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Fine Estimation Theory for Available Operation of Network Systems Extraordinarily Complicated and Diversified on Large-Scales

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Abstract—In this report, we shall construct mathematical theory based on the concept of set-valued mappings, suitable for available operation of network systems extraordinarily complicated and diversified on large scales, by introducing some connected-block structures. A fine estimation technique for availability of system behaviors of such network systems are obtained finally in the form of fixed point theorem for a special system of fuzzy-set-valued mappings.

1. Introduction

In large-scale network systems, so called as multi-media and/or integrated network systems, the precise evaluation and the perfect control, and also the ideal operation, of overall system behaviors cannot be necessarily expected by using any type of commonplace technologies for maintenance, which might be usually accomplished by simple measures in hierarchical network structures. Therefore, in order to evaluate, control and operate such complicated large-scale network systems, the quite separate type of methodologies becomes to be important, rather than the type of usual ones. They have to be introduced for procedure suitable to treating with undesirable uncertainties induced into those network systems [1].

For such a purpose, the author has recommended to introduce some connected-block structures: *i.e.*, whole networks might be separated into several blocks which are carefully self-evaluated, self-controlled and self-maintained by themselves, and so, which are originally self-sustained systems. In these network systems, whenever they observe and detect that some other blocks are in ill-conditions by some accidents, by always carefully watching each other, every block can repair and sustain those ill-conditioned blocks, through inter-block connections, at once. This style of maintenance of system is sometimes called as "locally autonomous", but the author recommends that only the ultimate responsibility on observation and regulation of whole system might be left for the headquarter itself, which is organized over all blocks[3, 2].

Here, let us consider complete metric linear spaces (X_i, ρ) $(i = 1, \cdots, n)$ and (Y_j, ρ) $(j = 1, \cdots, n)$, and their bounded convex closed subsets $X_i^{(0)}$ and $Y_j^{(0)}$, respectively, corresponding to each block, B_i and B_j of whole network system. Let us introduce mappings $f_{ij} : X_i \to Y_j$ such that $f_{ij}(X_i^{(0)}) \subset Y_j^{(0)}$ and let f_{ij} be completely continuous on $X_i^{(0)}$. For each block $B_i (i = 1, \cdots, n)$, dynamics of system behaviors can be represented originally by simple equations:

$$x_i = \alpha_i f_{ii}(x_i), \quad (i = 1, \cdots, n), \tag{1}$$

where α_i is a continuous operator: $Y_i^{(0)} \to X_i^{(0)}$. These equations have solutions x_i^* in every $X_i^{(0)} (i = 1, \cdots, n)$, according to the well-known Schauder's type of fixed point theorem[4]. Of course, these solutions represent original values of system behaviors.

But, in general conditions with inter-block connections, system dynamics may be represented in the form of system of equations:

$$x_i = \alpha_i f_{ii}(x_i) + \textstyle\sum_{j \neq i} \beta_{ji} f_{ij}(x_i) + \sum_{j \neq i} \gamma_{ji} f_{ji}(x_j), \quad (i = 1, \cdots, n), \tag{2}$$

where β_{ji} is a continuous operator: $Y_j \to X_i$ and γ_{ji} is a continuous operator: $Y_i \to X_i$. In the right-hand side of this system of equations, the first term represents the original performance of the i-th block itself, the second term represents the operation fed-back through all other blocks $(j \neq i)$ into the original i-th block, and the third term represents inter-block connections from all other blocks, in order to repair and sustain the i-th block performance. This system of equations also has solutions $x_i^* (i = 1, \cdots, n)$ in every $X_i^{(0)} (i = 1, \cdots, n)$, which represent resultant behaviors of block B_i, as a whole.

For more general situation of mutual connections between blocks, by newly introducing n composition-type mappings $g_i : X_i \times \Pi_j^n Y_j \times \Pi^n Y_i \to X_i$, where $\Pi_j^n Y_j$ means the direct product of n Y_j's for all $j \in \{1, \cdots, n\}$, and $\Pi^n Y_i$ means the direct product of n Y_i's for fixed i, we have general system of mapping equations:

$$x_i = g_i(x_i; f_{i1}(x_i), \cdots, f_{in}(x_i); f_{1i}(x_1), \cdots, f_{ni}(x_n)), \quad (i = 1, \cdots, n). \tag{3}$$

Here again, we can know that, under some natural conditions, these mappings g_i have at least one fixed point x_i^*

in the respective bounded convex closed subset $X_i^{(0)}$ $(i = 1, \cdots, n)$ [3].

These bounded closed subsets $X_i^{(0)}$ can be understood conveniently as "available" ranges of system behaviors $\{x_i^*\}$ $(i = 1, \cdots, n)$, if we can get them sufficiently small.

However, the fluctuation imposed on the actual system is nondeterministic rather than deterministic. In this case, even the effect due to a single cause is multi-valued, and the behavior is more naturally represented by a set of points, rather than a single point. Therefore, it is reasonable to consider some suitable subset of the range of system behavior, in place of single ideal point, as relevant target such that the behavior must reach under influence of system control. Now, we can newly name it as an "available range" of the system behavior. Thus, by the available range, we mean the range of behavior in which every behavior effectively satisfies tolerably good conditions beforehand specified, as a set of quasi-ideal behaviors. From such a point of view, the theory for fluctuation imposed on the system should be developed concerning the set-valued mapping [5, 6].

Here, by the set-valued mapping G defined on a space X is meant a correspondence in which a set $G(x)$ is specified in correspondence to any point x in X. In particular, when $G(X) \subset X$, and if there exists a point x^* such that $x^* \in G(x^*)$, x^* is called a fixed point of G[6].

2. A Fixed Point Theorem for a System of Set-Valued Nonlinear Mappings

Recently, the author presented a fixed point theorem for a general system of set-valued nonlinear mapping equations, as follows [2].

Corresponding to the original mappings g_i, let us introduce n set-valued mappings $G_i : X_i \times \Pi_j^n Y_j \times \Pi^n Y_i \rightarrow \mathcal{F}(X_i)$ (the family of all non-empty closed compact subsets of X_i) $(i = 1, \cdots, n)$.

Under some natural hypotheses, we can know that these mappings G_i have at least one fixed point x_i^* in the respective bounded convex closed subset $X_i^{(0)}$ such that,

$$x_i^* \in G_i(x_i^*; f_{i1}(x_i^*), \cdots, f_{in}(x_i^*); f_{1i}(x_1^*), \cdots, f_{ni}(x_n^*)), \quad (i = 1, \cdots, n). \tag{4}$$

This fixed point theorem can be applied to the analysis of complicated large-scale nonlinear networks, which consist of n nonlinear blocks originally represented by n mapping equations (3), undergone by some uncertain fluctuations so that system dynamics are overcome by the system of set-valued mapping equations (4). Here, bounded closed subsets $X_i^{(0)}$ are understood as "available" ranges of system behaviors $\{x_i^*\}$ $(i = 1, \cdots, n)$, as described at the last part of the preceding section.

Although this situation of system disturbance is originally crisp, at the next step, as a typical example of extremely refined methodologies using the concept of set - valued mappings, we will present a fixed point theorem for a system of fuzzy-set-valued nonlinear mapping equations, and will refer to available behaviors of such nonlinear network systems.

3. Fuzzy Set and Fuzzy-Set-Valued Mapping

First of all, let us consider a family of all fuzzy sets originally introduced by L. A. Zadeh [7], in a linear space X, and let any fuzzy set A be characterized by a membership function $\mu_A(x) : X \rightarrow [0, 1]$. Now, we can consider an α-level set A_α of the fuzzy set A as $A_\alpha \triangleq \{\xi \in X | \mu_A(\xi) \geq \alpha\}$, for any constant $\alpha \in (0, 1]$. The fuzzy set A is called compact, if all α-level sets are compact for arbitrary $\alpha \in (0, 1]$.

A fuzzy-set-valued mapping G from X into X is defined by $G : X \rightarrow \mathcal{F}(X)$, where $\mathcal{F}(X)$ is a family of all non-empty , bounded and closed fuzzy sets in X. If a point $x \in X$ is mapped to a fuzzy set $G(x)$, the membership function of $G(x)$ at the point $\xi \in X$ is represented by $\mu_{G(x)}(\xi)$.

For convenience' sake, let us introduce a useful notation: for an arbitrarily specified constant $\beta \in (0, 1]$, a point x belongs to the β-level set A_β of the fuzzy set A: $x \in A_\beta \triangleq \{\xi \in X \mid \mu_A(\xi) \geq \beta\}$ is denoted by $x \in_\beta A$ [8].

Here, let us introduce a new concept of β-level fixed point: for the fuzzy set $G(x)$, if there exists a point x^* such that $x^* \in_\beta G(x^*)$, then x^* is called β-level fixed point of the fuzzy-set-valued mapping G [8].

Now, let us remember that we have introduced a new metric into the space of fuzzy sets [8].

Definition 1 *Let us consider a metric linear space (X, ρ). For any fixed constant $\beta \in (0, 1]$, the β-level metric ρ_β between a point $x \in X$ and a fuzzy set A is defined as follows:*

$$\rho_\beta(x, A) \triangleq \inf_{\beta \leq \alpha \leq 1} d_\alpha(x, A), \tag{5}$$

where

$$d_\alpha(x, A) \triangleq \begin{cases} \inf_{y \in A_\alpha} \rho(x, y) & \text{if } \alpha \leq \alpha_A, \\ \inf_{y \in A_{\alpha_A}} \rho(x, y) & \text{if } \alpha > \alpha_A. \end{cases} \tag{6}$$

Here, $\alpha_A \triangleq \sup_{x \in X} \mu_A(x)$. And also, for any fixed constant $\beta \in (0, 1]$, by means of the Hausdorff metric d_H, the β-level metric $\mathcal{H}_\beta$ between two fuzzy sets A and B is introduced as follows:

$$\mathcal{H}_\beta(A, B) \triangleq \sup_{\beta \leq \alpha \leq 1} D_\alpha(A, B), \tag{7}$$

where D_α is defined as

$$D_\alpha(A, B) \triangleq \begin{cases} d_H(A_\alpha, B_\alpha) \\ \qquad \text{if } \alpha \leq \min\{\alpha_A, \alpha_B\}, \\ d_H(A_{\alpha_A}, B_\alpha) \\ \qquad \text{if } \alpha_A < \alpha \leq \alpha_B, \\ d_H(A_\alpha, B_{\alpha_B}) \\ \qquad \text{if } \alpha_A \geq \alpha > \alpha_B, \\ d_H(A_{\alpha_A}, B_{\alpha_B}) \\ \qquad \text{if } \alpha > \max\{\alpha_A, \alpha_B\}. \end{cases} \tag{8}$$

Here, $\alpha_B \triangleq \sup_{x\in X}\mu_B(x)$, and the Hausdorff metric d_H between two sets S_1 and S_2 is defined by

$$d_H(S_1,S_2) \;\triangleq\; \max\{\sup\{d(x_1,S_2)\mid x_1\in S_1\},\ \sup\{d(x_2,S_1)\mid x_2\in S_2\}\},$$

where $d(x,S)\triangleq\inf\{\rho(x,y)\mid y\in S\}$ is the distance between a point x and a set S.

In order to give a new methodology for the discussion more sophisticated than the one by usual set-valued mappings, the author presented mathematical theories based on the concept of β-level fixed point, by establishing fixed point theorems for β-level fuzzy-set-valued nonlinear mappings which describe detailed characteristics of such fuzzy-set-valued nonlinear mapping equations, for every level $\beta\in(0,1]$ [8].

4. A Fixed Point Theorem for a System of Fuzzy-Set-Valued Mappings

Here, we will present a fixed point theorem for a more general system of fuzzy-set-valued mapping equations.

Now, by using the same notation with crisp sets G_i, let us introduce n fuzzy-set-valued mappings $G_i : X_i\times\Pi_j^n Y_j\times\Pi^n Y_i\to\mathcal{F}(X_i)$ (the family of all non-empty closed compact fuzzy subsets of X_i) $(i=1,\cdots,n)$.

Moreover, let us introduce arbitrary constants $\beta_i\in(0,1]$, for every i $(i=1,\cdots,n)$, separately. Here, for any fixed constants $\beta_i\in(0,1]$, let G_i satisfy Lipschitz conditions with respect to the β_i-level metric $\mathcal{H}_{\beta_i}$: *i.e.*, there are three kinds of constants $0<a_i\equiv a_i(\beta_i)<1$, $b_{ji}\equiv b_{ji}(\beta_i)>0$ and $c_{ji}\equiv c_{ji}(\beta_i)>0$ such that for any $x_i^{(1)},x_i^{(2)}\in X_i^{(0)}$, for any $y_{ji}^{(1)},y_{ji}^{(2)}\in Y_i^{(0)}$, and for any $y_{ij}^{(1)},y_{ij}^{(2)}\in Y_j^{(0)}$, G_i's satisfy inequalities:

$$\begin{aligned}&\mathcal{H}_{\beta_i}(G_i(x_i^{(1)};y_{i1}^{(1)},\cdots,y_{in}^{(1)};y_{1i}^{(1)},\cdots,y_{ni}^{(1)}),\\ &\qquad G_i(x_i^{(2)};y_{i1}^{(2)},\cdots,y_{in}^{(2)};y_{1i}^{(2)},\cdots,y_{ni}^{(2)}))\\ &\le a_i\cdot\rho(x_i^{(1)},x_i^{(2)})+\sum_{j=1}^n b_{ji}\cdot\rho(y_{ij}^{(1)},y_{ij}^{(2)})\\ &\qquad+\sum_{j=1}^n c_{ji}\cdot\rho(y_{ji}^{(1)},y_{ji}^{(2)}),\\ &(i=1,\cdots,n).\end{aligned}\tag{9}$$

Further, let us assume that there is a unique β_i-level projection point $\tilde{x}_i^{(0)}\in X_i^{(0)}$ of an arbitrary point $x_i^{(0)}\in X_i^{(0)}$ upon the fuzzy set $G_i(x_i;y_{i1},\cdots,y_{in};y_{1i},\cdots,y_{ni})$, such that

$$\begin{aligned}\rho(\tilde{x}_i^{(0)},x_i^{(0)})=\min\{&\rho(\xi,x_i^{(0)})\mid\\ &\xi\in_{\beta_i}G_i(x_i;y_{i1},\cdots,y_{in};y_{1i},\cdots,y_{ni})\}.\end{aligned}\tag{10}$$

For convenience' sake, let us introduce closed subsets:

$$\begin{aligned}&G_i^{(0)}(x_i;y_{i1},\cdots,y_{in};y_{1i},\cdots,y_{ni})\\ &\triangleq G_i(x_i;y_{i1},\cdots,y_{in};y_{1i},\cdots,y_{ni})\cap X_i^{(0)}\neq\phi.\end{aligned}\tag{11}$$

Under this hypothesis, we have an important lemma on the system of β_i-level fuzzy-set-valued mapping equations:

$$\begin{aligned}&x_i\in_{\beta_i}G_i^{(0)}(x_i;f_{i1}(x_i),\cdots,f_{in}(x_i);f_{1i}(x_1),\cdots,f_{ni}(x_n)),\\ &(i=1,\cdots,n).\end{aligned}\tag{12}$$

Lemma 1 *For all i $(i=1,\cdots,n)$ and for any β_i-level, let us adopt arbitrary points $x_i^0\in X_i^{(0)}$, and also fix all values of $f_{ij}(x_i^0)$ and $f_{ji}(x_j^0)$, $(j=1,\cdots,n)$. Next, for every i, let us introduce a sequence $\{x_i^\nu\}(\nu=0,1,\cdots)$, starting from the above-adopted point x_i^0, and with each $x_i^\nu\in X_i^{(0)}$ as a β_i-level projection point of $x_i^{\nu-1}\in X_i^{(0)}$ upon the fuzzy set $G_i(x_i^{\nu-1};f_{i1}(x_i^0),\cdots,f_{in}(x_i^0);f_{1i}(x_1^0),\cdots,f_{ni}(x_n^0))$. Then, this sequence $\{x_i^\nu\}$ $(\nu=0,1,2,\cdots)$ is a Cauchy sequence, having its own limit point $\bar{x}_i\in X_i^{(0)}$, such that*

$$\begin{aligned}&\bar{x}_i\in_{\beta_i}G_i^{(0)}(\bar{x}_i;f_{i1}(x_i^0),\cdots,f_{in}(x_i^0);f_{1i}(x_1^0),\cdots,f_{ni}(x_n^0)),\\ &(i=1,\cdots,n).\end{aligned}\tag{13}$$

All limit points $\bar{x}_i(i=1,\cdots,n)$ depend on their starting points $x_i^0\in X_i^{(0)}$ and common parameters $f_{i1}(x_i^0)$, $\cdots$,$f_{in}(x_i^0)$ and $f_{1i}(x_1^0),\cdots,f_{ni}(x_n^0)$, respectively. These correspondences can be represented by single-valued continuous mappings defined on each domain:

$$\begin{aligned}&\bar{x}_i\triangleq\lambda_i(x_i^0;y_{i1},\cdots,y_{in};y_{1i},\cdots,y_{ni}),\\ &(i=1,\cdots,n),\end{aligned}\tag{14}$$

where $y_{ij}\triangleq f_{ij}(x_i^0)$ and $y_{ji}\triangleq f_{ji}(x_j^0)$ $(j=1,2,\cdots,n)$. Thus, we can add natural properties on these mappings: for any starting points $x_i^{01},x_i^{02}\in X_i^{(0)}$, let us denote

$$\begin{aligned}&\bar{x}_i^{(1)}\triangleq\lambda_i(x_i^{01};y_{i1}^{(1)},\cdots,y_{in}^{(1)};y_{1i}^{(1)},\cdots,y_{ni}^{(1)}),\\ &\bar{x}_i^{(2)}\triangleq\lambda_i(x_i^{02};y_{i1}^{(2)},\cdots,y_{in}^{(2)};y_{1i}^{(2)},\cdots,y_{ni}^{(2)}),\end{aligned}\tag{15}$$

where $y_{ij}^{(1)}\triangleq f_{ij}(x_i^{01})$, $y_{ij}^{(2)}\triangleq f_{ij}(x_i^{02})$ and $y_{ji}^{(1)}\triangleq f_{ji}(x_j^{01})$, $y_{ji}^{(2)}\triangleq f_{ji}(x_j^{02})$ $(j=1,\cdots,n)$. Then, we can assume that there exist constants $\xi_i\equiv\xi_i(\beta_i)>0$ and $\eta_i\equiv\eta_i(\beta_i)>0$ such that

$$\begin{aligned}&\rho(\bar{x}_i^{(1)},\bar{x}_i^{(2)})\\ &\le\mathcal{H}_{\beta_i}(G_i(\bar{x}_i^{(1)};y_{i1}^{(1)},\cdots,y_{in}^{(1)};y_{1i}^{(1)},\cdots,y_{ni}^{(1)}),\\ &\qquad G_i(\bar{x}_i^{(2)};y_{i1}^{(2)},\cdots,y_{in}^{(2)};y_{1i}^{(2)},\cdots,y_{ni}^{(2)}))\\ &\quad+\xi_i\sum_{j=1}^n\rho(y_{ij}^{(1)},y_{ij}^{(2)})+\eta_i\sum_{j=1}^n\rho(y_{ji}^{(1)},y_{ji}^{(2)}),\\ &\qquad(i=1,\cdots,n).\end{aligned}\tag{16}$$

From these inequalities, we can find that small values of $\rho(\bar{x}_i^{(1)},\ \bar{x}_i^{(2)})$ are suppressed by small values of $\rho(f_{ji}(x_j^{01}),\ f_{ji}(x_j^{02}))$ and $\rho(f_{ij}(x_i^{01}),\ f_{ij}(x_i^{02}))$ $(j=1,\cdots n)$. Thus, by standard procedures common in the functional analysis, from the complete continuity of mappings f_{ji} and f_{ij}, the complete continuity of mappings λ_i are proved on their own domains, *i.e.*, on bounded convex closed subsets $X_i^{(0)}$, respectively. Therefore, again, by the Schauder's type of fixed point theorem [4], we can know that these mappings λ_i have at least one fixed point x_i^* in the respective bounded convex closed subset $X_i^{(0)}$, *i.e.*,

$$x_i^* = \lambda_i(x_i^*; y_{i1}^*, \cdots, y_{in}^*; y_{1i}^*, \cdots, y_{ni}^*) \in X_i^{(0)}, \quad (i = 1, \cdots, n), \tag{17}$$

where, $y_{ij}^* \triangleq f_{ij}(x_i^*)$ and $y_{ji}^* \triangleq f_{ji}(x_j^*)$ $(i, j = 1, \cdots, n)$. These relations imply that

$$x_i^* \in_{\beta_i} G_i^{(0)}(x_i^*; f_{i1}(x_i^*), \cdots, f_{in}(x_i^*)); f_{1i}(x_1^*), \cdots, f_{ni}(x_n^*)), \quad (i = 1, \cdots, n). \tag{18}$$

This result means that the solution set $\{x_i^*\} \in X_i^{(0)}$ $(i = 1, \cdots, n)$ of the system of β_i-level fuzzy-set-valued mapping equations (12) can be obtained in connection with the set of limit points $\{\bar{x}_i\} \in X_i^{(0)}$ $(i = 1, \cdots, n)$ of Cauchy sequences $\{x_i^\nu\}$ $(i = 1, \cdots, n)$ $(\nu = 0, 1, 2, \cdots)$.

5. Applications to Analysis of Mutually Connected Blocks

As a result, this fixed point theorem can be applied to analysis of nonlinear networks, which consist of n nonlinear blocks originally represented by n mapping equations (3), but undergone by some uncertain fluctuations so that system dynamics are overcome by the system of β_i-level fuzzy-set-valued mapping equations (12). Here, bounded closed subsets $X_i^{(0)}$ are understood as "available" ranges of system behaviors $\{x_i^*\}$ $(i = 1, \cdots, n)$. Thus, we can apply this fixed point theorem immediately to "available operation" of system behaviors that appear in every block of general type of complicated large-scale network systems, as a whole.

If there exists a set of β_i-level fixed points $\{x_i^*\}$ in $X_i^{(0)}$ $(i = 1, \cdots, n)$, which satisfy the system of β_i-level fuzzy-set-valued mapping equations (12), each x_i^* can be considered as a β_i-level likelihood behavior of the individual block (i), being affected by uncertain fuzzy fluctuation, $(i = 1, \cdots, n)$. Here, this β_i-level likelihood behavior x_i^* can be found in a closed domain in which the membership function $\mu_{G_i}(\xi_i)$ has value larger than or equal to β_i.

On the one hand, when the signal x_i^* is found in a sufficiently small preassigned closed subset $V_i^{(0)} \subset X_i^{(0)}$, containing the desired behavior $x_i^{(0)}$, x_i^* can be considered as "available".

If we select $\beta_i \in (0, 1]$ sufficiently high, *i.e.*, near to unity, then the β_i-level set $G_{i\beta_i} \triangleq \{\xi_i \in X_i | \mu_{G_i}(\xi_i) \geq \beta_i\}$ is so small that $G_{i\beta_i} \subset V_i^{(0)}$, and as a result, the solution x_i^* becomes to be available, as a β_i-level likelihood behaviors of individual block (i).

Thus, the fluctuation analysis of this type of networks in connected blocks, undergone by undesirable uncertain fluctuations, can be successfully accomplished at arbitrarily fine-level of estimation, by immediate application of the here-presented fixed point theorem for the system of β_i-level fuzzy-set-valued nonlinear mappings, with consciously selected value of the parameter β_i.

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