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Analysis of Performance of Noncoherent DCSK Communication Systems Over a Multipath Rayleigh Fading Channel with Delay Spread

Yongxiang Xia, Chi K. Tse and Francis C. M. Lau

Department of Electronic and Information Engineering, Hong Kong Polytechnic University, Hong Kong, China
http://chaos.eie.polyu.edu.hk

Abstract— The performance of the noncoherent differential chaos-shift keying (DCSK) communication system over a multipath fading channel with delay spread is evaluated. Analytical expressions of the bit error rates are derived under the assumption of an independent Rayleigh fading two-ray channel model. Analytical and simulated results are presented and compared. The multipath performance of the DCSK system is also compared with that of the coherent chaos-shift-keying (CSK) system.

1. Introduction

The performance of chaos-based digital communication systems under an additive white Gaussian noise (AWGN) environment has been thoroughly studied [1]–[4]. However, the multipath performance analysis and data for chaos-based communication systems are generally unavailable. The earliest study of multipath performance of chaos-based communication systems was performed by Kolumbán and Kis [5] for the frequency-modulated differential chaos-shift-keying (FM-DCSK) system. Their study was simulation-based and each path in the two-ray channel model was assumed an ideal constant gain value. In practice, however, each path suffers from random fading, which should be duly incorporated in the channel model [3]. Recently, Mandal and Banerjee [6] analyzed the performance of the differential chaos-shift-keying (DCSK) system over a channel with Rayleigh fading or Ricean fading. However, the multipath time delay has not been considered. In a spread spectrum communication system such as DCSK, it is necessary to model the effects of multipath delay spread as well as fading. In this paper we evaluate the performance of the DCSK system over a practical multipath fading channel, incorporating multipath fading for each path and the effects of delay spread. Results will be compared with the benchmark data obtained earlier for the coherent chaos-shift-keying (CSK) system [7].

2. System Model

Fig. 1 shows the block diagram of the DCSK communication system. The lth transmitted symbol is denoted by b_l, which is either $+1$ or -1, and we assume that $+1$ and -1 occur with equal probabilities. During the lth symbol duration, the transmitted signal, s_k, is

$$s_k = \begin{cases} x_k & k = 2(l-1)\beta + 1, \cdots, (2l-1)\beta \\ b_l x_{k-\beta} & k = (2l-1)\beta + 1, \cdots, 2l\beta \end{cases} \tag{1}$$

where 2β is the spreading factor.

In studying spread spectrum wireless communication systems, a commonly used channel model is the *two-ray Rayleigh fading channel model* [3], as shown in Fig. 2. Using the discrete-time baseband equivalent model, the output of the channel is represented as

$$\text{output} = \alpha_1 s_k + \alpha_2 s_{k-\tau} \tag{2}$$

where α_1 and α_2 are independent and Rayleigh distributed random variables, τ is the time delay between two rays, and

$$s_{k-\tau} = \begin{cases} b_{l-1} x_{k-\beta-\tau} & k{=}2(l-1)\beta + 1, \cdots, 2(l-1)\beta + \tau \\ x_{k-\tau} & k{=}2(l-1)\beta + \tau + 1, \cdots, (2l-1)\beta \\ x_{k-\tau} & k{=}(2l-1)\beta + 1, \cdots, (2l-1)\beta + \tau \\ b_l x_{k-\beta-\tau} & k{=}(2l-1)\beta + \tau + 1, \cdots, 2l\beta. \end{cases}$$

In the receiver, a correlation-based detection is used, as shown in Fig. 1. After going through the two-ray Rayleigh fading channel, the signal received by the receiver (i.e., input to the correlator) is given by

$$r_k = \alpha_1 s_k + \alpha_2 s_{k-\tau} + \xi_k \tag{3}$$

where ξ_k is AWGN with mean equal to zero and variance $N_0/2$. Considering the lth symbol, the decision variable is the output of the correlator, which is given by

$$\begin{aligned} c_l &= \sum_{k=(2l-1)\beta+1}^{2l\beta} r_k r_{k-\beta} \\ &= \sum_{k=(2l-1)\beta+1}^{(2l-1)\beta+\tau} (\alpha_1 b_l x_{k-\beta} + \alpha_2 x_{k-\tau} + \xi_k) \\ &\qquad \times (\alpha_1 x_{k-\beta} + \alpha_2 b_{l-1} x_{k-2\beta-\tau} + \xi_{k-\beta}) \\ &\quad + \sum_{k=(2l-1)\beta+\tau+1}^{2l\beta} (\alpha_1 b_l x_{k-\beta} + \alpha_2 b_l x_{k-\beta-\tau} + \xi_k) \\ &\qquad \times (\alpha_1 x_{k-\beta} + \alpha_2 x_{k-\beta-\tau} + \xi_{k-\beta}). \end{aligned} \tag{4}$$

Then, the lth decoded symbol is determined according to the following rule:

$$\tilde{b}_l = \begin{cases} +1 & \text{if } c_l \geq 0 \\ -1 & \text{if } c_l < 0. \end{cases} \tag{5}$$

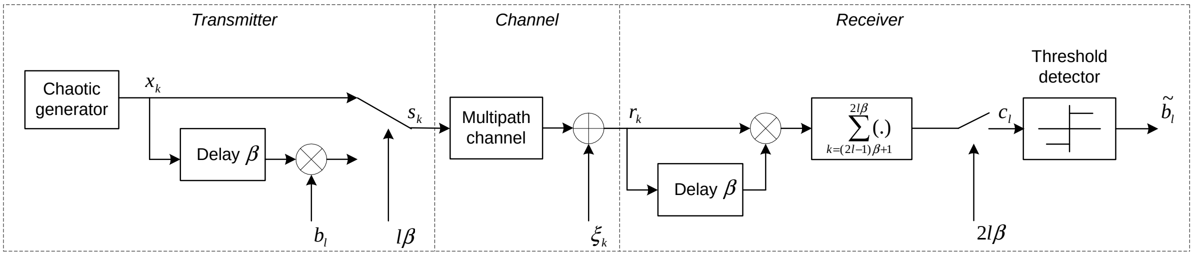

Figure 1: Block diagram of the DCSK communication system.

3. Analysis of Bit Error Performance

In the following analysis we assume that the multipath time delay is much shorter than the bit duration, i.e., $0 < \tau \ll 2\beta$. With such an assumption, the inter-symbol interference (ISI) is negligible compared with the interference within each symbol due to multipath time delay. Also, (4) may be approximated as

$$\begin{aligned} c_l \approx & \sum_{k=(2l-1)\beta+1}^{2l\beta} (\alpha_1 b_l x_{k-\beta} + \alpha_2 b_l x_{k-\beta-\tau} + \xi_k) \\ & \times (\alpha_1 x_{k-\beta} + \alpha_2 x_{k-\beta-\tau} + \xi_{k-\beta}). \end{aligned} \quad (6)$$

For large β and a given chaotic map (e.g., logistic map), we have

$$\sum_{k=(2l-1)\beta+1}^{2l\beta} x_{k-\beta} x_{k-\beta-\tau} \approx 0. \quad (7)$$

Thus, c_l may be simplified as

$$\begin{aligned} c_l \approx & \sum_{k=(2l-1)\beta+1}^{2l\beta} \alpha_1^2 b_l x_{k-\beta}^2 + \sum_{k=(2l-1)\beta+1}^{2l\beta} \alpha_2^2 b_l x_{k-\beta-\tau}^2 \\ & + \sum_{k=(2l-1)\beta+1}^{2l\beta} (\alpha_1 x_{k-\beta} + \alpha_2 x_{k-\beta-\tau})(\xi_k + b_l \xi_{k-\beta}) \\ & + \sum_{k=(2l-1)\beta+1}^{2l\beta} \xi_k \xi_{k-\beta} \\ = & A + B + C \end{aligned}$$

where

$$\begin{aligned} A &= \sum_{k=(2l-1)\beta+1}^{2l\beta} \alpha_1^2 b_l x_{k-\beta}^2 + \sum_{k=(2l-1)\beta+1}^{2l\beta} \alpha_2^2 b_l x_{k-\beta-\tau}^2 \\ B &= \sum_{k=(2l-1)\beta+1}^{2l\beta} (\alpha_1 x_{k-\beta} + \alpha_2 x_{k-\beta-\tau})(\xi_k + b_l \xi_{k-\beta}) \\ C &= \sum_{k=(2l-1)\beta+1}^{2l\beta} \xi_k \xi_{k-\beta}. \end{aligned}$$

Assuming "+1" is transmitted (i.e., $b_l = +1$), the following statistics are easily obtained

$$\begin{aligned} \mathrm{E}\{A|(\alpha_1,\alpha_2,b_l=+1)\} &= (\alpha_1^2+\alpha_2^2)\beta\mathrm{E}\{x_k^2\} \end{aligned}$$

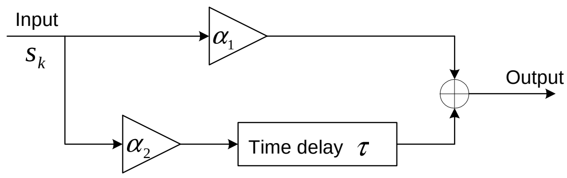

Figure 2: Two-ray Rayleigh fading channel model.

$$\begin{aligned} \mathrm{E}\{B|(\alpha_1,\alpha_2,b_l=+1)\} &= \mathrm{E}\{C|(\alpha_1,\alpha_2,b_l=+1)\} \\ &= 0 \\ \mathrm{var}\{A|(\alpha_1,\alpha_2,b_l=+1)\} &= (\alpha_1^4+\alpha_2^4)\beta\mathrm{var}\{x_k^2\} \\ \mathrm{var}\{B|(\alpha_1,\alpha_2,b_l=+1)\} &= (\alpha_1^2+\alpha_2^2)\beta\mathrm{E}\{x_k^2\}N_0 \\ \mathrm{var}\{C|(\alpha_1,\alpha_2,b_l=+1)\} &= \frac{1}{4}\beta N_0^2 \\ \mathrm{cov}\{A,B|(\alpha_1,\alpha_2,b_l=+1)\} &= \mathrm{cov}\{B,C \\ &\quad |(\alpha_1,\alpha_2,b_l=+1)\} \\ &= \mathrm{cov}\{A,C \\ &\quad |(\alpha_1,\alpha_2,b_l=+1)\} \\ &= 0 \end{aligned}$$

where E[·] and var[·] represent the expectation and variance operators, respectively, and $\mathrm{cov}[X,Y]$ denotes the covariance of X and Y. Then, we have

$$\mathrm{E}\{c_l|(\alpha_1,\alpha_2,b_l=+1)\} = \beta(\alpha_1^2+\alpha_2^2)\mathrm{E}\{x_k^2\} \quad (8)$$

$$\begin{aligned} \mathrm{var}\{c_l|(\alpha_1,\alpha_2,b_l=+1)\} = & \beta(\alpha_1^4+\alpha_2^4)\mathrm{var}\{x_k^2\} \\ & + \beta(\alpha_1^2+\alpha_2^2)\mathrm{E}\{x_k^2\}N_0 \\ & + \frac{1}{4}\beta N_0^2. \end{aligned} \quad (9)$$

The case of sending a symbol of "-1" may be computed in a likewise fashion, i.e.,

$$\mathrm{E}\{c_l|(\alpha_1,\alpha_2,b_l=-1)\} = -\mathrm{E}\{c_l|(\alpha_1,\alpha_2,b_l=+1)\}$$

$$\mathrm{var}\{c_l|(\alpha_1,\alpha_2,b_l=-1)\} = \mathrm{var}\{c_l|(\alpha_1,\alpha_2,b_l=+1)\}.$$

Using (8) and (9), and assuming that c_l follows a normal distribution under the given conditions, the conditional BER may be computed as

$$\begin{aligned} \mathrm{BER}(\alpha_1,\alpha_2) = & \frac{1}{2}\mathrm{Prob}(c_l < 0|(\alpha_1,\alpha_2,b_l=+1)) \\ & + \frac{1}{2}\mathrm{Prob}(c_l \geq 0|(\alpha_1,\alpha_2,b_l=-1)) \\ = & \frac{1}{2}\mathrm{erfc}\left(\left[\frac{2(\alpha_1^4+\alpha_2^4)\mathrm{var}\{x_k^2\}}{(\alpha_1^2+\alpha_2^2)^2\beta\mathrm{E}^2\{x_k^2\}}\right.\right. \end{aligned}$$

$$+ \quad \frac{4N_0}{(\alpha_1^2+\alpha_2^2)E_b} + \frac{2\beta N_0^2}{(\alpha_1^2+\alpha_2^2)^2 E_b^2}\Bigg]^{-\frac{1}{2}}\Bigg)$$

where E_b is the bit energy and is represented by

$$E_b = 2\beta \mathrm{E}\left\{(x_k)^2\right\} \tag{10}$$

and $\mathrm{erfc}(\psi) \equiv \frac{2}{\sqrt{\pi}}\int_\psi^\infty e^{-\lambda^2} d\lambda$.

If the logistic map is used, we have $\mathrm{var}\{x_k^2\} = 1/8$, $\mathrm{E}\{x_k^2\} = 1/2$, and

$$\begin{aligned}\mathrm{BER}(\alpha_1,\alpha_2) &= \frac{1}{2}\mathrm{erfc}\left(\left[\frac{(\alpha_1^4+\alpha_2^4)}{(\alpha_1^2+\alpha_2^2)^2\beta}\right.\right. \\ &+ \left.\left.\frac{4N_0}{(\alpha_1^2+\alpha_2^2)E_b} + \frac{2\beta N_0^2}{(\alpha_1^2+\alpha_2^2)^2 E_b^2}\right]^{-\frac{1}{2}}\right)\end{aligned}$$

For large β, the first term within the bracket in the above expression may be neglected. Thus, the conditional BER may be simplified as

$$\mathrm{BER}(\alpha_1,\alpha_2) = \frac{1}{2}\mathrm{erfc}\left(\left(\frac{4}{\gamma_b}+\frac{2\beta}{\gamma_b^2}\right)^{-\frac{1}{2}}\right) = \mathrm{BER}(\gamma_b)$$

where $\gamma_b = \frac{E_b}{N_0}(\alpha_1^2+\alpha_2^2) = \gamma_1+\gamma_2$, $\gamma_1 = \frac{E_b}{N_0}\alpha_1^2$, and $\gamma_2 = \frac{E_b}{N_0}\alpha_2^2$. Denoting $\bar{\gamma}_1 = \mathrm{E}\{\gamma_1\} = \frac{E_b}{N_0}\mathrm{E}\{\alpha_1^2\}$ and $\bar{\gamma}_2 = \mathrm{E}\{\gamma_2\} = \frac{E_b}{N_0}\mathrm{E}\{\alpha_2^2\}$, the probability density function of γ_b may be computed as

$$f(\gamma_b) = \begin{cases} \frac{\gamma_b}{\bar{\gamma}_1^2} e^{-\gamma_b/\bar{\gamma}_1} & \mathrm{E}\{\alpha_1^2\} = \mathrm{E}\{\alpha_2^2\} \\ \frac{1}{\bar{\gamma}_1-\bar{\gamma}_2}\left(e^{-\gamma_b/\bar{\gamma}_1} - e^{-\gamma_b/\bar{\gamma}_2}\right) & \mathrm{E}\{\alpha_1^2\} \neq \mathrm{E}\{\alpha_2^2\}. \end{cases} \tag{11}$$

Finally, the BER can be obtained by averaging the conditional BER, i.e.,

$$\mathrm{BER} = \int_0^\infty \mathrm{BER}(\gamma_b) f(\gamma_b) d\gamma_b. \tag{12}$$

This formula will be used in the next section for evaluating the bit error performance of the system under different channel conditions. Clearly, as the form of (12) does not permit a closed-form solution, we have to resort to a numerical integration procedure for finding the BERs.

4. Results

We consider three cases corresponding to different path gain ratios and a fixed path delay.

Case I: The two paths have identical average power gain.

Case II: The average power gain of the second path is 3 dB below that of the first path.

Case III: The average power gain of the second path is 10 dB below that of the first path.

Numerical calculation of the BERs as well as computer simulation of the BERs are performed. In particular, Fig. 3 shows the effect of β on the BER performance. Here, we set $E_b/N_0 = 25$ dB and $\tau = 2$. From the figure we can see that BER increases with β. This is because for a fixed E_b/N_0, the noise power increases with β and the degradation due to the increased noise power will overwhelm any gain in symbol detection that might have resulted when β is large. In Fig. 4, the calculated and simulated BERs are plotted for $\beta = 50$.

The effect of τ on the BER performance is shown in Fig. 5, in which $E_b/N_0 = 25$ dB, and $\beta = 50$. The figure shows that when τ is large, the numerical results do not agree with the simulated ones. This is because in the derivation of the BER (12), we have assumed that the multipath time delay is much less than the bit duration ($\tau \ll 2\beta$) and hence ISI can be neglected. Under this assumption the numerical BER result is independent of τ. However, in the simulations, ISI is present and it increases with τ. Therefore, as τ increases, ISI increases and the simulated BER deteriorates. Fortunately, in most practical applications, the condition $\tau \ll 2\beta$ holds, such that neglecting the ISI is justifiable. For example, in typical wireless local area network (WLAN) applications, the time delay τ is some tens of ns, which is very small compared with the duration of a transmitted symbol [5].

The coherent CSK system, being a generic form of chaos-based digital communication systems, may serve as a benchmark system for comparison [7]. Fig. 6 shows the simulated results of the BER performance of the coherent CSK as well as the noncoherent DCSK over the AWGN channel and the two-ray Rayleigh fading channel. In the 2-ray Rayleigh fading case, we assume that the average power gain of the second path is 3 dB below that of the first path. Under an AWGN channel, the coherent CSK system performs much better compared with DCSK. In a two-ray fading environment, the performance of the coherent CSK system degrades dramatically [7] but for the noncoherent DCSK, the performance degradation is much less severe. Thus, the advantage of the coherent CSK system over the DCSK system diminishes. The same observation is made when E_b/N_0 is large (say > 21 dB). This fact also verifies Kolumbán's conclusion that the DCSK system can be used even under poor channel propagation conditions [8].

5. Conclusions

We have studied the multipath performance of the noncoherent DCSK system based on a two-ray Rayleigh fading model. Results from this study show that the DCSK system can perform better than the coherent CSK system and achieve a reasonable BER in a multipath environment.

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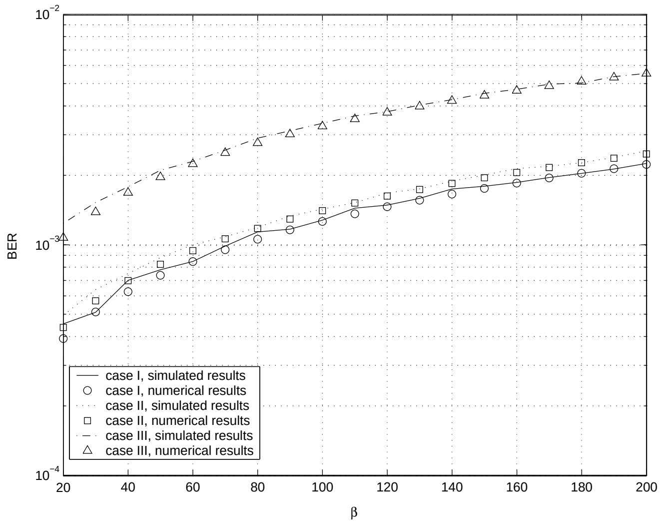

Figure 3: BER performance of the DCSK system over a two-ray Rayleigh fading channel (BER versus β) with $E_b/N_0 = 25$ dB and $\tau = 2$.

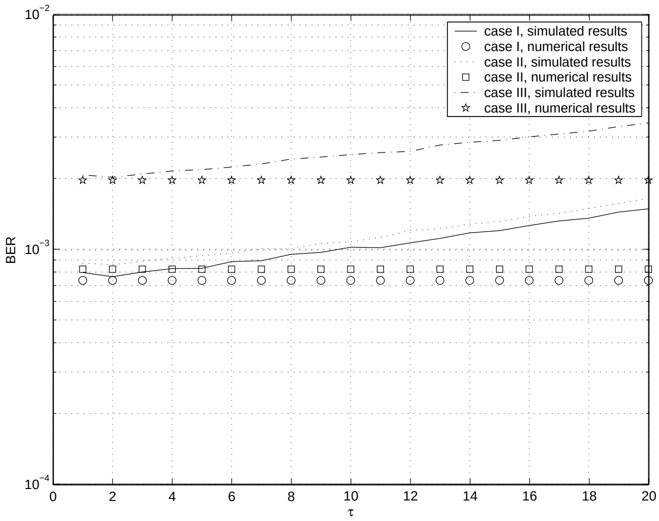

Figure 5: BER performance of the DCSK system over a two-ray Rayleigh fading channel (BER versus τ) with $E_b/N_0 = 25$ dB $\beta = 50$.

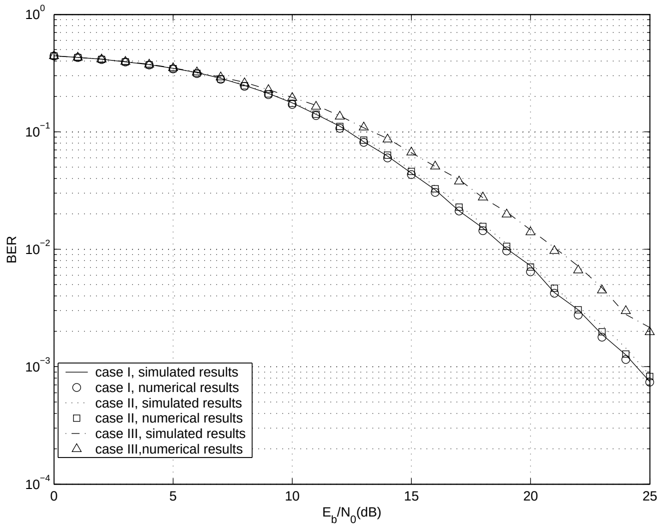

Figure 4: BER performance of the DCSK system over a two-ray Rayleigh fading channel, with $\beta = 50$ and $\tau = 2$.

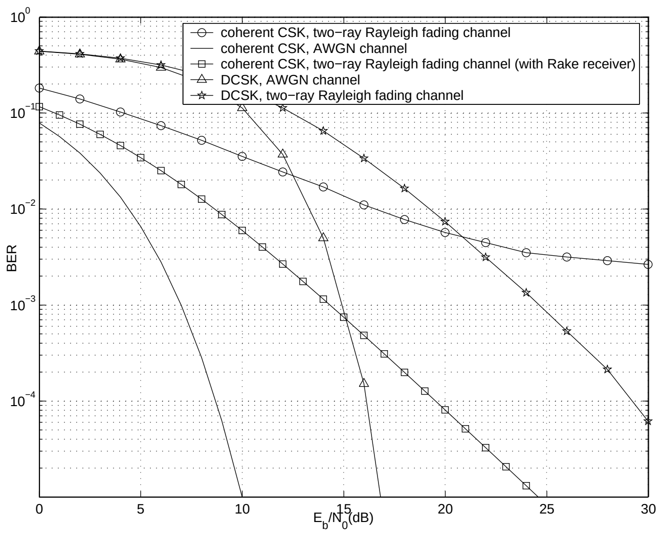

Figure 6: Simulated results of the BER performance of the coherent CSK and noncoherent DCSK systems.

References

[1] F. C. M. Lau and C. K. Tse, *Chaos-based Digital Communication Systems,* Heidelberg: Springer-Verlag, 2003.

[2] G. Kolumbán, "Theoretical noise performance of correlator-based chaotic communications schemes," *IEEE Trans. Circ. Syst. I,* vol. 47, no. 12, pp. 1692–1701, Dec. 2000.

[3] T. S. Rappaport, *Wireless Communications: Principles and Practice,* Englewood Cliffs NJ: Prentice-Hall, 1996.

[4] G. Kolumbán, "Basis function description of chaotic modulation scheme," *Proc. Int. Workshop on Nonlinear Dynamics of Electronic Systems,* 2000, pp. 165–169.

[5] G. Kolumbán and G. Kis, "Multipath performance of FM-DCSK chaotic communications system," *Proc. IEEE International Symposium on Circuits and Systems,* 2000, vol. IV, pp. 433–436.

[6] S. Mandal and S. Banerjee, "Performance of differential chaos shift keying over multipath fading channels," *Indian National Conf. Nonlinear Systems and Dynamics,* 2003.

[7] Y. Xia, C. K. Tse, and F. C. M. Lau, "Some benchmark multipath performance data of coherent CSK digital communication systems," *Proc. Int. Workshop on Nonlinear Circuits Signal Processing,* 2004, pp. 261–264.

[8] G. Kolumbán, M. P. Kennedy, and L. O. Chua, "The role of synchronization in digital communications using chaos—part II: Chaotic modulation and chaotic synchronization," *IEEE Trans. Circ. Syst. I,* vol. 45, no. 11, pp. 1129–1140, Nov. 1998.