} + \mathcal{B}_iD_{21_j} \\ C_{1_i}X - C_j & 0 \end{bmatrix} \\ \Psi_{22_{ij}} & = & \begin{bmatrix} A_i^TY + Y^TA_i + \mathcal{B}_iC_{2_j} + C_{2_i}^T\mathcal{B}_j^T & C_{1_i}^T \\ C_{1_i} & -\gamma I \end{bmatrix}, \end{array}$$

then there exists a sufficiently small $\hat{\varepsilon} > 0$ *such that for* $\varepsilon \in (0, \hat{\varepsilon}]$*, the prescribed* $\mathcal{H}_\infty$ *performance* $\gamma > 0$ *is guaranteed. Furthermore, a suitable filter* $(\hat{A}_{ij}, \hat{B}_i$ *and* $\hat{C}_i)$ *is given as follows:*

$$\begin{array}{rcl} \hat{A}_{ij} & = & N^{-1}[\mathcal{A}_{ij} - Y^TA_iX - \mathcal{B}_iC_{2j}X](M^T)^{-1} \\ \hat{B}_i & = & N^{-1}\mathcal{B}_i, \\ \hat{C}_i & = & C_i(M^T)^{-1} \end{array} \tag{20}$$

where $SM^T = I - YX$*,* M *and* S *are square and nonsingular matrices.* S *is an upper triangular matrix, that is,* $S = \begin{pmatrix} S_1 & 0 \\ S_2 & S_3 \end{pmatrix}$ *and* $N = \begin{pmatrix} S_1^T & 0 \\ S_2^T & S_3^T \end{pmatrix}$.

Proof: Due to the page limited, the detail of the proof is omitted. ■

4. Conclusion

This paper has investigated the problem of designing an $\mathcal{H}_\infty$ filter for a class of nonlinear singularly perturbed systems described by TS fuzzy model that guarantees the $\mathcal{L}_2$-gain from an exogenous input to a filter error being less than or equal to a prescribed value. An LMI approach has been used to derive sufficient conditions for the existence of an $\mathcal{H}_\infty$ filter. The sufficient conditions are given in terms of a family of ε-independent linear matrix inequalities. The proposed approach does not involve the separation of states into slow and fast ones and it can be applied not only to standard, but also to nonstandard singularly perturbed nonlinear systems.

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