Robust Adaptive Control of Rossler and Chen Chaotic Systems

Zhi Li [a, b] Chongzhao Han [a] Jiuqiang Han [a]

[a]*School of Electronic and Information Engineering Xi'an Jiaotong University, Xi'an* 710049, *China*
[b]*Department of Automatic Control Engineering Xidian University, Xi'an 710071, China*

E-mail: zhilih@hotmail.com

Abstract — This paper considers the robust adaptive control problem of Rossler and Chen chaotic systems with time-varying unknown parameters. A novel controller is derived on the basis of Lyapunov stability theory when system's unknown parameters vary in a bound interval and the bound of interval is unknown. The proposed controller make the states of Rossler and Chen chaotic systems asymptotically track arbitrary smooth trajectory. Simulation result shows effectiveness of the proposed controller.

Keywords: robust adaptive control; uncertain chaotic systems; Rossler system; Chen attractor

1. INTRODUCTION

Chaos control has been studied extensively in recent years and numerous achievements [1-8] have been made. Recently, two identical Rossler and Chen systems synchronized by using active control [4]. As we know, active control is an effective method for making two identical Rossler and Chen systems be synchronized. However this method works only for a certain class of chaotic systems with known parameters both in drive systems and response systems. In order to overcome this limitation We have proposed a modification measure based on Lyapunov stability theory in our published paper [9] in which an adaptive synchronization controller, which can make the states of two identical Rossler and Chen systems be asymptotically synchronized in the presence of system's unknown parameters, is derived. In paper [9], we assumed that system's unknown parameters are constant. When these system's unknown parameters are not constant, how do we deal with the problems of control of nonlinear chaotic systems? Solving these problems is more challenging.

In this paper, taking the examples of Rossler and Chen systems, we study the robust adaptive control problems of the Rossler and Chen systems with time-varying unknown parameters. We assume that system's unknown parameters vary in bound intervals, and the bounds of intervals are unknown. A robust adaptive controller based on Lyapunov stability theory is derived, which can make both Rossler and Chen systems asymptotically track arbitrary smooth trajectory. Correctness of our design approach has been proved on Lyapunov stability theory and validated via computer simulation.

2. ROBUST ADAPTIVE CONTROL OF ROSSLER SYSTEMS

The Rossler system [11] is described by the following ordinary differential equations

$$\begin{aligned}\dot{x} &= -y - z,\\ \dot{y} &= x + a(t)y,\\ \dot{z} &= b(t) + z(x - c(t)),\end{aligned} \tag{1}$$

where x, y, and z are state variables and $a(t)$, $b(t)$ and $c(t)$ are three varing-time uncertain paremeters and vary in bound intervals and their bounds of the intervals are unknown, which is described as follows:

$$a(t) \in [\underline{a}, \overline{a}],\quad b(t) \in [\underline{b}, \overline{b}],\quad c(t) \in [\underline{c}, \overline{c}] \tag{2}$$

where $\underline{a}, \overline{a}, \underline{b}, \overline{b}, \underline{c}$, and $\overline{c}$ are unknown constants.

Let $\alpha = max\{|\underline{a}|, |\overline{a}|\}$, $\beta = max\{|\underline{b}|, |\overline{b}|\}$,

$$\gamma = max\{|\underline{c}|, |\overline{c}|\}. \tag{3}$$

We have $$|a(t)| \le \alpha\ \ |b(t)| \le \beta\ , |c(t)| \le \gamma\ . \tag{4}$$

After introducing the control input into Eq. (1), we obtain the following ordinary differential equations

$$\begin{aligned}\dot{x} &= -y - z + u_1,\\ \dot{y} &= x + a(t)y + u_2,\\ \dot{z} &= b(t) + z(x - c(t)) + u_3,\end{aligned} \tag{5}$$

where $u = [u_1, u_2, u_3]^T$ are controller we introduced Eq. (5).

Let $x_d(t)=[x_{d1}(t),x_{d2}(t),x_{d3}(t)]^T$ be an arbitrarily desired smooth trajectory, we obtain the error dynamical system between Eq. (5) and the desired trajectory

$$\begin{aligned}\dot{e}_1 &= -e_2 - e_3 - \dot{x}_{d1} - x_{d2} - x_{d3} + u_1,\\ \dot{e}_2 &= e_1 + a(t)(e_2 + x_{d2}) - \dot{x}_{d2} + x_{d1} + u_2,\\ \dot{e}_3 &= b(t) + e_1e_3 + x_{d3}e_1 + x_{d1}e_3 + x_{d1}x_{d3}\\ &- c(t)(e_3 + x_{d3}) - \dot{x}_{d3} + u_3,\end{aligned} \tag{6}$$

where $e_1 = x - x_{d1}, e_2 = y - x_{d2}, e_3 = z - x_{d3}$.

Our goal of control is to find a robust adaptive controller u so that the solutions of Eq. (6) are robust asymptotically stable at $e_i = 0, i = 1,2,3$, which is

$\lim_{t\to\infty} e_i(t) = 0, i = 1,2,3$ for $a(t)$, $b(t)$ and $c(t)$ satisfy Eq. (2)-(4). (7)

There many choices of controller for achieving the goal of Eq. (7), we choose a controller and a parameter estimation update law $\dot{\hat{a}}, \dot{\hat{\beta}}, \dot{\hat{\gamma}}$ as follows:

$$\begin{aligned}u_1 &= -k_1e_1 + \dot{x}_{d1} + x_{d2} + x_{d3},\\ u_2 &= -k_2e_2 - \hat{\alpha}(e_2 + |x_{d2}|signe_2) + \dot{x}_{d2} - x_{d1},\\ u_3 &= -k_3e_3 - \hat{\beta}signe_3 - e_1e_3 - x_{d3}e_1 - x_{d1}e_3\\ &- x_{d1}x_{d3} + e_1 + \dot{x}_{d3} - \hat{\gamma}(e_3 + |x_{d3}|signe_3),\\ \dot{\hat{\alpha}} &= e_2^2 + |x_{d2}||e_2|,\\ \dot{\hat{\beta}} &= |e_3|,\\ \dot{\hat{\gamma}} &= e_3^2 + |x_{d3}||e_3|,\end{aligned} \tag{8}$$

where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ are estimate value of the unknown constant parameters α, β, γ and $k_i > 0, i = 1,2,3$.

Constructed Lyapunov function for Eq.(6)

$$V(e,\tilde{\alpha},\tilde{\beta},\tilde{\gamma}) = \frac{1}{2}\sum_{i=1}^{3} e_i^2 + \frac{1}{2}(\tilde{\alpha}^2 + \tilde{\beta}^2 + \tilde{\gamma}^2), \tag{9}$$

where $\tilde{\alpha} = \alpha - \hat{\alpha}, \tilde{\beta} = \beta - \hat{\beta}, \tilde{\lambda} = \gamma - \hat{\gamma}$.

We have

$$\frac{dV}{dt} \le -k_1e_1^2 - k_2e_2^2 - k_3e_3^2. \tag{10}$$

This leads to $\lim_{t\to\infty} \|e(t)\| = 0$,

where $a(t)$, $b(t)$ and $c(t)$ satisfy Eq.(2)-(4).

Hence, the control goal of Rossler system is achieved under the controller and a parameter estimation update law Eq. (8).

3. ROBUST ADAPTIVE CONTROL OF CHEN SYSTEMS

Recently, on studying anticontrol of chaos, Chen [10] introduced a new chaotic attractor (Chen attractor), which can be described by the following nonlinear differential equations

$$\begin{aligned}\dot{x} &= a(t)(y - x),\\ \dot{y} &= (c(t) - a(t))x - xz + c(t)y\\ \dot{z} &= xy - b(t)z\end{aligned} \tag{11}$$

where x, y, and z are state variables where x, y, and z are state variables and $a(t)$, $b(t)$ and $c(t)$ are three varing-time uncertain paremeters and vary in bound intervals and their bounds of the intervals are unknown, which satisfy Eq. (2)- Eq. (4).

For Chen chaotic attractor (11), we introduce controller u into Eq. (11) and have

$$\begin{aligned}\dot{x} &= a(t)(y - x) + u_1,\\ \dot{y} &= (c(t) - a(t))x - xz + c(t)y + u_2,\\ \dot{z} &= xy - b(t)z + u_3\end{aligned} \tag{12}$$

where $u = [u_1, u_2, u_3]^T$ is controller we introduced in Eq. (12). Then, error dynamical system between Eq. (12) and desired smooth trajectory can be expressed by

$$\begin{aligned}\dot{e}_1 &= a(t)(e_2 + x_{d2} - e_1 - x_{d1}) - \dot{x}_{d1} + u_1\\ \dot{e}_2 &= (c(t) - a(t))(e_1 + x_{d1}) + c(t)(e_2 + x_{d2})\\ &- (e_1 + x_{d1})(e_3 + x_{d3}) - \dot{x}_{d2} + u_2\\ \dot{e}_3 &= (e_1 + x_{d1})(e_2 + x_{d2}) - b(t)(e_3 + x_{d3})\\ &- \dot{x}_{d3} + u_3\end{aligned} \tag{13}$$

where $e_1 = x - x_{d1}, e_2 = y - x_{d2}, e_3 = z - x_{d3}$.

Our popurse is to find a robust adaptive controller u so that the solutions of Eq. (12) robust asymptotically track arbitrary smooth tajectory $x_d(t) = [x_{d1}(t), x_{d2}(t), x_{d3}(t)]^T$, which is that the solutions of Eq. (13) robust asymptotically stable at $e_i = 0, i = 1,2,3$, that is $\lim_{t\to\infty} e_i(t) = 0, i = 1,2,3$,

for $a(t)$, $b(t)$ and $c(t)$ satisfy Eq. (2)-(4). (14)

There many choices of controller for achieving the goal of Eq. (14), we choose a controller and a parameter estimation update law $\dot{\hat{a}}, \dot{\hat{\beta}}, \dot{\hat{\gamma}}$ as follows:

$$u_1 = -k_1 e_1 - \hat{\alpha}(|x_{d2} - x_{d1}| signe_1 + e_1) + \dot{x}_{d1},$$
$$u_2 = -k_2 e_2 - \hat{\gamma}(e_2 + |e_1 + x_{d1} + x_{d2}| signe_2)$$
$$- \hat{\alpha}|x_{d1}| signe_2 + x_{d1}x_{d3} + x_{d3}e_1 + \dot{x}_{d2}, \quad (15)$$
$$u_3 = -k_3 e_3 - x_{d2}e_1 - x_{d1}x_{d2} -$$
$$\hat{\beta}(e_3 + |x_{d3}| signe_3) + \dot{x}_{d3},$$
$$\dot{\hat{\alpha}} = e_1^2 + |(x_{d2} - x_{d1})e_1| + |x_{d1}e_2|,$$
$$\dot{\hat{\beta}} = e_3^2 + |x_{d3}e_3|,$$
$$\dot{\hat{\gamma}} = e_2^2 + |(e_1 + x_{d1} + x_{d2})e_2|.$$

where $\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ are estimate values of the unknown paremeters α, β, γ and $k_i > 0, \quad i = 1,2,3$.

For Eq.(13) with Eq. (15), consider a Lyapunov function candidate as

$$V(e, \tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}) = \frac{1}{2}\sum_{i=1}^{3} e_i^2 + \frac{1}{2}(\tilde{\alpha}^2 + \tilde{\beta}^2 + \tilde{\gamma}^2), \quad (16)$$

where $\tilde{\alpha} = \alpha - \hat{\alpha}, \tilde{\beta} = \beta - \hat{\beta}, \tilde{\lambda} = \gamma - \hat{\gamma}$.

We have

$$\frac{dV}{dt} \le -k_1 e_1^2 - k_2 e_2^2 - k_2 e_3^2. \quad (17)$$

This trnslates to $\lim_{t \to \infty} \|e(t)\| = 0$,

where $a(t), b(t)$ and $c(t)$ satisfy Eq. (2)-(4).

Thus, the control goal of Rossler system is achieved under the controller and a parameter estimation update law Eq. (15).

4. NUMERICAL RESULT

In this section, taking the example of Rossler chaotic system, we show the numerical simulation to demonstrate the effectiveness of the proposed control approach when unknown uncertain paramete- rs $a(t)$, $b(t)$ satisfy Eq. (2)-(4). It has been proved that the Rössler system (1) presents chaos when $a(t)=b(t)=0.2$ and $c(t)=5.7$.

In simulation, we choose the true values of "unknown" time-varying parameters as $a(t) = b(t) = 0.2(1 + \sin t), c(t) = 5.7(2 - \cos t)$.

We obtain

$$|a(t)| = |b(t)| \le \max\{0,0.4\}, \quad |c(t)| \le \max\{0,17.1\}$$

and unknown parameters $\alpha = \beta = 0.4 \ \gamma = 17.1$.

For desired smooth trajectory

$$x_d(t) = [\cos t, \sin 2t, \cos 0.5t]^T,$$

the initial values of the error states between states of Eq. (1) and the desired trajectory and initial value of estimate for "unknown" parameter α, are taken as $e_1(0) = e_2(0) = e_3 = 5,$ and $\hat{\alpha}(0) = \hat{\beta}(0) = 0.1$, $\hat{\gamma}(0) = 0.5$. According to Eq. (8), we construct the controller and updating law. From Fig. 1, we can see that the states of the Rossler chaotic system globally robust asymptotically track desired smooth trajectory $x_d(t) = [\cos t, \sin 2t, \cos 0.5t]^T$ under the controller designed by our proposed control approach.

5. CONCLUSION

In conclusion, a novel robust adaptive contol method is proposed for Rossler and Chen chaotic systems with unknown time-varying parameters in this paper. Based on Lyapunov stability theory, a robust adaptive controller without the knowledge of the bounds of systems' unknown time-varying parameters, which can make the states of Rossler and Chen systems asymptotically track desired smooth trajectory is derived. All results are proved using a well-known Lyapunov stability theorem. Numerical simulations are given to validate the proposed control approach.

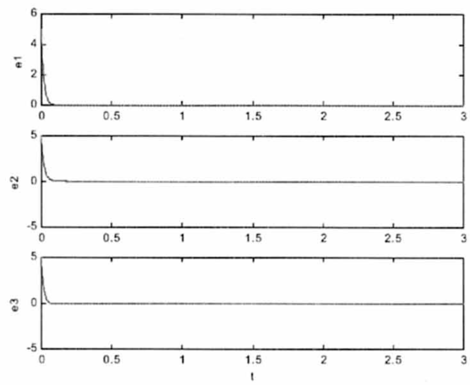

Fig. 1 The time response of error states beween Rossler system and desired trajectory when the controller is activated at t=0s.

REFERENCES

[1] E. Ott, C. Grebogi, J.A. Yorke, Phys. Rev. Lett. **64** (11) (1990)821.

[2] Liao T L and Tsai S H. 2000 *Chaos, Solitons & Fractals* **11** 1387.

[3] C. Wang, S.S. Ge, Chaos, Solitons & Fractals **12** (2001)1199.
[4] H.N. Agiza, M.T. Yassen, 2001 *Phys. Lett. A*, **278**, 191.
[5] Femat, R., Alvarez-Ramiez, J., 1997 *Phys. Lett. A*, **236** 307.
[6] Parmananda, P., 1998 *Phys. Lett. A*, **240** 55.
[7] Li Zhi, Han Chongzhao . Chin . Phys . **10**(2001) 494.
[8] Li Zhi, Han Chongzhao . Chin . Phys . **11**(2002) 9.
[9] Zhi Li, Chongzhao Han and Songjiao Shi . 2002 Phys . Lett. A, (In press).
[10] G. Chen, Int. J. Bifurcation Chaos 9 (7) (1999) 1465.
[11] X. Yu, *Chaos, Solitons & Fractals* 8 (9) (1997) 1577.