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Modeling and Classification of Approximately Periodic Signals Using Chaotic Systems

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***Abstract* —** A new chaos-based technique for modeling the diversity of approximately periodic signals is introduced and exploited, combined with generalized chaotic synchronization phenomena, for the solution of temporal pattern recognition problems.

1. Introduction

The problem addressed in this paper is to establish black-box models from real-word data, in our case from measured time-series. This is an old subject and consistent literature is available on it. In a classification context, the main difficulty is to express the diversity of data that has essentially the same origin without creating confusion with data that has a different origin.

Normally, the diversity of time-series is modeled by a stochastic process, such as filtered white noise, a Hidden Markov Model, or a stochastic differential equation. Often, it is reasonable to assume that the time series is generated by a deterministic dynamical system rather than a stochastic process. In this case, the diversity of the data is expressed by the variability of the parameters of the dynamical system. The parameter variability itself is then, once again, modeled by a stochastic process. In both cases the diversity is generated by some form of exogenous noise.

In this paper a further step is taken. A single chaotic dynamical system is used to model the data and its diversity. Indeed, a chaotic system produces a whole family of trajectories that are all different but nonetheless very similar. It is believed that chaotic dynamics not only are a convenient means to represent diversity but that in many cases the origin of diversity actually stems from chaotic dynamics. However, whether or not this is the case is not important for the classification task considered in this paper [1].

Since this approach explores completely new grounds, the most suitable kind of data is considered, namely approximately periodic signals. In nature such signals are rather common, in particular physiological signals of living beings, such as the electrocardiograms (ECG), parts of speech signals, electroencephalograms (EEG), etc. are of this kind. Since there are often strong arguments in favor of the chaotic nature of these signals, they appear to be the best candidates for modeling diversity by chaos. It is repeated, however, that this modeling approach is thought to be quite general and whether or not a chaotic system has produced the signals under consideration is not crucial for being able to perform the classification task.

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2. Classification Problem

For simplicity, we consider the case where two classes of approximately periodic signals are given and we have to find an algorithm that decides to which class a given signal belongs. Such a classification problem can be easy or hard, depending on:

- how "close" the two classes are;
- how the two classes are defined.

In our case, the classes are defined indirectly by a representative set of examples, in the form of a database of recorded signals, labeled with the class symbol. From the examples, a model for the classes has to be deduced. This operation is called "supervised learning" [2]. In fact, the learning is supervised, because for each recorded signal the class is known, the "teacher" or "supervisor" tells us what the class is.

To fix the ideas, we give two examples. The first concerns vowels in speech recognition. A database of 50 recorded and labeled [a]'s and [e]'s is given and the task consists of distinguishing, in a given speech segment that is supposed to represent either a spoken [a] or a spoken [e], which one of the two vowels actually has been pronounced. An example of a spoken [a] is represented in Fig. 1(a), and an example of a spoken [e] in Fig. 1(b).

Since these signals are approximately periodic, they can be decomposed into "pseudo-periods". Since they are not precisely periodic, the pseudo-periods are slightly different and the signals within the various pseudo-periods also differ slightly, even if they are time-aligned. Averaging over the time-aligned signals, one obtains a periodic "generating" cycle for the [a] and for the [e] spoken vowels. The set of time-aligned signal portions are represented in Fig. 2, with a highlighted generating cycle.

The second example concerns electrocardiograms (ECG). The first class of ECG signals has been taken from persons

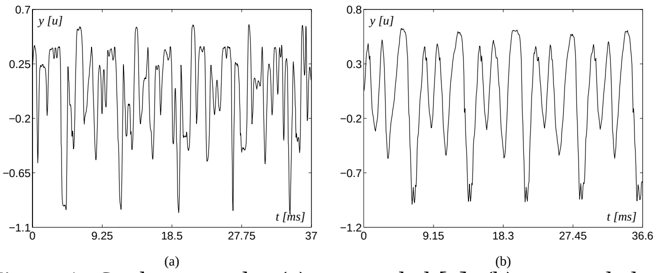


Figure 1: Spoken vowels: (a) – recorded [a]; (b) – recorded [e].

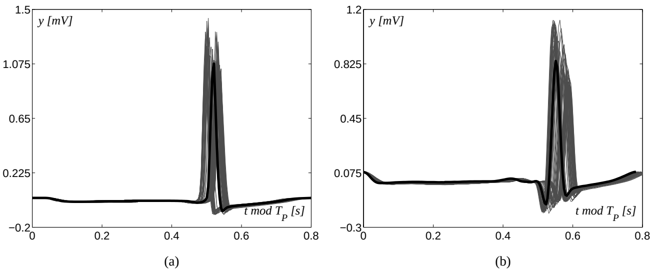


Figure 4: Recorded ECG's: (a) – time-aligned pseudo-periods of all 20 recorded pathological ECG's; (b) – time-aligned pseudo-periods of all 20 recorded healthy ECG's. The generating cycles are represented in bold.

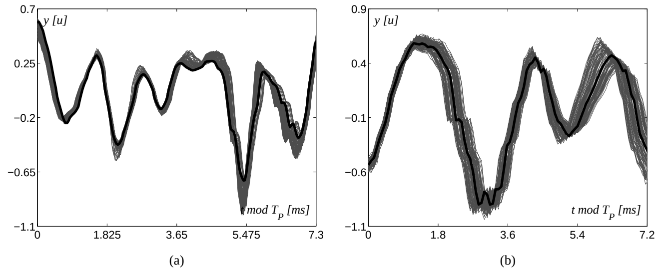


Figure 2: Spoken vowels: (a) – time-aligned pseudo-periods of all 50 recorded signals [a]; (b) – time-aligned pseudo-periods of all 50 recorded signals [e]. The generating cycles are represented in bold.

that have a certain pathology. An example of such an ECG is given in Fig. 3(a). They have to be distinguished from healthy persons. A corresponding ECG is given in Fig. 3(b).

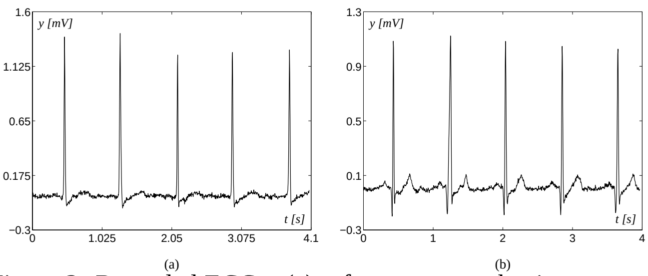


Figure 3: Recorded ECGs: (a) – from a person having a certain pathology; (b) – from a healthy person.

Again, both healthy and pathological ECG's are approximately periodic signals. The pseudo-periods can be normalized, time-aligned signals can be computed and corresponding waveforms within a pseudo-period can be superposed in order to illustrate the time-variability within a single signal, or the variability among different signals (Fig. 4). Also, a periodic generating cycle can be computed.

3. Modeling by Nonlinear Dynamic System Identification

Instead of modeling a signal class by a stochastic process, we use a single dynamical system as a model for the whole class [3]. The diversity of the different signals is represented by the attractor of the system. The attractor of a dynamical system may be very simple, such as a closed curve or a torus, or rather complicated such as a chaotic attractor. In our examples, the diversity of the classes is such that apparently a chaotic attractor is needed for modeling.

We chose a Lur'e system as a reference model (upper part of Fig. 5). Its ring structure composed of a nonlinear static dynamic and a linear system has distinct advantage for the modeling process. For computational convenience, we restrict the nonlinearity to be one-dimensional. If it were not for this constraint, the Lur'e systems would actually represent the most general class of finite dimensional nonlinear dynamical systems. To be precise, the 1-dimensional nonlinear function we use is a piecewise linear function composed of 5 pieces, whose angles have been smoothed to second order.

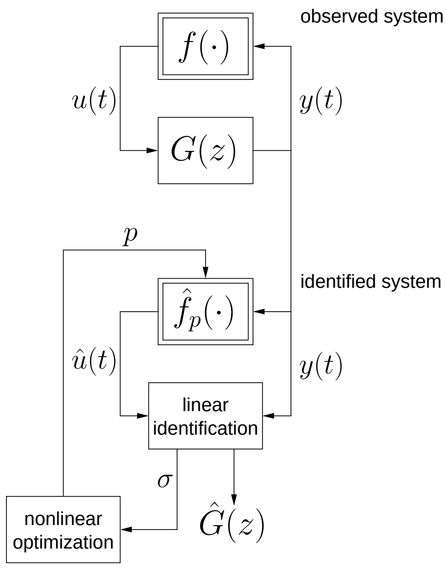


Figure 5: Operating scheme of the Lur'e model based nonlinear identification algorithm.

The model is established by an identification process, using the examples from the corresponding database. This is also called supervised learning [4]. In the lower part of Fig. 5 the identification or learning algorithm is represented schematically. The loop of the Lur'e system is cut open and a recorded signal is injected into an initial guess of the nonlinearity ($f_p(\cdot)$) obtaining the approximated input ($\widehat{u}(t)$) of the linear dynamical system ($G(z)$). Afterwards, a parametric linear identification technique [5] is applied on the pair $(\widehat{u}(t), y(t))$, obtaining an estimation ($\widehat{G}(z)$) of the linear system and a measure of its quality (σ). Hence, the nonlinearity parameters (p) can be adjusted, by a suitable optimization method, to improve the identification quality. The procedure is iteratively repeated until the best possible pair $(p, \widehat{G}(z))$ is determined. A certain number of constraints has to be applied to the identification process, however, in order to avoid that it converges to the trivial solution, where the loop functions act as the identity operator.

Profiting from the Lur'e structure, we have used alternated linear and nonlinear system identification. Keeping the nonlinear function fixed, we adjust the linear dynamic part using a standard algorithm. Inversely, keeping the linear part fixed, we adjust the nonlinear function by a genetic algorithm. The result of this identification procedure is shown in Figs. 6 to 9. It can be seen that the synthetic signals produced by the Lur'e systems resemble closely the recorded signals.

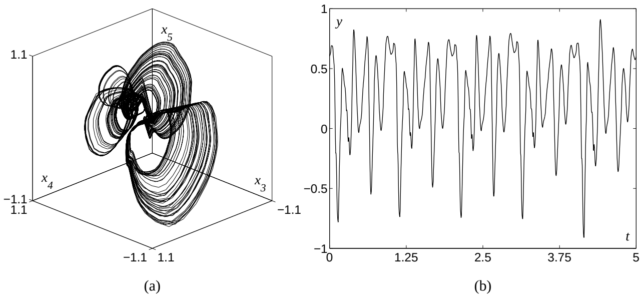


Figure 6: Result of the identification of the acoustic signals [a]: (a) – 3-dimensional projection of the attractor of the 5-dimensional identified model; (b) – example of a synthetic signal produced by the corresponding system.

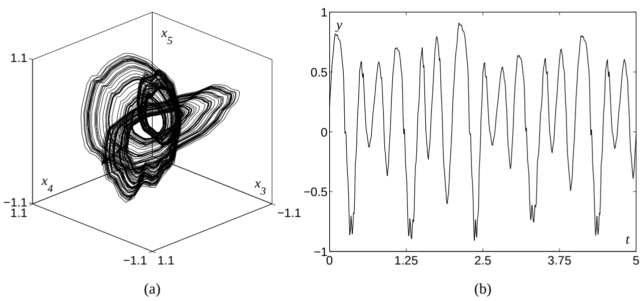


Figure 7: Result of the identification of the acoustic signals [e]: (a) – 3-dimensional projection of the attractor of the 5-dimensional identified model; (b) – example of a synthetic signal produced by the corresponding system.

4. Classification by Synchronization

Having obtained a nonlinear dynamical system that autonomously produces signals that resemble closely the

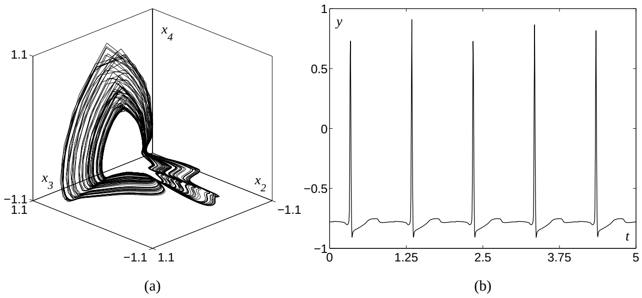


Figure 8: Result of the identification of the pathological ECG signals: (a) – 3-dimensional projection of the attractor of the 4-dimensional identified model; (b) – example of a synthetic signal produced by the corresponding system.

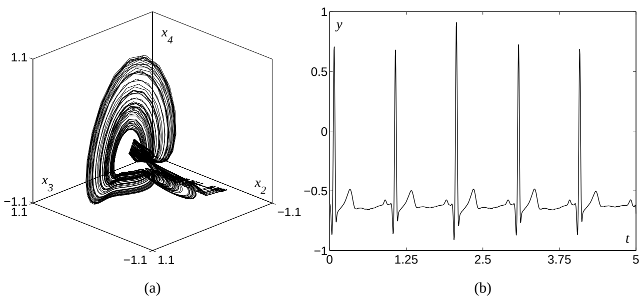


Figure 9: Result of the identification of the healthy ECG signals: (a) – 3-dimensional projection of the attractor of the 4-dimensional identified model; (b) – example of a synthetic signal produced by the corresponding system.

recorded signals of the class it represents, we now add an input and error feedback to the system (right part of Fig. 10). This is done in such a way that the system synchronizes approximately with a signal that belongs to the class it models, whereas synchronization does not take place for other signals.

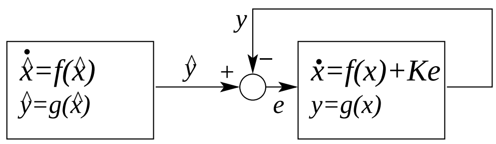


Figure 10: Master-slave configuration for synchronization controlling the slave by error feedback.

The idea behind this approach to classification is the following [6]. If we connect two identical nonlinear dynamical systems in a master-slave configuration as shown in Fig. 10, then by suitably adjusting the feedback coefficients the slave system will synchronize with the master system. This synchronization is caused by the output signal of the master system alone, because no other information reaches the slave system. Thus, if the output of the master system is recorded, and later replayed at the input of the slave system, the latter will still synchronize with the replayed signal. Now, if the signal at the input of the slave system is only approximately like an output signal from the master system, the slave system will still approximately synchronize with the incoming signal. Hence, if the master system (and thus the slave system without input and feedback) models a signal class then the slave system with input and feedback will approximately synchronize with input signals from the class and not synchronize with other signals.

It turns out [7] that even though the systems that were obtained by our learning/identification process reproduce quite faithfully the signals of their class in the autonomous mode, they are not yet suitable for the classification process. The reason is that their dynamics have a rather rigid underlying approximate periodicity. If the incoming signal is out of phase with the internal dynamics of the system, the feedback will not be able to lock the system onto this signal. The remedy is to modify the system by carefully changing its parameters until it has a homoclinic loop. The presence of the homoclinic loop introduces phase-slips into the output signals of the free-running system (Fig. 11). Even though the modified system in the autonomous mode produces signals that are not so similar to the recorded signals anymore, the system is much more flexible for synchronizing with a suitable input signal.

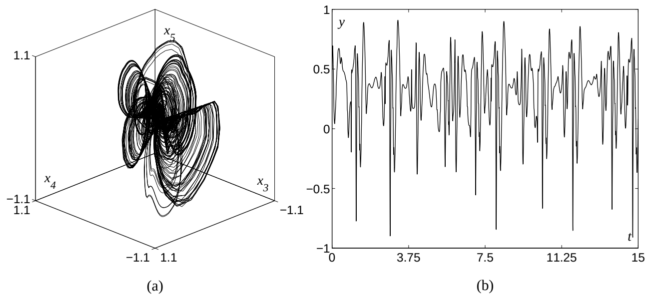


Figure 11: Modified system, to have a homoclinic loop, of the acoustic signals [a]: (a) – 3-dimensional projection of the attractor of the 5-dimensional modified model (with homoclinic loop); (b) – example of the corresponding system output signal, comparing with Fig. 6(b), the phase slips are visible.

The feedback coefficients in Fig. 10 have to be set in such a way that approximate synchronization takes place for the signals the class models, and no synchronization for the signals of the wrong class. If the feedback is too weak, synchronization hardly ever happens, whereas if the feedback is too strong, the system will synchronize also with wrong signals. The following idea helps to find the right feedback. The crucial trajectories in the attractor of the autonomous system are the periodic generating cycle and the homoclinic loop. They are represented in Fig. 12 for a system that serves just for illustration purposes.

Close to the generating cycle, the dynamics produce output signals similar to those of the class the system models. However, the trajectories of the free-running system explore the whole attractor and therefore they cannot remain close to the generating cycle forever. On the other hand, in the system with input, when a signal of the right class is injected,

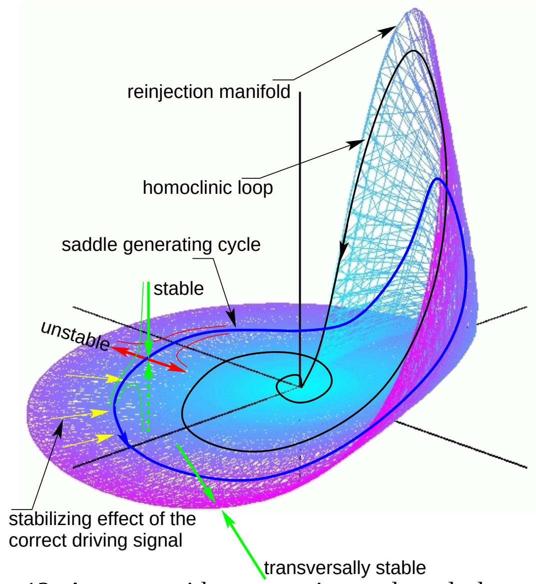


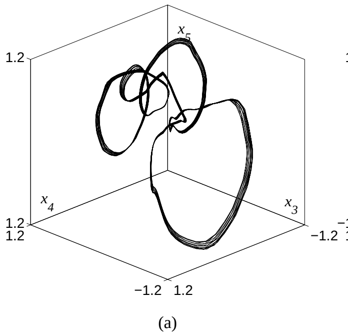
Figure 12: Attractor with a generating cycle and a homoclinic loop, highlighted in bold.

the feedback control should keep it close to the generating cycle once it enters into its vicinity, whereas a signal of the wrong class should not be captured by the generating cycle. The corresponding feedback coefficients are determined by periodic control theory applied to the system linearized about the generating cycle. In Figure 13(a) the attractor of the modified system for the acoustic signals [a] with the right input signal is represented. With respect to the attractor of the autonomous system shown in Fig. 11, it is much thinner. Indeed, it is concentrated around the generating cycle. In Figure 13(b), the attractor of the same system is shown, when the input is a signal from the wrong class. It clearly fills out much more of the state space and it does not stay close to the generating cycle. The two situations are easy to distinguish, either by checking the degree of synchronization of the system output with the input, or by checking the “thickness” of the attractor.

The classification results are given in Table 1 for the examples mentioned above. They are quite reasonable. Classical methods trimmed to the specific application can certainly achieve still better results. However, we have been able to show the feasibility of this entirely different approach. We also have been able to classify EEG signals belonging to two different sleeping states, where the learning procedure was still much more difficult.

5. Conclusions

We have shown that the diversity of approximately periodic signals found in nature can be modeled by means of chaotic dynamics. Furthermore, we have illustrated how to exploit this kind of modeling technique, together with selective properties of the synchronization of chaotic systems, for pattern recognition purposes.



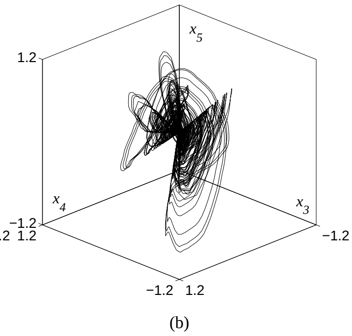


Figure 13: Classification by synchronization for the acoustic signals [a]: (a) – 3-dimensional projection of the attractor of the 5-dimensional modified model that has approximately synchronized with a signal from the class it models; (b) – 3-dimensional projection of the attractor of the 5-dimensional modified model with a signal from the wrong class as input.

in \ as	P	H
P	88.19%	11.81%
H	14.98%	85.02%

(a)

in \ as	[a]	[e]
[a]	85.33%	8.80%
[e]	6.05%	87.38%

(b)

Table 1: Classification results, only the vectors not used for learning are classified: (a) – ECG signals; (b) – vowels signals.

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