CHAOS SYNCHRONIZATION IN THE MODIFIED CHUA'S CIRCUIT WITH $x|x|$

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Abstract: This paper studies chaos synchronization in the modified Chua's circuit with $x|x|$ using linear state feedback control scheme. A new sufficient condition is derived for chaos synchronization between the two modified Chua's circuits with $x|x|$ based on the Lyapunov stability theory and the Linear Quadratic Optimal Control theory. A simulation example is given for demonstration.

1. Introduction

Today, chaos synchronization has become popular [4]. Chaos synchronization can be considered as an observer design problem, in the sense that the response system is the observer of the drive system [1, 6]. On the other hand, chaos synchronization can also be viewed from a dynamical control perspective [2, 3]. This is clear because chaos synchronization can be regarded as a model-tracking problem, in which the response system can track the drive system asymptotically, particularly if state feedback is used as a means of control.

In the present authors' paper [2], we studied chaos synchronization in the classic Chua's circuit by using linear state feedback control scheme. Based on the Lyapunov stability theory and the Linear Quadratic optimal control theory, a new, simple and yet easily verified sufficient condition is established for global chaos synchronization, which is applicable to a large class of general chaotic systems.

In this paper, we studied chaos synchronization in the modified Chua's circuit with $x|x|$ [5] by using linear state feedback control scheme. A new sufficient condition is established for global chaos synchronization in the modified Chua's circuit.

2. A Global Chaos Synchronization Criterion

Consider a chaotic system in the form of:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + g(\mathbf{x}), \tag{1}$$

where $\mathbf{x} \in R^n$ is the state vector, $\mathbf{A} \in R^{n\times n}$ is a constant matrix, and $g(\mathbf{x})$ is a continuous non-linear function. Assume that

$$g(\mathbf{x}) - g(\tilde{\mathbf{x}}) = \mathbf{M}_{\mathbf{x},\tilde{\mathbf{x}}}(\mathbf{x} - \tilde{\mathbf{x}}), \tag{2}$$

for a bounded matrix $\mathbf{M}_{\mathbf{x},\tilde{\mathbf{x}}}$, in which the elements are dependent on $\mathbf{x}$ and $\tilde{\mathbf{x}}$. It is noted that, most chaotic systems, including Lur'e nonlinear systems and Lipschitz nonlinear systems, can be described by (1) and (2) [1].

From the state feedback control approach, a slave system for (1) is constructed as follows:

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}\tilde{\mathbf{x}} + g(\tilde{\mathbf{x}}) + \mathbf{B}(\mathbf{K}\mathbf{x} - \mathbf{K}\tilde{\mathbf{x}}), \tag{3}$$

where $\mathbf{u} = \mathbf{BK}(\mathbf{x} - \tilde{\mathbf{x}}) \in R^n$ is a linear state feedback control input vector, $y = \mathbf{Kx}$ is a scalar chaotic signal, which is a linear combination of the state variables, $\mathbf{K} = [k_1, k_2, \cdots, k_n]$ is the feedback gain to be designed, and $\mathbf{B} = [b_1, b_2, \cdots, b_n]'$ is chosen such that $(\mathbf{A}, \mathbf{B})$ is controllable. It follows from (1) and (3) that

$$\begin{aligned}\dot{\mathbf{e}} &= \mathbf{Ae} + g(\mathbf{x}) - g(\tilde{\mathbf{x}}) - \mathbf{BK}(\mathbf{x} - \tilde{\mathbf{x}}) \\ &= \mathbf{Ae} - \mathbf{BKe} + g(\mathbf{x}) - g(\tilde{\mathbf{x}}) \\ &= (\mathbf{A} - \mathbf{BK})\mathbf{e} + g(\mathbf{x}) - g(\tilde{\mathbf{x}}) \\ &= (\mathbf{A} - \mathbf{BK} + \mathbf{M}_{\mathbf{x},\tilde{\mathbf{x}}})\mathbf{e}\end{aligned} \tag{4}$$

where $\mathbf{e} = \mathbf{x} - \tilde{\mathbf{x}}$ is the error vector.

Theorem 1 (2). *If a positive constant, r, and a (semi-)positive definite symmetric matrix, $\mathbf{Q}$, exists such that*

$$\mathbf{Q} - (\mathbf{M}_{\mathbf{x},\tilde{\mathbf{x}}}^T\mathbf{P} + \mathbf{PM}_{\mathbf{x},\tilde{\mathbf{x}}}) \geq \mu\mathbf{I} > \mathbf{0}, \tag{5}$$

where $\mathbf{I}$ *is the identity matrix,* μ *is a positive constant, and* $\mathbf{P}$ *is a solution of the following Algebraic Riccati Equation (ARE):*

$$\mathbf{A}^T\mathbf{P} + \mathbf{PA} - \mathbf{PB}r^{-1}\mathbf{B}^{\mathbf{T}}\mathbf{P} + \mathbf{Q} = \mathbf{0}, \tag{6}$$

then, with the feedback gain

$$\mathbf{K} = \frac{1}{2}r^{-1}\mathbf{B}^T\mathbf{P} \tag{7}$$

the error dynamical system (4) is globally asymptotically stable, implying that the coupled systems (1) and (3) are globally synchronized.

3. Synchronization of Coupled modified Chua's Circuits with $x|x|$

To illustrate the use of the new criterion for chaos synchronization, the modified Chua's circuit with $x|x|$ is considered, for which the condition reduces to a fairly simple form. The modified Chua's circuit with $x|x|$ is described by

$$\begin{cases}\dot{x} = \alpha\,(y - f(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y,\end{cases} \tag{8}$$

where $\alpha > 0$, $\beta > 0$, and $f(x) = ax + bx|x|$ where a and b are chosen as –1/6 and 1/16.

Eq. (8) can be written by

$$\begin{cases}\dot{x} = \alpha\,(y - ax - f_1(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y,\end{cases} \tag{9}$$

where $f_1(x) = bx|x|$. Thus, we have

$$f_1(x) - f_1(\tilde{x}) = k_{x,\tilde{x}}(x - \tilde{x}), \tag{10}$$

where $k_{x,\tilde{x}}$ is dependant on x and $\tilde{x}$, and varies within the interval $\left[0, \frac{1}{8}x_{\max}\right]$ for $t \geq 0$. Hence, $k_{x,\tilde{x}}$ is bounded as $0 \leq k_{x,\tilde{x}} \leq \frac{1}{8}x_{\max}$.

Referring to (3), a response system is constructed as below for the drive system (8) or (9), with linear unidirectional coupling:

$$\begin{cases}\dot{\tilde{x}} = \alpha(\tilde{y} - a\tilde{x} - f_1(\tilde{x})) + b_1\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}) \\ \dot{\tilde{y}} = \tilde{x} - \tilde{y} + \tilde{z} + b_2\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}) \\ \dot{\tilde{z}} = -\beta\tilde{y} + b_3\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}).\end{cases} \tag{11}$$

Subtracting (11) from (9) gives

$$\begin{cases}\dot{e}_x = \alpha(e_y - ae_x - k_{x,\tilde{x}}e_x) - b_1\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}) \\ \dot{e}_y = e_x - e_y + e_z - b_2\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}) \\ \dot{e}_z = -\beta e_y - b_3\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}),\end{cases} \tag{12}$$

where $e_x = x - \tilde{x}$, $e_y = y - \tilde{y}$, $e_z = z - \tilde{z}$ are the errors. Furthermore, (12) can be rewritten as

$$\dot{\mathbf{e}} = \mathbf{Ae} + g(\mathbf{x}) - g(\tilde{\mathbf{x}}) - \mathbf{BKe}, \tag{13}$$

where $\mathbf{A} = \begin{bmatrix} -a\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$,

$\mathbf{K} = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$, $\mathbf{e} = \begin{bmatrix} x - \tilde{x} \\ y - \tilde{y} \\ z - \tilde{z} \end{bmatrix}$ and

$$g(\mathbf{x}) = \begin{bmatrix} -\alpha f(x) \\ 0 \\ 0 \end{bmatrix}.$$

Consider

$$g(\mathbf{x}) - g(\tilde{\mathbf{x}}) = \begin{bmatrix} -\alpha(f_1(x) - f_1(\tilde{x})) \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\alpha k_{x,\tilde{x}}(x - \tilde{x}) \\ 0 \\ 0 \end{bmatrix}, \qquad (14)$$

$$= \begin{bmatrix} -\alpha k_{x,\tilde{x}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x - \tilde{x} \\ y - \tilde{y} \\ z - \tilde{z} \end{bmatrix} = \mathbf{M}_{\mathbf{x},\tilde{\mathbf{x}}}\mathbf{e}$$

where $\mathbf{M}_{\mathbf{x},\tilde{\mathbf{x}}} = \begin{bmatrix} -\alpha k_{x,\tilde{x}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then, (13) can be rewritten as

$$\begin{aligned} \dot{\mathbf{e}} &= \mathbf{Ae} + \mathbf{g}(\mathbf{x}) - \mathbf{g}(\tilde{\mathbf{x}}) - \mathbf{BKe} \\ &= (\mathbf{A} + \mathbf{M})_{\mathbf{x},\tilde{\mathbf{x}}}\mathbf{e} - \mathbf{BKe} \end{aligned} \qquad (15)$$

The vector B is not unique, and can be chosen as any other values provided that $(\bar{\mathbf{A}}, \mathbf{B})$ is controllable. In particular, if $\mathbf{B} = [b_1 \quad 0 \quad 0]^T$, namely, $b_1 \neq 0$, $b_2 = 0$, $b_3 = 0$, then $(\bar{\mathbf{A}}, \mathbf{B})$ is controllable. In this case, the response system (11) can be simplified as

$$\begin{cases} \dot{\tilde{x}} = \alpha(\tilde{y} - a\tilde{x} - f_1(\tilde{x})) + b_1\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}) \\ \dot{\tilde{y}} = \tilde{x} - \tilde{y} + \tilde{z} \\ \dot{\tilde{z}} = -\beta\tilde{y}. \end{cases} \qquad (16)$$

In the following, chaos synchronization between the driving system (8) and response system (16) is demonstrated. The parameters of the circuit used are $\alpha = 9.78$, $\beta = 14.97$, $a = -1/6$, $b = 1/16$, for which the system exhibits chaotic behavior as shown in Fig. 1. From the figure, we get $x_{\max} < 4$, so $0 \le k_{x,\tilde{x}} < 0.5$.

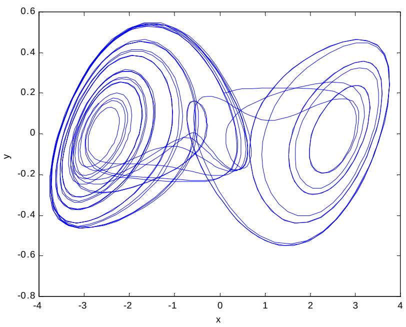

Fig.1 The attractor of chaotic circuit (8)

Choose $r = 0.0028$, $\mathbf{Q} = diag(2, 1, 1)$, $b_1 = 1$ and $\mu = 0.1$. Then one obtains $\mathbf{P} = \begin{bmatrix} 0.0857 & 0.1711 & -0.0107 \\ 0.1711 & 4.1290 & -0.4797 \\ -0.0107 & -0.4797 & 0.3444 \end{bmatrix}$ by solving the ARE (6) using the LQR function in MATLAB, and obtains $\mathbf{K} = [15.2989 \quad 30.5471 \quad -1.9027]$ from (7). Then, it follows that $\mathbf{Q} - (\mathbf{M}_{\mathbf{x},\tilde{\mathbf{x}}}{}^T\mathbf{P} + \mathbf{PM}_{\mathbf{x},\tilde{\mathbf{x}}}) - \mu\mathbf{I} > 0$.

According to Theorem 1, the two coupled modified Chua's systems (8) and (16) are globally asymptotically synchronized, as shown in Fig. 2.

4. Conclusions

In this paper, based on the linear state feedback control scheme, we studied the chaos synchronization in the two coupled modified Chua's circuit with $x|x|$, and developed a sufficient criterion for global synchronization

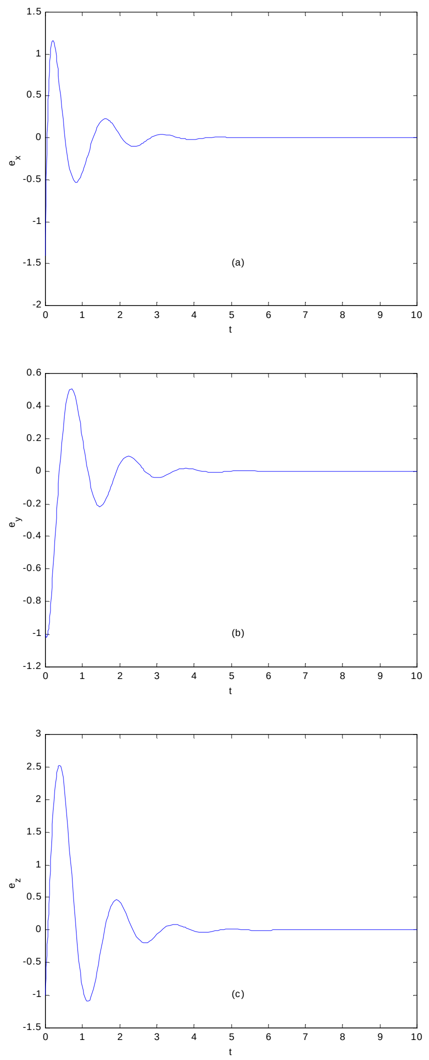

Fig. 2 Chaos synchronization between the two coupled chaotic circuits (8) and (16)

between two chaotic systems coupled through a linear state feedback connection.

It is remarked that this new criterion can be applied to a large class of chaotic systems. Under a unified framework developed by the present authors [1], similar approach can be applied to the Murali-Lakshmanan-Chua (MLC) circuit, modified Chua's circuit with a sine function, Rössler system Lorenz system, Chen's system, and so on.

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