2004 International Symposium on Nonlinear Theory and its Applications (NOLTA2004)
Fukuoka, Japan, Nov. 29 - Dec. 3, 2004

Element-Value Determination with Measurement Errors

Isao Yamaguchi* and Shoji Shinoda**

* Department of Electrical and Electronic Engineering, Tokai University
1117 Kitakaname, Hiratsuka, Kanagawa 259-1292 Japan, E-mail isao@keyaki.cc.u-tokai.ac.jp
** Department of Electrical, Electronic and Communication Engineering, Chuo University
1-13-27 Kasuga, Bunkyo-ku, Tokyo 112-8551 Japan, E-mail shinoda@elect.chuo-u.ac.jp

Abstract- This paper deals with a linear analog circuit which satisfies some sufficient conditions for the unique element-value determination, and proposes a method for actual computation of the element-values taking into account the measurement errors.

1. Introduction

It is of significantly importance in relation to the problem of diagnosis of deviation faults in linear analog circuits to check whether or not it is possible to uniquely determine the element-values in a given linear analog circuit from the measurements performed at its accessible nodes. The problem of checking the unique determinability of the element-values is called the *element-value determinability problem*. This problem was first considered by Berkowitz[1] in 1962, and subsequently was studied by many researchers[2]. However, the techniques presented in most of the papers are not promising, because they are given on the impractical assumption that the actual values of the good circuit elements are exactly on the respective design values. At present, the most promising approach to diagnosis of deviation faults is the element-value identification technique under the assumption that (a) a given linear analog circuit is of known topology (and of known element-kinds if possible) and (b) the actual value of each element-value of the circuit almost always deviates from the design value and is not known exactly. The key part in this element-value identification technique is to check whether or not it is possible to uniquely determine the element-values from the measurements and then to give a method for actual computation of the element-values if it is possible. Some useful sufficient conditions for the element-value determinability of a linear analog circuit and a method for actual computation of the element-values were developed([3],[4]). The sufficient conditions are characterized by using the equivalent circuit transformation with the repeated application of a generalized star-to-polygon transformation and its inverse equivalent circuit transformation.

In this paper, we deal with a linear analog circuit which satisfies such sufficient conditions for the unique element-value determination, and propose a method for actual computation of the element-values taking into account the measurement errors.

2. Unistor Circuit Model and Generalized Star-to-Polygon Transformation

We consider a linear analog circuit N made up of resistors, inductors, capacitors, voltage-controlled current sources and/or independent current sources. Nodes at which voltages and/or currents can be applied and/or measured in N are called the *accessible nodes* of N, and the remaining nodes are designated as *inaccessible nodes* where the inaccessible nodes are nodes at which neither voltages nor currents can be applied or measured. We assume that the inaccessible nodes of N are numbered consecutively from 1 to m, and the accessible nodes of N are numbered consecutively from m+1 to m+n+1, where m and n+1 denote the numbers of inaccessible nodes and accessible nodes, respectively. The node m+n+1 is designated as the reference node of N.

A *unistor* is a directed branch such that its current can only flow in the direction of the arrow through the branch and is proportional to the node-voltage of its initial node, independently of the node-voltage of its terminal node. The proportional constant is called the admittance of the unistor. The unistor is represented by an ordered pair of the form (p,q) if its initial node is q and its terminal node is p, and is shown in Fig.1, where i is the current of the unistor, y_{pq} is the admittance of the unistor, and v_q is the node-voltage of node q with respect to the reference node, and the voltage-current characteristic of the unistor is represented by

$$i = y_{pq} v_q . \quad (1)$$

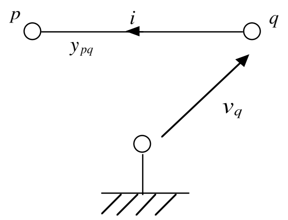


Fig.1 Unistor.

It should be noted that the unistor is an abstract circuit element but is a powerful tool for treating the element-value determination problem. Now let $\boldsymbol{V}$ be the node-voltage vector, let $\boldsymbol{I}$ be the current-source vector, and let $\boldsymbol{Y}$ be the nodal admittance matrix with respect to the reference node of N. Then the nodal equation of N is represented by

$$\boldsymbol{YV}=\boldsymbol{I} . \quad (2)$$

Let y_{pq} be the negative of the p-th row and q-th column element of $\boldsymbol{Y}$, and let y_{m+n+1q} be the sum of all the elements in the q-th column vector of $\boldsymbol{Y}$. Then, for N, we can construct a unistor circuit N^* in such a way that (i) N^* and N have the same node set, (ii) there exists in N^* a unistor (p,q) whose admittance is y_{pq} if and only if $y_{pq} \neq 0$ for $p \neq q$, where p, $q = 1, 2, ..., m+n$, (iii) there exists in N^* a unistor $(m+n+1,q)$ whose admittance is y_{m+n+1q} if and only if $y_{m+n+1q} \neq 0$, where $q = 1, 2, ..., m+n$, and (iv) N^* and N have the same independent current sources. Such a unistor circuit N^* is called *associated unistor circuit* of N. Two unistors are called *parallel unistors* if they have the same initial nodes and the same terminal nodes. In this construction, N^* has no parallel unistors. It should be noted that the topology of N^* can be determined directly from both the topology of N and the distribution of voltage-controlled current sources in N. The unistor circuit representation of a non-source branch with admittance y is a pair of unistors with equal admittance y. If each pair of unistors with equal admittance in N^* is drawn in a figure by a single branch with no direction, then the topology of N^* is identical with the topology of N if N does not contain voltage-controlled current sources.

Let us consider a sequence of circuits $N_0, N_1, ..., N_{m-1}$ and N_m, which are obtained from N^* by eliminating the inaccessible nodes one by one in an elimination order π by a generalized star-to-polygon transformation, where parallel unistors generated by eliminating each inaccessible node are assumed to be replaced with a single unistor with the sum of admittances of the parallel unistors. N_0 is the original unistor circuit N^* itself, $N_k (k=1,2, ..., m)$ is the unistor circuit obtained from N_{k-1} by eliminating inaccessible node $\pi(k)$ by the transformation, and N_m is the final unistor circuit which contains the accessible nodes only and whose nodal admittance matrix can be calculated from measurements performed at the accessible nodes. It should be noted that N_m is unique, independently of the order π. Let V_k be the set of nodes of N_k and let E_k be the set of unistors of N_k. With respect to node $\pi(k)$ in N_{k-1}, let V_k^- be the set of terminal nodes of unistors directed from the node $\pi(k)$ in N_{k-1}, let V_k^+ be the set of initial nodes of unistors directed to the node $\pi(k)$ in N_{k-1}, let A_k be the set of unistors connected to the node $\pi(k)$ in N_{k-1} (*i.e.*, directed either from or to the node $\pi(k)$ in N_{k-1}), let B_k be the set of all the unistors in N_{k-1} such that the initial (resp.,terminal) node of each of unistors is in V_k^+ (resp., in V_k^-), and let C_k be the set of all the unistors in N_k such that, for the initial node, say b, and the terminal node, say a, of each of the unistors, there exists no unistor (a,b) in N_{k-1}. Let $y_{pq}^{(k)}$ be the admittance of unistor (p,q) of N_k and let $y_{pq}^{(0)} = y_{pq}$. Then, for every $k=1, 2, ..., m$, the following relations holds;

(i) $V_k = V_{k-1} - \{\pi(\mathrm{k})\}$ and $E_k = (E_{k-1} - A_k) \bigcup C_k$

(ii) for every unistor (p,q) in $E_{k-1} - A_k \bigcup B_k$ in N_k

$$y_{pq}^{(k)} = y_{pq}^{(k-1)} \tag{3}$$

(iii) for every unistor (p,q) in B_k in N_k

$$y_{pq}^{(k)} = y_{pq}^{(k-1)} + y_{p\pi(k)}^{(k-1)} y_{\pi(k)q}^{(k-1)} / \Delta^{(k-1)} \tag{4}$$

(iv) for every unistor (p,q) in C_k in N_k

$$y_{pq}^{(k)} = y_{p\pi(k)}^{(k-1)} y_{\pi(k)q}^{(k-1)} / \Delta^{(k-1)} \tag{5}$$

where

$$\Delta^{(k-1)} = \sum_{p \in V_k^-} y_{p\pi(k)}^{(k-1)} \tag{6}$$

A set of equations of (4) and (5) is called a *generalized star-to-polygon transformation* which obtains N_k from N_{k-1} by eliminating inaccessible node $\pi(k)$.

3. A Solution of Element-Value Determinability Problem of the Unistor Circuit Model

Assume that $D_k = V_k^+ \bigcap V_k^-$ is not empty. If all the unistor admittances in N_k are known, and if the following set of equations:

$$\left.\begin{array}{ll} y_{p\pi(k)}^{(k-1)} y_{\pi(k)q}^{(k-1)} / \Delta^{(k-1)} = y_{pq}^{(k)} & \text{for all } (p,q) \in C_k \\ y_{p\pi(k)}^{(k-1)} = y_{\pi(k)q}^{(k-1)} & \text{for all } p \in D_k \end{array}\right\} \tag{7}$$

contains as the variables the admittances (of the forms $y_{p\pi(k)}^{(k-1)}$ and/or $y_{\pi(k)q}^{(k-1)}$) of all the fundamental unistor (at least one of its endnodes is inaccessible) of N_{k-1}, and furthermore if it has a unique solution with respect to the variables, then the remaining unistor admittances in N_{k-1} are obtained from sets of equations of (4) and (5), where $\Delta^{(k-1)}$ is given by (6). That is, if (7) can be uniquely solved, then N_{k-1} can be restored from N_k. For convenience's sake of unified description, let us introduce new notations: $x_p^{(k-1)} = y_{p\pi(k)}^{(k-1)} = y_{\pi(k)p}^{(k-1)}$ for every p in D_k; $x_p^{(k-1)} = y_{p\pi(k)}^{(k-1)}$ for every p in $V_k^- - D_k$; $x_{p+}^{(k-1)} = y_{\pi(k)p}^{(k-1)}$ for every p in $V_k^+ - D_k$; $a_{pq}^{(k)} = y_{pq}^{(k)}$ for every (p,q) in $\{(p,q) \in C_k \mid p, q \in D_k\}$; $a_{pq+}^{(k)} = y_{pq}^{(k)}$ for every (p,q) in $\{(p,q) \in C_k \mid p \in D_k, q \in (V_k^+ - D_k)\}$; $a_{p-q}^{(k)} = y_{pq}^{(k)}$ for every (p,q) in $\{(p,q) \in C_k \mid p \in (V_k^- - D_k), q \in D_k\}$; $a_{p-q+}^{(k)} = y_{pq}^{(k)}$ for every (p,q) in $\{(p,q) \in C_k \mid p \in (V_k^- - D_k), q \in (V_k^+ - D_k)\}$; and $\overline{C}_k = \{(p,q) \mid (p,q) \in C_k, p, q \in D_k\} \bigcup \{(p,q^+) \mid (p,q) \in C_k, p \in D_k, q \in (V_k^+ - D_k)\} \bigcup \{(p^-,q) \mid (p,q) \in C_k, p \in (V_k^- - D_k), q \in D_k\} \bigcup \{(p^-,q^+) \mid (p,q) \in C_k, p \in (V_k^- - D_k), q \in (V_k^+ - D_k)\}$.

Then, (7) can be rewritten simply as:

$$x_u^{(k-1)} x_v^{(k-1)} / \Delta^{(k-1)} = a_{uv}^{(k)} \quad \text{for all } (u,v) \text{ in } \overline{C}_k \tag{8}$$

where (6) can be rewritten as

$$\Delta^{(k-1)} = \sum_p x_p^{(k-1)} \tag{9}$$

As a graph associated with (8), we define an undirected graph G_k such that the set of nodes in $V(G_k) = \{p \mid p \in D_k\} \bigcup \{p^- \mid p \in (V_k^- - D_k)\} \bigcup \{p^+ \mid p \in (V_k^+ - D_k)\}$ and the set of edges is $E(G_k) = \{[u,v] \mid (u,v) \in \overline{C}\}$ where [u,v] denotes an undirected edge whose both endnodes are u and v. G_k is called the *associated graph* of (8), or the graph associated with the elimination of inaccessible node $\pi(k)$ in N_{k-1}. A particular subgraph of G_k such that (a) the subgraph contains all the nodes of G_k and (b) each of the connected components of the subgraph contains exactly one cycle with an odd number of edges (greater than or equal to three) is a *dendroid* of G_k, where a cycle is loop whose nodes are all distinct. Also, G_k is said to be *dendroidal* if either (a) it has a connected dendroid or (b) it has a disconnected dendroid such that the signs of the real and imaginary parts of the unistor admittance associated with a certain node of each connected component are known.

In [3], the following theorem was obtained:

Theorem 1: Suppose that the topology of N^* and all the unistor admittances of N_m are known. Then, $N^*(=N_0)$ can be uniquely restored from N_m if there exists an elimination order π of inaccessible nodes such that every G_k is dendroidal for $k = 1, 2, ..., m$. //

4. Element-Value Determination with Measurement Errors

Theorem 1 gave us a method for actual computation of the element-values of N_0[3,4]. We can obtain N_0 from N_m by reviving all the eliminated nodes one by one in the reverse order of elimination. In such backward process, we use admittances corresponding to edges of the dendroid to solve (8). However, actual computation of element-values has not taken measurement errors into consideration. Suppose that all the admittances of N_k are subjected to measurement errors and G_k has several dendroids. Then by using admittances corresponding the respective dendroids, we obtain several N_{k-1}'s whose admittances are different from one another, but we have no means of knowing which one is suitable for N_{k-1}.

Suppose that G_k has s edges and the dendroid consists of t edges, where s>t≥3. Then (8) constitutes a system of overdetermined equations with respect to variables $x_w^{(k-1)}$'s ($w \in V(G_k)$), that is, the number of equations is larger than the number of unknown variables. One of the common methods to resolve this problem is the least-squares method. We shall determine all the variables by minimizing

$$S^{(k-1)} = \sum_{[u,v] \in E(G_k)} (z_u^{(k-1)} z_v^{(k-1)} - a_{uv}^{(k)})^2 \qquad (10)$$

where $z_w^{(k-1)} = x_w^{(k-1)} / \sqrt{\Delta^{(k-1)}}$'s ($w = u, v$). The minimum is specified by setting all the partial derivatives $\partial S^{(k-1)} / \partial z_w^{(k-1)}$'s ($w \in V(G_k)$) equal to zero, *i.e.*,

$$\left.\begin{aligned} \partial S^{(k-1)} / \partial z_u^{(k-1)} &= 2 \sum_{[u,v] \in E(G_k)} (z_u^{(k-1)} z_v^{(k-1)} - a_{uv}^{(k)}) z_v^{(k-1)} = 0 \\ \partial S^{(k-1)} / \partial z_v^{(k-1)} &= 2 \sum_{[u,v] \in E(G_k)} (z_u^{(k-1)} z_v^{(k-1)} - a_{uv}^{(k)}) z_u^{(k-1)} = 0 \end{aligned}\right\} \qquad (11)$$

(11) is a system of nonlinear equations, and can be solved by Newton-Raphson method starting from the element-values of N_{k-1} obtained based on a dendroid as the initial guess. Then, substituting the solution into $\Delta^{(k-1)}$, $x_w^{(k-1)}$'s ($w \in V(G_k)$) are obtained as follows;

$$x_w^{(k-1)} = K^{(k-1)} z_w^{(k-1)} \qquad (12)$$

where $K^{(k-1)} = \sum_p z_p^{(k-1)}$.

A system of nonlinear equations (11) can be linearized by taking logarithms before summing squares. Taking logarithms of both sides of (8), the following sum of squares is obtained instead of (10),

$$\tilde{S}^{(k-1)} = \sum_{[u,v] \in E(G_k)} (\tilde{z}_u^{(k-1)} + \tilde{z}_v^{(k-1)} - \tilde{a}_{uv}^{(k)})^2 \qquad (13)$$

where $\tilde{z}_w^{(k-1)} = \ln(x_w^{(k-1)} / \sqrt{\Delta^{(k-1)}})$ and $\tilde{a}_{uv}^{(k)} = \ln(a_{uv}^{(k)})$ (w=u, v). Setting all the partial derivatives $\partial \tilde{S}^{(k-1)} / \partial \tilde{z}_w^{(k-1)}$'s $(w \in V(G_k))$ equal to zero gives

$$\left.\begin{aligned} \partial \tilde{S}^{(k-1)} / \partial \tilde{z}_u^{(k-1)} &= 2 \sum_{[u,v] \in E(G_k)} (\tilde{z}_u^{(k-1)} + \tilde{z}_v^{(k-1)} - \tilde{a}_{uv}^{(k)}) = 0 \\ \partial \tilde{S}^{(k-1)} / \partial \tilde{z}_v^{(k-1)} &= 2 \sum_{[u,v] \in E(G_k)} (\tilde{z}_u^{(k-1)} + \tilde{z}_v^{(k-1)} - \tilde{a}_{uv}^{(k)}) = 0 \end{aligned}\right\} \qquad (14)$$

(14) is a system of linear equations with respect to $\tilde{z}_w^{(k-1)}$ $(w \in V(G_k))$. Solving (14), $x_w^{(k-1)}$'s are obtained as follows;

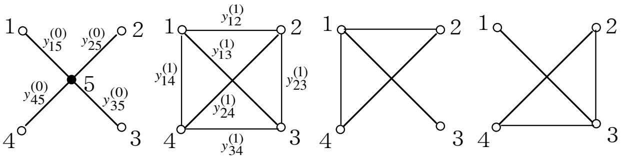
$$x_w^{(k-1)} = \tilde{K}^{(k-1)} \exp(\tilde{z}_w^{(k-1)}) \qquad (15)$$

where $\tilde{K}^{(k-1)} = \sum_p \exp(\tilde{z}_p^{(k-1)})$.

Example 1. We work out a resistive network N_0 of Fig.2(a) to illustrate the procedure. Let us assume that nodes 1, 2, 3 and 4 are accessible and node 5 is inaccessible. Fig.2(b) illustrates N_1 obtained from N_0 by eliminating node 5 by a generalized star-to-polygon transformation, where

$$\left.\begin{aligned} y_{12}^{(1)}(= y_{21}^{(1)}) &= y_{15}^{(0)} y_{25}^{(0)} / \Delta^{(0)}, y_{13}^{(1)}(= y_{31}^{(1)}) = y_{15}^{(0)} y_{35}^{(0)} / \Delta^{(0)} \\ y_{14}^{(1)}(= y_{41}^{(1)}) &= y_{15}^{(0)} y_{45}^{(0)} / \Delta^{(0)}, y_{23}^{(1)}(= y_{32}^{(1)}) = y_{25}^{(0)} y_{35}^{(0)} / \Delta^{(0)} \\ y_{24}^{(1)}(= y_{42}^{(1)}) &= y_{25}^{(0)} y_{45}^{(0)} / \Delta^{(0)}, y_{34}^{(1)}(= y_{43}^{(1)}) = y_{35}^{(0)} y_{45}^{(0)} / \Delta^{(0)} \end{aligned}\right\} \qquad (16)$$

and $\Delta^{(0)} = y_{15}^{(0)} + y_{25}^{(0)} + y_{35}^{(0)} + y_{45}^{(0)}$.



(a) N_0 (b) N_1 (c) G_1 (d) G_1'

Fig.2 Circuits and associated graphs for example 1.

Since the associated graph has the same structure as N_1 and has several connected dendroids, we can see from theorem 1 that N_0 can be uniquely restored from N_1, if all the admittances of N_1 are free of measurement errors. The original circuit, denoted by N_0, can be uniquely obtained by using admittances of N_1 corresponding to edges of G_1 of Fig.2(c). Also, the another original circuit, denoted by N_0', can be uniquely obtained by using admittances of N_1 corresponding to edges of the another dendroid G_1' of Fig.2(d). If all the admittances of N_1 are subject to measurement errors, then N_0 and N_0' are different from each other. Next we shall determine all $y_{pq}^{(0)}$'s by minimizing the sum of squares,

$$S^{(0)} = \sum_{p,q=1}^{4} (z_p^{(0)} z_q^{(0)} - a_{pq}^{(1)})^2 \qquad (p < q) \qquad (17)$$

where $z_p^{(0)} = y_{p5}^{(0)} / \sqrt{\Delta^{(0)}}$ and $a_{pq}^{(1)} = y_{pq}^{(1)}$. At the minimum for $S^{(0)}$, all the partial derivatives $\partial S^{(0)} / \partial z_p^{(0)}$'s vanish. Writing the equations for these gives four equations;

$$\left.\begin{aligned} \partial S^{(0)} / \partial z_1^{(0)} &= 2\{ f_{12}^{(0)} z_2^{(0)} + f_{13}^{(0)} z_3^{(0)} + f_{14}^{(0)} z_4^{(0)} \} = 0 \\ \partial s^{(0)} / \partial z_2^{(0)} &= 2\{ f_{12}^{(0)} z_1^{(0)} + f_{23}^{(0)} z_3^{(0)} + f_{24}^{(0)} z_4^{(0)} \} = 0 \\ \partial S^{(0)} / \partial z_3^{(0)} &= 2\{ f_{13}^{(0)} z_1^{(0)} + f_{23}^{(0)} z_2^{(0)} + f_{34}^{(0)} z_4^{(0)} \} = 0 \\ \partial S^{(0)} / \partial z_4^{(0)} &= 2\{ f_{14}^{(0)} z_1^{(0)} + f_{24}^{(0)} z_2^{(0)} + f_{34}^{(0)} z_3^{(0)} \} = 0 \end{aligned}\right\} \qquad (18)$$

where $f_{pq}^{(0)} = z_p^{(0)} z_q^{(0)} - a_{pq}^{(1)}$. The system of (18) is solved by using Newton-Raphson method starting from the element-values of N_0 or N_0' as the initial guess. Then, substituting the solution into $\Delta^{(0)}$, admittances of N_0 is obtained as follows;

$$y_{p5}^{(0)} = K^{(0)} z_p^{(0)} \qquad (p = 1,2,3,4) \qquad (19)$$

where $K^{(0)} = z_1^{(0)} + z_2^{(0)} + z_3^{(0)} + z_4^{(0)}$.

Taking logarithms of both sides of (16), the following sum of squares is obtained instead of (17),

$$\tilde{S}^{(0)} = \sum_{p,q=1}^{4} (\tilde{z}_p^{(0)} + \tilde{z}_q^{(0)} - \tilde{a}_{pq}^{(1)})^2 \tag{20}$$

where $\tilde{z}_p^{(0)} = \ln(y_{p5}^{(0)} / \sqrt{\Delta^{(0)}})$ and $\tilde{a}_{pq}^{(1)} = \ln(y_{pq}^{(1)})$. Setting all the partial derivatives $\partial\tilde{S}^{(0)} / \partial\tilde{z}_p^{(0)}$'s (p=1,2,3,4) equal to zero gives

$$\begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 \\ 1 & 1 & 3 & 1 \\ 1 & 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} \tilde{z}_1^{(0)} \\ \tilde{z}_2^{(0)} \\ \tilde{z}_3^{(0)} \\ \tilde{z}_4^{(0)} \end{bmatrix} = \begin{bmatrix} \tilde{a}_{12}^{(1)} + \tilde{a}_{13}^{(1)} + \tilde{a}_{14}^{(1)} \\ \tilde{a}_{12}^{(1)} + \tilde{a}_{23}^{(1)} + \tilde{a}_{24}^{(1)} \\ \tilde{a}_{13}^{(1)} + \tilde{a}_{23}^{(1)} + \tilde{a}_{34}^{(1)} \\ \tilde{a}_{14}^{(1)} + \tilde{a}_{24}^{(1)} + \tilde{a}_{34}^{(1)} \end{bmatrix} \tag{21}$$

Solving (21), $y_{p5}^{(0)}$ are obtained as follows;

$$y_{p5}^{(0)} = \tilde{K}^{(0)} \exp(\tilde{z}_p^{(0)}) \quad \text{(p=1,2,3,4)} \tag{22}$$

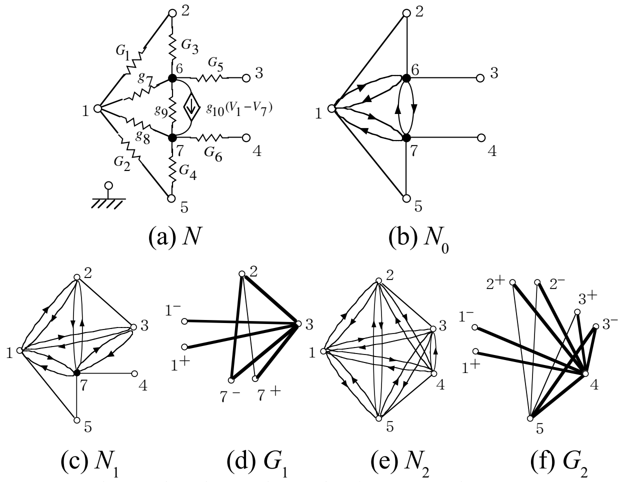
where $\tilde{K}^{(0)} = \exp(\tilde{z}_1^{(0)}) + \exp(\tilde{z}_2^{(0)}) + \exp(\tilde{z}_3^{(0)}) + \exp(\tilde{z}_4^{(0)})$.

When all the admittances of N_1 contain measurement errors within ±1 %, calculation results are given in Table 1. We see that computation errors resulting from the method of least-squares are smaller than those based on the dendroids.

Table 1 Calculation results.

		$G1$		$G1'$		Newton method		Linear eqs.	
	exact values [S]	calculated values [S]	errors [%]	calculated values [S]	errors [%]	calculated values [S]	errors [%]	calculated values [S]	errors [%]
$y_{15}^{(0)}$	1.000	0.994	0.60	1.026	2.64	0.996	0.39	0.997	0.34
$y_{25}^{(0)}$	2.000	2.028	1.41	2.012	0.60	2.007	0.35	1.993	0.34
$y_{35}^{(0)}$	3.000	3.104	3.46	2.958	1.39	2.961	1.29	2.990	0.34
$y_{45}^{(0)}$	4.000	4.056	1.41	4.024	0.60	3.998	0.04	3.986	0.34

Example 2. We illustrate DC small-signal equivalent circuit of a single-stage transistor amplifier of Fig.3(a), considered in [5]. The associated unistor circuit $N^* = N_0$ is shown in Fig.3(b). First, eliminating node 6 by a generalized star-to-polygon transformation, we obtain as N_1 and G_1 a circuit and a graph shown in (c) and (d) of Fig.3, respectively. Subsequently, eliminating node 7 by a generalized star-to-polygon transformation, we obtain as N_2 and G_2 a circuit and a graph shown in (e) and (f) of Fig.3. Since both G_2 and G_1 have the respective connected dendroids indicated by the bold lines in the figure, we can see from theorem 1 that N_0 can be restored from N_2. Since N_0 has two inaccessible nodes 6 and 7, the sum of squares, $S^{(1)} = \sum (z_p^{(1)} z_q^{(1)} - a_{pq}^{(2)})^2$, is required to be minimum in the restoration of N_1 from N_2, where $z_u^{(1)} = x_u^{(1)} / \sqrt{\Delta^{(1)}}\ (u \in V(G_2))$, $a_{pq}^{(2)} = y_{pq}^{(2)}\ ((p,q) \in E(G_2))$ and $\sum$ stands for the summation taken over all $(p,q) \in E(G_2)$. Subsequently, the sum of squares, $S^{(0)} = \sum (z_p^{(0)} z_q^{(0)} - a_{pq}^{(1)})^2$ is required to be minimum in the restoration of N_0 from N_1, where $z_u^{(0)} = x_u^{(0)} / \sqrt{\Delta^{(0)}}\ (u \in V(G_1))$, $a_{pq}^{(1)} = y_{pq}^{(1)}((p,q) \in E(G_1))$ and $\sum$ stands for the summation taken over all $(p,q) \in E(G_1)$. The system similar to (11), obtained by setting the partial derivatives $\partial S^{(1)} / \partial z_p^{(1)}$'s (resp., $\partial S^{(0)} / \partial z_p^{(0)}$'s) equal to zero is solved by using Newton-Raphson method starting from the element-values of N_2(resp., N_1) obtained based on dendroid G_2(resp., G_1) as the initial guess. In the case where all the admittances of N_2 are assumed to be subjected to measurement errors within ±1 %, calculation results with resistances are given in Table 2. We see that the computation errors resulting from the method of least-squares are smaller.



(a) N (b) N_0

(c) N_1 (d) G_1 (e) N_2 (f) G_2

Fig.3 Circuits and graphs for example 2.

Table 2 Calculation results.

		dendroids		Newton method		Linear eqs.	
	exact values [kΩ]	calculated values[kΩ]	errors [%]	calculated values[kΩ]	errors [%]	calculated values[kΩ]	errors [%]
R_1	90.00	89.11	0.99	89.11	0.99	89.11	0.99
R_2	10.00	9.901	0.99	9.901	0.99	9.901	0.99
R_3	5.00	5.033	0.67	5.050	1.01	5.046	0.93
R_4	1.00	0.987	0.91	0.991	0.91	0.998	0.24
R_5	1.00	1.027	2.70	1.010	0.99	1.014	1.43
R_6	1.00	1.007	0.67	1.009	0.93	1.003	0.26
r_7	2.360×10^9	2.329×10^9	1.32	2.337×10^9	0.99	2.335×10^9	1.07
r_8	3.670×10^8	3.609×10^8	1.65	3.632×10^8	1.03	3.622×10^8	1.32
r_9	1.670	1.648	1.32	1.655	0.91	1.666	0.24
r_{10}	1.670×10^{-2}	1.642×10^{-2}	1.68	1.654×10^{-2}	1.00	1.656×10^{-2}	0.83

5. Conclusion

In this paper, it has been shown that the element-value determination taking into account the measurement errors constitutes a system of overdetermined equations. We have proposed the least-squares method for actual computation of element-values. Newton-Raphson method converges rapidly because the initial guess is from the element-values previously determined based on dendroids.

References

[1] R.S.Berkowitz,"Conditions for network-element-value solvability",IRE Trans. Circuit Theory,CT-9, pp.24-29, 1962.

[2] S.Shinoda and K.Okada,"On solutions of the Element-value determinability problem of linear analog circuits," IEICE Trans. Fundamentals, vol. E77-A, No.7, pp1132-1143, 1994.

[3] I.Yamaguchi, S.Shinoda and T. Ozawa,"Conditions on the parameter-value determinability of linear analog circuits," Trans. IEICE, vol. J70-A, No.5, pp.839-855, 1987.

[4] I.Yamaguchi and S.Shinoda,"A sufficient condition for the unique restorability of circuits and its related graph-theoretic problems," Trans. IEICE, J73-A, No.4, pp.839-855, 1990.

[5] N.Navid and A.N.Willson, Jr.,"A theory and an algorithm for analog circuit fault diagnosis," IEEE Trans. Circuit and System, CAS-26, pp.440-457, 1979.